Calculo de integrales

1. ⓐ $\int_1^4 3x-2\,dx = 3\underbrace{\int_1^4 x\,dx}_{} - 2\int_1^4 1\,dx = \frac{45}{2} - 6$

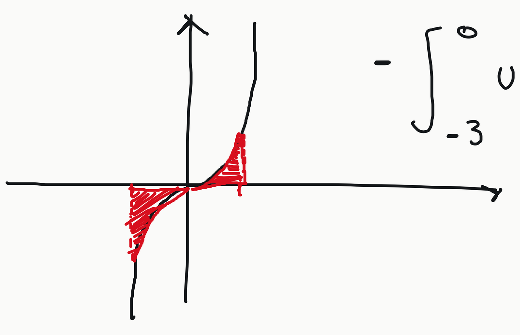
$\frac{3(4^2-1^2)}{2} - 2(4-1)$

ⓖ $\int_0^2 (u+3)(u+1)\,du = \int_0^2 u^2+4u+3\,du = \int_0^2 u^2du + 4\int_0^2 u\,du + 3\int_0^2 du = \frac{8}{3}+6+6 = \frac{8+18+18}{3} = \frac{44}{3}$

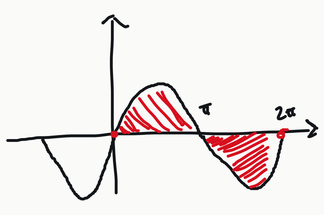
$\frac{2^3-0^3}{3} + 4\frac{(2^2-0^2)}{2} + 3.2$

5. ⓐ $\int_{-3}^{3} u^{|u|}\,du = \int_0^3 u^{|u|}\,du + \int_{-3}^{0} u^{|u|}\,du = 0$

$-\int_{-3}^{0} u^{|u|}\,du$



ⓓ $\int_0^{2\pi} \text{sen}(kt)+5 = \underbrace{\int_0^{2\pi} \text{sen}(kt)\,dt}_{=0} + 5\int_0^{2\pi} dt = 5(2\pi-0) = 10\pi$



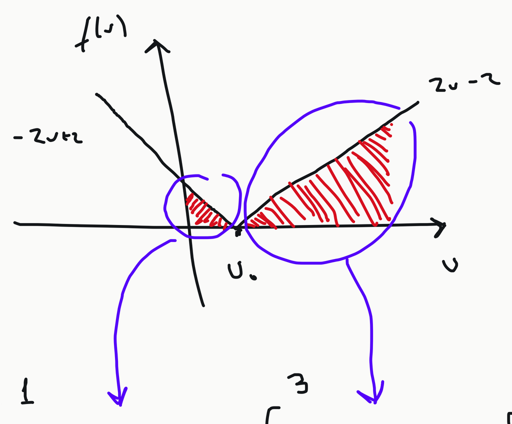
8. ⓐ $\int_3^4 \sqrt{3x}\,dx = \sqrt{3}\int_3^4 \sqrt{x}\,dx = \sqrt{3}\,\frac{2}{3}\left(4^{3/2}-3^{3/2}\right) = \frac{16}{\sqrt{3}} - \sqrt{3}\cdot\frac{2}{3}\cdot 3\sqrt{3} = \frac{16}{\sqrt{3}} - 6$

($\sqrt{3x} = \sqrt{3}\sqrt{x}$; $4^{1+1/2} = 4\sqrt{4}$; $3^{3/2} = 3\sqrt{3}$)

ⓓ $\int_1^2 \sqrt{x+1}\,dx \quad \left(\int_a^b f(t)\,dt = \int_{a+p}^{b+p} f(t-p)\,dt\right) \quad f(x) = \sqrt{x+1} \quad f(x-1) = \sqrt{x-1+1} = \sqrt{x}$

$\int_2^3 \sqrt{x}\,dx = \frac{2}{3}\left(3^{3/2}-2^{3/2}\right) = \frac{2\,3\sqrt{3}}{3} - \frac{2\sqrt{2}\cdot 2}{3} = 2\sqrt{3} - \frac{4\sqrt{2}}{3}$

