8_ $f(x) = \begin{cases} 0 & si \quad x=0 \\ x^2 sen(1/x) & si \quad x \neq 0 \end{cases}$

$g(x) = x^2 sen(1/x) \to g'(x) = (x^2)' sen(1/x) + \cancel{x^2}(sen(1/x))' = 2x\, sen(1/x) - cos(1/x)$

$[sin(1/x)]' = [g(f(x))]' = g'(f(x))f'(x) = cos(1/x)\left(-\frac{1}{\cancel{x^2}}\right)$

$g(x) = sen(x) \to g'(x) = cos(x)$

$f(x) = 1/x \to f'(x) = (x^{-1})' = -1(x^{-1-1}) = -x^{-2} = \frac{-1}{x^2}$

$\lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h} \overset{x_0=0}{=} \lim_{h\to 0} \frac{f(h) - f(0)}{h} \; (f(0)=0) = \lim_{h\to 0}\frac{f(h)}{h} = \lim_{h\to 0}\frac{h^2 sen(1/h)}{h} = \lim_{h\to 0} \underbrace{h}_{\to 0} \underbrace{sen(1/x)}_{acotado} = 0$

¿$f'(x)$ es continua? $f'(x) = 2x\, sen(1/x) - cos(1/x)$

$\left.\begin{array}{l} \lim_{x\to 0^+} \underbrace{2x}_{\to 0}\, \underbrace{sen(1/x)}_{acotado} - cos(1/x) \\ \lim_{x\to 0^-} 2x\, sen(1/x) - cos(1/x) \end{array}\right\}$ $\nexists \lim_{x\to 0} f'(x) \Rightarrow$ la derivada de $f(x)$ no es continua

f no es dos veces derivable porque f' no es cont. $\Rightarrow f'$ no es derivable

6.4_ Regla de L'Hopital

$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0} = \lim_{x\to a}\frac{f'(x)}{g'(x)} = L$

$\lim_{x\to 0} \frac{1/g(x)}{1/f(x)} = \frac{\infty}{\infty}$

$\lim_{x\to 0} e^{f(x)} - 1 = f(x)$ $\qquad \frac{de^x}{dx} = e^x$

$\lim_{x\to 0} sen(x) = x$

(1) a_ $\lim_{x\to 0} \frac{1-e^{4x}}{sen(x)} = \lim_{x\to 0} \frac{-(e^{4x}-1)}{sen(x)} = \lim_{x\to 0}\frac{-4x}{x} = -4$

$\lim_{x\to 0}\frac{1-e^{4x}}{sen(x)} = \lim_{x\to 0} \frac{-(4x)'e^{4x}}{cos(x)} = \lim_{x\to 0}\frac{-4e^{4x}}{cos(x)} = -4$

b_ $\lim_{x\to 1}\frac{log(x)}{cos(\pi x)+1} = \lim_{x\to 1}\frac{1/x}{-\pi\, sen(\pi x)} = \lim_{x\to 1}\frac{1}{-\pi x\, sen(\pi x)} = -\infty$

c_ $\lim_{x\to 0}\frac{e^x - cos(x)}{sen(\pi x)} = \lim_{x\to 0}\frac{e^x + sen(x)}{\pi\, cos(\pi x)} = \frac{1}{\pi}$

(2) a_ $\lim_{x\to \frac{\pi}{2}^+} \frac{log(sen(x))}{cos(x)} = \lim_{x\to\frac{\pi}{2}^+} \frac{\frac{cos(x)}{sen(x)}}{-sen(x)} = \lim_{x\to\frac{\pi}{2}^+} \frac{\overset{\to 0}{cos(x)}}{\underset{\to 1}{-sen^2(x)}} = 0$

$[log(sen(x))]' = [g(f(x))]' = g'(f(x))f'(x) = \frac{1}{sen(x)} \cdot cos(x)$

$g(x) = log(x) \to g'(x) = 1/x$

$f(x) = sen(x) \to f'(x) = cos(x)$

b_ $\lim_{x\to 1}\frac{sen(log(x))}{sen(\pi x)} = \lim_{x\to 1} \frac{\overset{\to 1}{cos(log(x))}}{\underset{\to -\pi}{x\pi\, cos(\pi x)}} = \frac{1}{-\pi}$

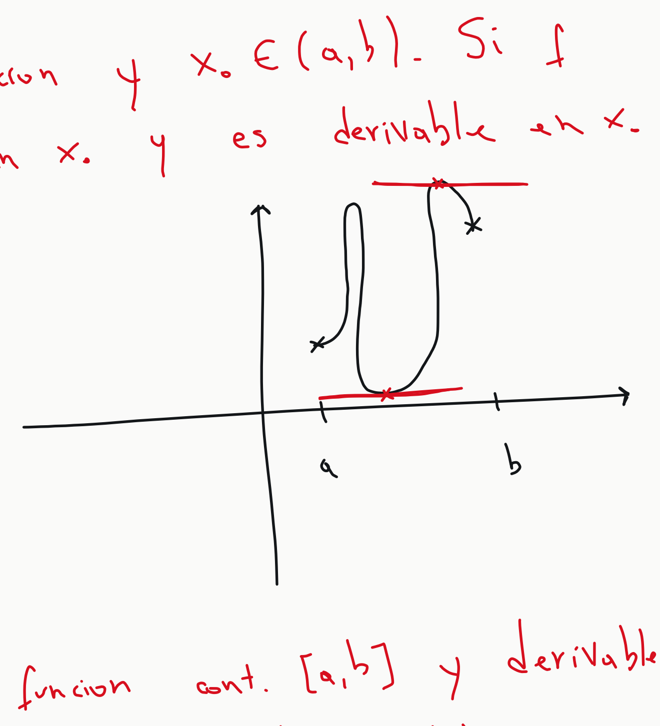
$[sen(log(x))]' = [g(f(x))]' = g'(f(x))f'(x) = cos(log(x))\, 1/x$

