Teo. de Valor Medio

- f continua en $[a,b]$
- f derivable en (a,b)

$\Big\} \exists c \in (a,b) / f'(c) = \dfrac{f(b)-f(a)}{b-a}$

③ c_ $f(x) = \sqrt[3]{x}$ en $[-1,1]$

$$f'(x) = (x^{1/3})' = \frac{1}{3} x^{1/3-1} = \frac{x^{-2/3}}{3} = \frac{1}{3x^{2/3}} = \frac{1}{3\sqrt[3]{x^2}}$$

$f(x)$ no es derivable para $x=0$ ($f(x)$ es derivable $\forall x / \{0\}$)

$\Rightarrow$ no cumple las hip del teorema de Lagrange porque

$\{0\} \in (-1,1)$

d_ $f(x) = \sqrt{x-1}$ en $[1,3]$ $\qquad f'(x) = \dfrac{1}{2\sqrt{x-1}}$

$\lim_{x \to 1^-} \sqrt{x-1} = 0$

$\lim_{x \to 1^+} \sqrt{x-1} = 0$

$f(1) = 0$

$\Big\}$ es continua

$f(g(x)) = \sqrt{x-1}$

$f(x) = \sqrt{x} \to f'(x) = \dfrac{1}{2\sqrt{x}}$

$g(x) = x-1 \to g'(x) = 1$

$[f(g(x))]' = f'(g(x))\, g'(x)$

puede tener "problemas" en $x=1$ pero las hip. piden que sea derivable solo en $(1,3) \Rightarrow$ cumple Lagrange

$f(3) = \sqrt{2}$

$f(1) = 0$

$\Big\} \; f'(c) = \dfrac{\sqrt{2}}{2} = \dfrac{1}{2\sqrt{c-1}}$ $\quad$ (numerador $= \sqrt{2} - 0$, denominador $= 3-1$)

$$\Rightarrow \sqrt{2} = \frac{1}{\sqrt{c-1}} \Rightarrow 2 = \frac{1}{c-1} \Rightarrow c-1 = \frac{1}{2} \Rightarrow \boxed{c = \frac{3}{2}}$$

4_ $f(x) = 1 - \sqrt[3]{x^2}$ $\qquad$ ¿Porque no se contradice el teo. de Rolle?

$f(1) = f(-1) = 0$

$\Downarrow$

$\nexists c / f'(c) = 0$

- f cont. $[a,b]$
- f derivable en (a,b)
- $f(a) = f(b)$

$\Big\} \exists c / f'(c) = 0$

$\lim_{h \to 0} \dfrac{f(x_0+h) - f(x_0)}{h}$ en $x_0 = 0$; $f(0) = 1$

$$\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^+} \frac{f(h) - 1}{h} = \lim_{h \to 0^+} \frac{1 - \sqrt[3]{h^2} - 1}{h} = \lim_{h \to 0^+} \frac{\sqrt[3]{h^2}}{h} = \lim_{h \to 0^{(+)}} \frac{1}{\sqrt[3]{h}} = +\infty$$

$$\lim_{h \to 0^-} \frac{f(h) - 1}{h} = \lim_{h \to 0^-} \frac{\sqrt[3]{h^2}}{h} = \lim_{h \to 0^-} \frac{h^{2/3}}{h} = \lim_{h \to 0^-} h^{2/3-1} = \lim_{h \to 0^-} h^{-1/3} = \lim_{h \to 0^-} \frac{1}{h^{1/3}} = \lim_{h \to 0^{(-)}} \frac{1}{\sqrt[3]{h}} = -\infty$$

$\nexists \lim_{h \to 0} \dfrac{f(h) - f(0)}{h}$ no es derivable en $x = 0$

$\Rightarrow$ no cumple hip. de Rolle

6.6_ $f'(x_0) = 0 \Rightarrow x_0$ es punto crítico $\begin{cases} \text{maximo} \\ \text{minimo} \\ \text{punto de inflexion} \end{cases}$

$f'(x) \gtrless 0 \quad \forall x$ es monotona creciente / decreciente

① a_ $f(x) = |x| = \begin{cases} x & \text{si } x > 0 \\ -x & \text{si } x < 0 \end{cases}$

Es derivable $\forall x / \{0\}$

$f(0) = 0$

$$f'(x) = \begin{cases} 1 & \text{si } x > 0 \Rightarrow \text{monotona creciente} \\ -1 & \text{si } x < 0 \Rightarrow \text{monotona decreciente} \end{cases}$$

$f(x) > f(0) \; \forall x \Rightarrow f(0)$ es minino

b_ $f(x) = \sqrt{|x|}$

Es derivable $\forall x / \{0\}$

$$\lim_{h \to 0^+} \frac{f(h) - \overset{=0}{f(0)}}{h} = \lim_{h \to 0^+} \frac{\sqrt{h}}{h} \neq \lim_{h \to 0^-} \frac{f(h) - \overset{=0}{f(0)}}{h} = \lim_{h \to 0^-} \frac{\sqrt{-h}}{h}$$

$f(x) > \overbrace{f(0)}^{0} \; \forall x \Rightarrow f(0)$ es minimo

f_ $f(x) = x^3$ es derivable $f'(x) = 3x^2$

$f'(x) = 0 \Rightarrow 3x^2 = 0$ para $x = 0$

en $x=0$ punto de inflexion

+ + + 0 + + + (recta con 0 marcado) $\quad sig(f'(x))$

$f'(x) \geq 0 \; \forall x \Rightarrow$ monotona creciente $\Rightarrow$ no tiene extremos

3_ a_ $(arctg(x))' = \dfrac{1}{1+x^2}$ $\qquad tg'(x) = sec^2(x) = 1 + tg^2(x)$

$y(x) = arctg(x) \to tg(y(x)) = tg(arctg(x)) = x$

$$\frac{d\, tg(y(x))}{dx} = \frac{dx}{dx} = 1$$

$$\frac{d\,tg(y(x))}{dy} \cdot \frac{dy(x)}{dx} = 1$$

$$sec^2(y(x)) \frac{dy(x)}{dx} = 1$$

$$[1 + tg^2(y(x))] \frac{dy(x)}{dx} = 1$$

$x^2 = tg^2(arctg(x))$ $\qquad \dfrac{dy(x)}{dx} = (arctg(x))'$

$(1+x^2)(arctg(x))' = 1$

$\Rightarrow (arctg(x))' = \dfrac{1}{1+x^2}$

b_ $(arcsen(x))' = \dfrac{1}{\sqrt{1-x^2}}$ $\qquad 1 = cos^2(x) + sen^2(x)$

$\Rightarrow cos(x) = \sqrt{1 - sen^2(x)}$

$y = arcsen(x) \to sen(y) = x$

$$\underbrace{\frac{d\,sen(y)}{dy}}_{\sqrt{1-x^2}} \cdot \underbrace{\frac{dy}{dx}}_{(arcsen(x))'} = \underbrace{\frac{dx}{dx}}_{1} \to \sqrt{1-x^2}\,(arcsen(x))' = 1$$

$$\Rightarrow \boxed{(arcsen(x))' = \frac{1}{\sqrt{1-x^2}}}$$

$$\frac{d\,sen(y)}{dy} = cos(y(x)) = \sqrt{1 - sen^2(y)} = \sqrt{1 - x^2}$$

(y = arcsen(x))