9. ⓐ $\int_0^3 |2u-2|\,du$



$2u-2=0 \to u_0 = 1$

$f(u) = \begin{cases} 2u-2 & u \geq 1 \\ -2u+2 & u<1 \end{cases}$

$\int_0^1 -2u+2\,du + \int_1^3 2u-2\,du = -2\int_0^1 u\,du + 2\int_0^1 du + 2\int_1^3 u\,du - 2\int_1^3 du$

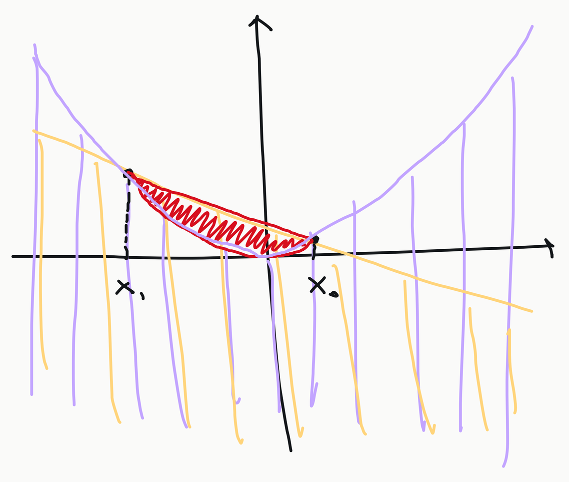
$\int_0^3 |2u-2|\,du = -\frac{2}{2} + 2 + \frac{2}{2}(3^2-1) - 2(3-1) = 5$

11. Calcular el area encerrada por los graficos $f(x)$ y $g(x)$

ⓐ $f(x) = x^2$

$g(x) = -2x+1$

$\int_{-1-\sqrt{2}}^{-1+\sqrt{2}} -2x+1 - \int_{-1-\sqrt{2}}^{-1+\sqrt{2}} x^2 -$



$x^2 = -2x+1$

$x = \frac{-2 \pm \sqrt{4+4}}{2} = -1 \pm \sqrt{2}$

$x_0 = -1+\sqrt{2} \quad x_1 = -1-\sqrt{2}$

Limites

$\lim_{x\to a} f(x) = L \iff \forall\, \varepsilon>0\ \exists\, \delta>0\ \forall x\ |x-a|<\delta \Rightarrow |f(x)-L|<\varepsilon$

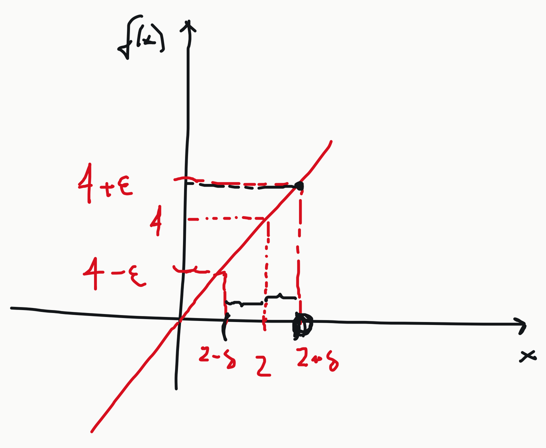
Def. y prop. (4.2)

1. Encontrar L y $\delta>0$ / $|f(x)-L|<\varepsilon\ \forall x$ con $\begin{cases}\varepsilon = 10^{-2} \\ \varepsilon = 10^{-3} \\ \varepsilon = 10^{-4}\end{cases}$

$f(x) = 2x \qquad L = \lim_{x\to 2} 2x = 4$

$a = 2$

$\varepsilon = 10^{-4}$



$f(2+\delta) = 4+\varepsilon$

$2(2+\delta) = 4+\varepsilon$

$\delta = \frac{4+\varepsilon}{2} - 2 = \frac{4+\varepsilon-4}{2} = \frac{\varepsilon}{2}$

