$f(x) \qquad f'(x)$

- $u(x)+v(x) \longrightarrow u'(x)+v'(x)$
- $u(x)-v(x) \longrightarrow u'(x)-v'(x)$
- $u(x)v(x) \longrightarrow u'(x)v(x)+u(x)v'(x)$
- $\dfrac{u(x)}{v(x)} \longrightarrow \dfrac{u'(x)v(x)-u(x)v'(x)}{v^2(x)}$
- $u(v(x)) \rightarrow u'(v(x))v'(x)$

4_ ⓒ $f(x)=\dfrac{x}{x^2+1}$ $\qquad m=f'(x_0)$ $\qquad f'(x)=\dfrac{\overbrace{(x)'}^{=1}(x^2+1)-x\overbrace{(x^2+1)'}^{2x}}{(x^2+1)^2}=\dfrac{x^2+1-2x^2}{(x^2+1)^2}$

$p=(0,0)$ (x_0, y_0) $\qquad m=\left.\dfrac{-x^2+1}{(x^2+1)^2}\right|_{x=0}=1$ $\qquad \boxed{y=x}$

5_ $f(x)=x^2+ax+b$ $\qquad f(3)=g(3)=3$ $\begin{cases}3^2+3a+b=3 \rightarrow \boxed{b=18} \\ 3c-3^2=3 \rightarrow \boxed{c=4}\end{cases}$

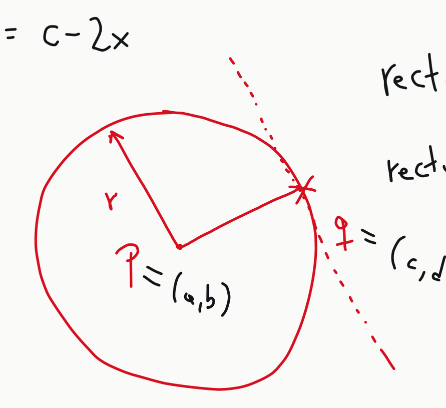
$g(x)=cx-x^2$

$p=(3,3)$ $\qquad f'(3)=g'(3) \rightarrow 2.3+a=c-2.3 \rightarrow \boxed{a=-8}$

$f'(x)=2x+a$

$g'(x)=c-2x$

6_



$\left.\begin{array}{l}\text{recta}^{PQ} = y=m(x-x_0)+y_0 \\ \text{recta}^{\text{tangente}} = y=m_p(x-x_1)+y_1\end{array}\right\}$ $m=\dfrac{-1}{m_p}$

$g(h(x))$ $\begin{cases}\sqrt{x}=g(x) \\ r^2-(x-a)^2=h(x)\end{cases}$

$(x-a)^2+(y-b)^2=r^2 \rightarrow f(x)=b\pm\sqrt{r^2-(x-a)^2}$

$(y-b)^2=r^2-(x-a)^2$ $\qquad f'(x)=\dfrac{\pm 2(x-a)}{2\sqrt{r^2-(x-a)^2}} \rightarrow f'(c)=\dfrac{\mp(c-a)}{\sqrt{r^2-(c-a)^2}}=m_p$

$y-b=\pm\sqrt{r^2-(x-a)^2}$

$y=b\pm\sqrt{r^2-(x-a)^2}$

$\rightarrow$ $\left.\begin{array}{l}(x-a)^2=g(h(x)) \\ g(x)=x^2 \rightarrow g'(x)=2x \\ h(x)=x-a \rightarrow h'(x)=1\end{array}\right\}$ $[(x-a)^2]'=g'(h(x)).h'(x)=2(x-a)$

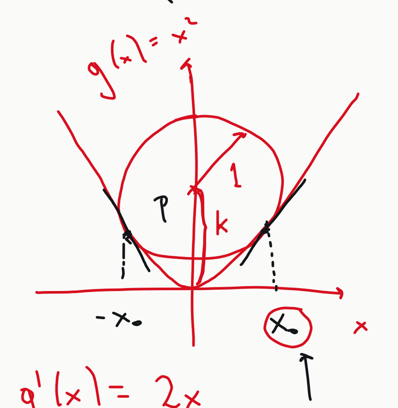
$\boxed{m=\dfrac{d-b}{c-a}}$ $\qquad m_p=\dfrac{\mp(c-a)}{\sqrt{r^2-(c-a)^2}}$

$q\in C \longrightarrow (c-a)^2+(d-b)^2=r^2$

$m_p=\dfrac{\mp(c-a)}{\sqrt{(c-a)^2+(d-b)^2-(c-a)^2}}=\mp\dfrac{(c-a)}{(d-b)}$

$\dfrac{-1}{m_p}=\dfrac{-1}{\frac{-(c-a)}{(d-b)}}=\dfrac{d-b}{c-a}=m=\dfrac{-1}{m_p}$ perpendiculares