$g(x) = sen(x) \to g'(x) = cos(x)$

$f(x) = log(x) \to f'(x) = 1/x$

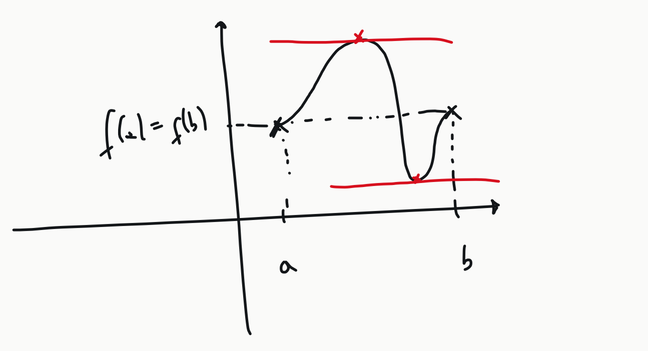
c_ $\lim_{x\to 0}\frac{sen(\pi e^x)}{e^{sen(x)} - 1} = \lim_{x\to 0}\frac{\overbrace{cos(\pi e^x)}^{\to -1}\overbrace{\pi e^x}^{\to \pi}}{\underbrace{e^{sen(x)} \cdot cos(x)}_{\to 1}} = -\pi$

Sean $f:[a,b]\to R$ un funcion y $x_0 \in (a,b)$. Si f tiene un extremo relativo en x_0 y es derivable en x_0 entonces $f'(x_0) = 0$



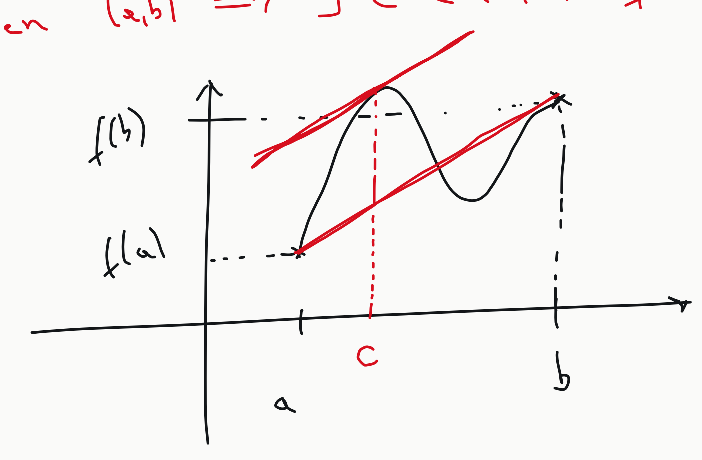
Teo. de Rolle

Sea $f:[a,b]\to R$ una funcion cont. $[a,b]$ y derivable en (a,b) tal que $f(a) = f(b) \Rightarrow \exists\, c \in (a,b) \mid f'(c) = 0$



Teo. de valor medio de Lagrange

Sea $f:[a,b]\to R$ una funcion cont. en $[a,b]$ y derivable en $(a,b) \Rightarrow \exists\, c \in (a,b) \mid f'(c) = \frac{f(b)-f(a)}{b-a}$



(3) a_ $f(x) = x + \frac{1}{x}$ en $[1, 3]$ ($a = 1$, $b = 3$) $\qquad f'(x) = 1 + \left(-\frac{1}{x^2}\right)$

$f'(c) = \frac{f(b) - f(a)}{b-a}$ $\qquad f(b) = 3 + 1/3$ $\qquad f(a) = 1+1$

$1 - \frac{1}{c^2} = \frac{3 + 1/3 - 2}{3-1} \Rightarrow 1 - \frac{1}{c^2} = \frac{\overbrace{1 + 1/3}^{4/3}}{2} = \frac{2}{3}$

$1 - \frac{2}{3} = \frac{1}{c^2} = \frac{1}{3} \Rightarrow c^2 = 3 \to c = \pm\sqrt{3}$ (+ circled)

b_ $f(x) = x\, log(x)$ en $[1, e]$ $\qquad f'(x) = (x)'\, log(x) + x[log(x)]'$

$f'(x) = log(x) + \frac{x}{x} = log(x) + 1$

$f'(c) = \frac{f(b) - f(a)}{b-a}$ $\qquad f(e) = e\, log(e) = e$ $\qquad f(1) = 1 . log(1) = 0$

$log(c) + 1 = \frac{e - 0}{e-1} \Rightarrow log(c) = \frac{e}{e-1} - 1 = \frac{e - e + 1}{e-1} = \frac{1}{e-1}$

$log(c) = \frac{1}{e-1} \Rightarrow \boxed{c = e^{1/e-1}}$