6.6.3 – ⓒ $(\arccos(x))' = \frac{-1}{\sqrt{1-x^2}}$

$$\boxed{1 = \operatorname{sen}^2(x) + \cos^2(x)}$$

$\rightarrow \operatorname{sen}(x) = \sqrt{1-\cos^2(x)}$

$\arccos(x) = y(x)$

$\boxed{\cos(\arccos(x)) = \cos(y)}$

$x = \cos(y)$

$$\underbrace{\frac{\partial x}{\partial x}}_{=1} = \underbrace{\frac{\partial \cos(y)}{\partial y}}_{-\operatorname{sen}(y)} \cdot \underbrace{\frac{\partial y}{\partial x}}_{(\arccos(x))'}$$

$$1 = -\sqrt{1-\cos^2(y)} \cdot \frac{\partial y}{\partial x}$$

$\cos^2(y) = x^2$, $y = \arccos(x)$

$$\frac{\partial y}{\partial x} = \frac{-1}{\sqrt{1-x^2}} = (\arccos(x))'$$

$$1 = -\sqrt{1-x^2} \cdot \frac{\partial y}{\partial x}$$

ⓔ $(\operatorname{sen}(\arccos(x)))' = \operatorname{sen}'(\arccos(x))\arccos'(x) = \underbrace{\cos(\arccos(x))}_{x} \cdot \underbrace{\arccos'(x)}_{\frac{-1}{\sqrt{1-x^2}}} = \frac{-x}{\sqrt{1-x^2}}$

6.7.1 – $f'(x) > 0 \quad \forall x \Rightarrow$ monotona creciente

$f'(x) < 0 \quad \forall x \Rightarrow$ " decreciente

ⓐ $f(x) = x^3 + 3x + 1$

$+ + + + +$ → sig $(3x^2+3)$

es monotona creciente

no tiene extremos

$f'(x) = 3x^2 + 3$

$0 = 3x^2 + 3 \rightarrow x = \pm\sqrt{-1}$

$f'(x) = 0$ punto critico (maximo, minimo, punto de inflexion)

ⓑ $f(x) = e^{-x^2}$, $g(x) = -x^2$, $h(x) = e^{g(x)}$

$f'(x) = h'(g(x))g'(x) = e^{-x^2} \cdot (-2x)$

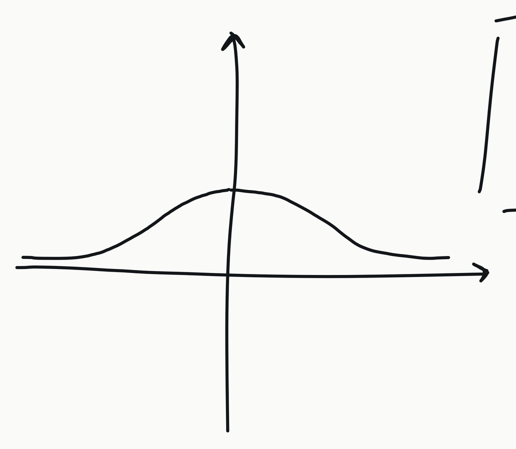
$+ + + \; 0 \; - - -$ → sig $(e^{-x^2}(-2x))$

monotona creciente ↗ 0 ↘ monotona decreciente

$\lim_{x\to+\infty} e^{-x^2} = \lim_{x\to\infty} \frac{1}{e^{x^2}} = 0^+$ ($e^{x^2} \to \infty$)

$\boxed{\text{max en } x=0 \text{ y es } 1 = f(0)}$

$\lim_{x\to-\infty} e^{-x^2} = \lim_{x\to-\infty} \frac{1}{e^{x^2}} = 0^+$ ($e^{x^2} \to \infty$)



② a – $f(x) = x^3 - x^2 - 8x + 1$ en $[-2, 1]$

Candidatos a extremos
* extremos del intervalo
* $f'(x_0) = 0$

$f'(x) = 3x^2 - 2x - 8$ buscar $x_0 \,/\, f'(x_0) = 0$

$$3x^2 - 2x - 8 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4+96}}{6} = \begin{cases} x = 2 \notin [-2,1] \\ x = -4/3 \in [-2,1] \end{cases}$$