10_



$p=(0,K)$

$(x-0)^2+(y-K)^2=1 \rightarrow K=y\pm\sqrt{1-x^2}$

$\rightarrow f(x)=y=K\pm\sqrt{1-x^2}$

$g'(x)=2x$ $\qquad f'(x)=\pm\dfrac{(-2x)}{2\sqrt{1-x^2}}=\dfrac{\mp x}{\sqrt{1-x^2}}$

$\boxed{2x_0=\dfrac{\mp x_0}{\sqrt{1-x_0^2}}} \rightarrow x_0=\pm\sqrt{\dfrac{3}{4}} \rightarrow y_0=x_0^2=\dfrac{3}{4}$

$1-x_0^2=\left(\dfrac{\mp 1}{2}\right)^2=\dfrac{1}{4}$

$K=y_0\pm\sqrt{1-x_0^2}=\dfrac{3}{4}\pm\sqrt{1-\left(\sqrt{\tfrac{3}{4}}\right)^2}=\dfrac{3}{4}\pm\sqrt{\dfrac{1}{4}}=\begin{cases}5/4 \\ 1/4\end{cases}$

6.3 Calculo de derivadas

$(x^\alpha)'=\alpha x^{\alpha-1}$ $\quad (\ln(x))'=\dfrac{1}{x}$ $\quad (e^x)'=e^x$

① a_ $\left(\dfrac{1}{x}+\dfrac{2}{x^2}+\dfrac{3}{x^3}\right)'=-\dfrac{1}{x^2}-\dfrac{4}{x^3}-\dfrac{9}{x^4}$

$\left(\dfrac{1}{x}\right)'=(x^{-1})'=-1\,x^{-1-1}=-x^{-2}=\dfrac{-1}{x^2}$

$\left(\dfrac{2}{x^2}\right)'=\dfrac{(2)'x^2-2(x^2)'}{(x^2)^2}=\dfrac{-4x}{x^4}=\dfrac{-4}{x^3}$

$\left(\dfrac{3}{x^3}\right)'=\dfrac{0-3x^2.3}{(x^3)^2}=\dfrac{-9x^2}{x^6}=\dfrac{-9}{x^4}$

d_ $\left(x\sqrt{1+x^2}\right)'=(x)'\sqrt{1+x^2}+x\left(\sqrt{1+x^2}\right)'=\sqrt{1+x^2}+\dfrac{x^2}{\sqrt{1+x^2}}$

$\underbrace{\left(\sqrt{1+x^2}\right)'}_{f(g(x))}=f'(g(x))g'(x)=\dfrac{1}{2\sqrt{1+x^2}}\cdot 2x=\dfrac{x}{\sqrt{1+x^2}}$

$f(x)=\sqrt{x} \rightarrow f'(x)=\dfrac{1}{2\sqrt{x}}$

$g(x)=1+x^2 \rightarrow g'(x)=2x$

g- $\underbrace{\left(\text{sen}^3(x)\right)'}_{f(g(x))}=3\,\text{sen}^2(x)\cos(x)$

$f(x)=x^3$

$g(x)=\text{sen}(x)$

J- $\underbrace{\left(\text{sen}\left(\dfrac{1}{x}+x^2\right)\right)'}_{f(g(x))}=\cos\left(\dfrac{1}{x}+x^2\right)\left(\dfrac{-1}{x^2}+2x\right)$

$f(x)=\text{sen}(x)$

$g(x)=\dfrac{1}{x}+x^2$

m_ $\left((\cos(x)-1)^{100}\right)'=-100\,(\cos(x)-1)^{99}\,\text{sen}(x)$

$f(x)=x^{100}$

$g(x)=\cos(x)-1$

$\text{sen}^2(x)+\cos^2(x)=1$

3_ $\text{senh}(x)=\dfrac{e^x-e^{-x}}{2}$ $\qquad \cosh(x)=\dfrac{e^x+e^{-x}}{2}$

$[\text{senh}(x)]'=\cosh(x)$ $\qquad \cosh^2(x)-\text{senh}^2(x)=1$

$[\cosh(x)]'=\text{senh}(x)$ $\qquad \left(\dfrac{e^x+e^{-x}}{2}\right)^2-\left(\dfrac{e^x-e^{-x}}{2}\right)^2=1$

$[\text{senh}(x)]'=\left(\dfrac{e^x-e^{-x}}{2}\right)'=\dfrac{e^x-(-x)'e^{-x}}{2}$ $\qquad \dfrac{e^{2x}+2e^xe^{-x}+e^{-2x}}{4}-\dfrac{e^{2x}-2e^xe^{-x}+e^{-2x}}{4}$

$=\dfrac{e^x+e^{-x}}{2}=\cosh(x)$ $\qquad \dfrac{2e^xe^{-x}+2e^xe^{-x}}{4}=\dfrac{4e^{x-x}}{4}=\dfrac{4e^0}{4}=1$