4.3_6) (a) $\lim\limits_{x\to 1} \frac{\log(x)}{x-1}$ teo. del sandwich $\frac{x-1}{x} \leq \log(x) \leq x-1$

$$\left.\begin{array}{l} \lim\limits_{x\to 1} \frac{\log(x)}{x-1} \geq \frac{x-1}{x} \cdot \frac{1}{x-1} = \frac{1}{x} \\ \lim\limits_{x\to 1} \frac{\log(x)}{x-1} \leq x-1 \cdot \frac{1}{x-1} = 1 \end{array}\right\} \quad 1 \geq \lim_{x\to 1} \frac{\log(x)}{x-1} \geq \lim_{x\to 1} \frac{1}{x} = 1 \qquad \lim_{x\to 1} \frac{\log(x)}{x-1} = 1$$

(b) $\lim\limits_{x\to 0^+} x^a \log(x) : a>1$

$\lim\limits_{x\to 0^+} x\log(x) \Rightarrow \forall x \in (0,1)\ \ \log(x) < 0 \to x\log(x) < 0 \ \ \forall x \in (0,1)$

Cambio de Variable $t = \frac{1}{\sqrt{x}}$ $\lim\limits_{x\to 0^+} x\log(x) = \underbrace{\lim\limits_{t\to\infty} \frac{1}{t^2} \log\left(\frac{1}{t^2}\right) = \lim\limits_{t\to\infty} \frac{1}{t^2} \cdot (-2\log(t))}_{\log\left(\frac{1}{x}\right) = -\log(x)\ ;\ \log(x^n) = n\log(x)}$

$\hookrightarrow x = \frac{1}{t^2}$

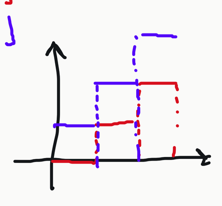
$\log(x) \leq x-1 < x \Rightarrow \log(x) < x \Rightarrow -\log(x) > -x$

$\lim\limits_{x\to 0^+} x\log(x) = \lim\limits_{t\to\infty} \frac{-2\log(t)}{t^2} > \lim\limits_{t\to\infty} \frac{-2t}{t^2} = 0$

$0 \leq \lim\limits_{x\to 0^+} x\log(x) \leq 0 \Rightarrow \lim\limits_{x\to 0^+} x\log(x) = 0$

$\lim\limits_{x\to 0^+} x^a\log(x) = \lim\limits_{x\to 0^+} \underbrace{x\log(x)}_{\to 0} \overbrace{x^{a-1}}^{\to 0} = 0$

4.4_6) a) $\lim\limits_{x\to\infty} \underbrace{\lfloor x+1 \rfloor}_{\lfloor x \rfloor + 1} - \lfloor x \rfloor = 1$



7) a) $\lim\limits_{x\to\infty} \frac{\operatorname{sen}(x)}{x} = \lim\limits_{x\to\infty} \underbrace{\operatorname{sen}(x)}_{\text{acotado}} \cdot \underbrace{\frac{1}{x}}_{\to 0} = 0$

d) $\lim\limits_{x\to\infty} \frac{x}{\sqrt{x^2+x+1}} = \lim\limits_{x\to\infty} \frac{x}{\sqrt{x^2}} = 1$

g) $\lim\limits_{x\to\infty} \frac{\log(x)}{x} = \lim\limits_{t\to 0^+} t\log(1/t) = \lim\limits_{t\to 0^+} -\log(t)\,t = 0$

$\hookrightarrow t = \frac{1}{x}$

Funciones Cont.

$\lim\limits_{x\to a^+} f(x) = \lim\limits_{x\to a^-} f(x) = f(a)$

5.1_2) a) $f(x) = \begin{cases} x^2+3x+2 & \text{si } x \leq 1 \\ ax^2+bx+1 & \text{si } x>1 \end{cases}$

$$\left.\begin{array}{l} \lim\limits_{x\to 1^+} ax^2+bx+1 = a+b+1 \\ \lim\limits_{x\to 1^-} x^2+3x+2 = 6 \end{array}\right\} \quad 6 = a+b+1 \Rightarrow \boxed{a+b=5} \qquad f(1) = 1^2+3.1+2 = 6$$

c) $f(x) = \begin{cases} \operatorname{sen}(\pi x) & \text{si } x<1 \\ ax+b & \text{si } 1 \leq x \leq 2 \\ x^2 & \text{si } x>2 \end{cases}$

$$\left.\begin{array}{l} \left.\begin{array}{l} \lim\limits_{x\to 1^-} \operatorname{sen}(\pi x) = 0 \\ \lim\limits_{x\to 1^+} ax+b = a+b \end{array}\right\} a+b=0 \\ \left.\begin{array}{l} \lim\limits_{x\to 2^-} ax+b = 2a+b \\ \lim\limits_{x\to 2^+} x^2 = 4 \end{array}\right\} 2a+b=4 \end{array}\right\} \quad \begin{array}{l} a=-b \\ 2a+b=4 \\ \boxed{a=4 \to b=-4} \end{array}$$

Teo de Bolzano Si $y=f(x)$ funcion continua en el intervalo cerrado $[a,b]$

$f(a)$ y $f(b)$ tengan signos opuestos $\Rightarrow \exists$ al menos un $c \in (a,b) \,/\, f(c)=0$

1_ Demostrar que la ecuacion $x+2\cos(x)=0$ tiene al menos una solucion

$f(x) = x+2\cos(x)$ suma de continuas

$x_1 + 2\cos(x_1) > 0 \to 2\cos(x_1) > -x_1$ para $x_1 = 0$ $2>0 \Rightarrow f(x_1)>0$

$x_2 + 2\cos(x_2) < 0 \to 2\cos(x_2) < -x_2$ para $x_2 = -\frac{\pi}{2}$ $0 < \frac{\pi}{2} \Rightarrow f(x_2) < 0$

Por Bolzano $\exists c \in \left(-\frac{\pi}{2}, 0\right) / f(c) = 0 \Rightarrow c + 2\cos(c) = 0$ entonces c es sol.

3. Sean $f, g : [a,b] \to \mathbb{R}$ continuas. Si $f(a) > g(a)$ y $f(b) < g(b)$

Demostrar $\exists c \in (a,b) \mid f(c) = g(c)$

$$\left.\begin{array}{l} h(x) = f(x) - g(x) \quad \text{resta de continuas} \\ h(a) = f(a) - g(a) > 0 \quad \text{por (H)} \\ h(b) = f(b) - g(b) < 0 \quad \text{por (H)} \end{array}\right\} \quad \begin{array}{l} \text{Por Bolzano } \exists c \in (a,b) / h(c) = 0 \\ h(c) = f(c) - g(c) = 0 \Rightarrow f(c) = g(c) \end{array}$$

5. Punto Fijo

Dada f una funcion, un punto fijo de f es un valor $c / f(c) = c$

(a) Sea $f : [0,1] \to [0,1]$ cont. Probar $\exists$ punto fijo

$$\left.\begin{array}{l} g(x) = f(x) - x \quad \forall x \in [0,1] \\ g(0) = f(0) \quad \text{como } f(0) \in [0,1] \to g(0) \geq 0 \\ g(1) = f(1) - 1 \quad \text{como } f(1) \in [0,1] \to g(1) \leq 0 \end{array}\right\} \quad \begin{array}{l} \text{Por Bolzano} \\ \exists c \in (0,1) \mid g(c) = 0 \\ g(c) = f(c) - c = 0 \Rightarrow f(c) = c \end{array}$$