Candidatos a extremos $\begin{cases} \boxed{x = -4/3} \\ x = -2 \\ \boxed{x = 1} \end{cases}$

$f(-4/3) = 7{,}518$ max

$f(-2) = 5$

$f(1) = -7$ min

| | $>0$ | $<0$ | $=0$ |
|---|---|---|---|
| $f'$ | creciente | decreciente | punto critico |
| $f''$ | concavidad positiva | concavidad negativa | punto de inflexion |

punto critico $\begin{cases} \max\ (f'' < 0) \\ \min\ (f'' > 0) \\ \text{punto de inflexion} \end{cases}$

e – $f(x) = \frac{x+1}{x^2+1}$ en $[-1, 1/2]$

estudiar existencia $(\exists\, \forall x \in \mathbb{R})$

$x^2 + 1 = 0 \Rightarrow$ para los $\mathbb{R}\ \nexists\, x \,/\, x^2+1=0$

$$f'(x) = \frac{1 \cdot (x^2+1) - 2x(x+1)}{(x^2+1)^2} = \frac{-x^2 - 2x + 1}{(x^2+1)^2}$$

$x_0 \,/\, f'(x_0) = 0 \rightarrow 0 = \frac{-x^2-2x+1}{(x^2+1)^2} \Rightarrow -x^2 - 2x + 1 = 0$

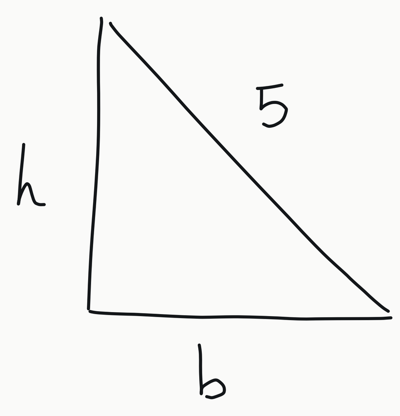
$$x = \frac{2 \pm \sqrt{4+4}}{-2} = -1 \pm \sqrt{2} \begin{cases} \simeq 0{,}4\ldots \quad (-1+\sqrt{2}) \\ \simeq -2{,}4\ldots \end{cases}$$

$f(-1+\sqrt{2}) = \frac{\sqrt{2}}{4-2\sqrt{2}} \simeq 1{,}03\ldots$

$f(-1) = 0 \longrightarrow$ min

$f(1/2) = \frac{6}{5} = 1{,}2 \longrightarrow$ max

6.8.1 – ⓐ ¿Cual es la mayor area de un triangulo rectangulo cuya hip. mide 5cm?




$A = \frac{b \cdot h}{2}$ $\quad 5^2 = h^2 + b^2$, $\quad h = \sqrt{5^2 - b^2}$

$A(b) = \frac{b \cdot \sqrt{5^2 - b^2}}{2}$

$A'(b) = \frac{1}{2}\left[1 \cdot \sqrt{5^2 - b^2} + b\,\frac{(-b)}{\sqrt{5^2-b^2}}\right]$

$A'(b) = 0$

$\sqrt{5^2 - x^2} = f(g(x))$

$f(x) = \sqrt{x} \rightarrow f'(x) = \frac{1}{2\sqrt{x}}$

$g(x) = 5^2 - x^2 \rightarrow g'(x) = -2x$

$(\sqrt{5^2 - x^2})' = f'(g(x))\,g'(x) = \frac{1\,(-2x)}{2\sqrt{5^2 - x^2}}$

$$\frac{1}{2}\left[\sqrt{5^2 - b^2} - \frac{b^2}{\sqrt{5^2 - b^2}}\right] = 0 \Rightarrow \sqrt{5^2 - b^2} = \frac{b^2}{\sqrt{5^2 - b^2}}$$

$(\sqrt{5^2 - b^2})^2 = b^2$

$5^2 - b^2 = b^2 \rightarrow 2b^2 = 25$

$\boxed{b = \pm\sqrt{\frac{25}{2}}}$

$\boxed{A\left(b = \sqrt{\frac{25}{2}}\right) = \frac{25}{4}}$

$$\frac{\sqrt{\frac{25}{2}} \cdot \sqrt{25 - \frac{25}{2}}}{2} = \frac{25}{2} \Big/ 2 = \frac{25}{4}$$