$f(2-\delta) = 4-\varepsilon$

$2(2-\delta) = 4-\varepsilon$

$\delta = 2-2+\frac{\varepsilon}{2} = \frac{\varepsilon}{2}$

ⓐ $f(x) = x^2 \qquad L = \lim_{x\to 0} x^2 = 0$

$a = 0$

$f(0+\delta) = 0+\varepsilon$

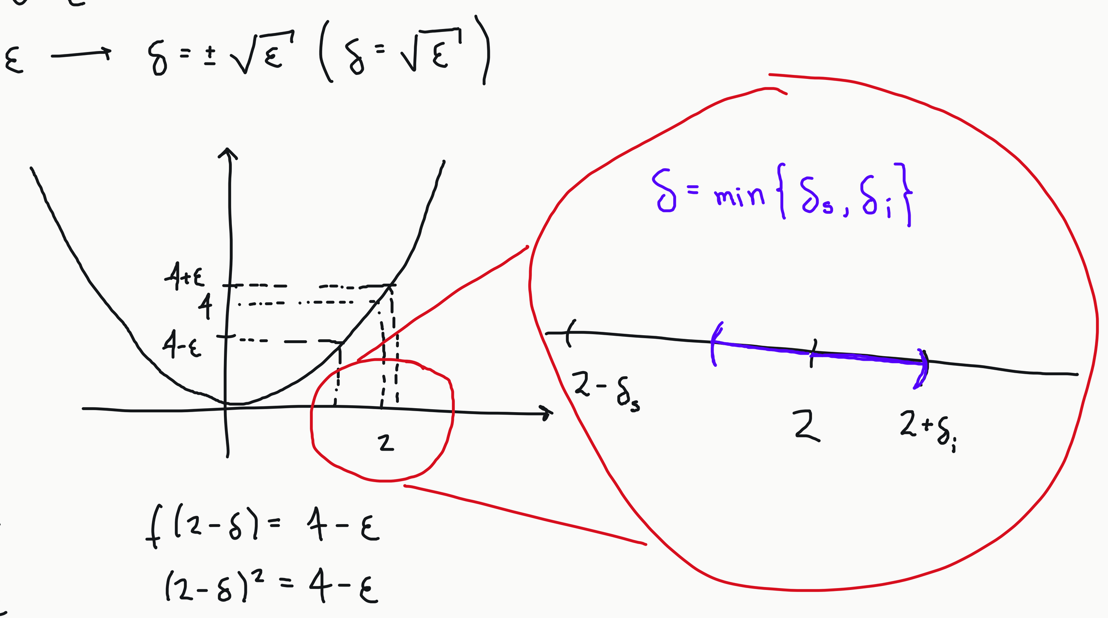
$\delta^2 = \varepsilon \longrightarrow \delta = \pm\sqrt{\varepsilon}\ (\delta = \sqrt{\varepsilon})$

$f(x) = x^2$

$a = 2$

$\varepsilon = 10^{-4}$

$L = \lim_{x\to 2} f(x) = 4$



$f(2+\delta) = 4+\varepsilon$

$(2+\delta)^2 = 4+\varepsilon$

$\delta_0 = \pm\sqrt{4+\varepsilon} - 2$

$f(2-\delta) = 4-\varepsilon$

$(2-\delta)^2 = 4-\varepsilon$

$\delta_i = 2 \pm \sqrt{4-\varepsilon}$

$\delta = -2+\sqrt{4+\varepsilon}$

ⓑ $f(x) = 1/x \qquad L = \lim_{x\to 1} 1/x = 1 \qquad f(1+\delta_0) = 1+\varepsilon$

$a = 1 \qquad \frac{1}{1+\delta_0} = 1+\varepsilon$

para $\varepsilon = 0{,}1$

$\Rightarrow \delta_0 = \frac{1}{1+\varepsilon} - 1 = 0{,}09 \qquad f(1-\delta_i) = 1-\varepsilon \qquad \delta = \min\{\delta_0, \delta_i\}$

$\frac{1}{1-\delta_i} = 1-\varepsilon \qquad \delta = 0{,}09$

$\delta_i = 1 - \frac{1}{1-\varepsilon} = 0{,}11$