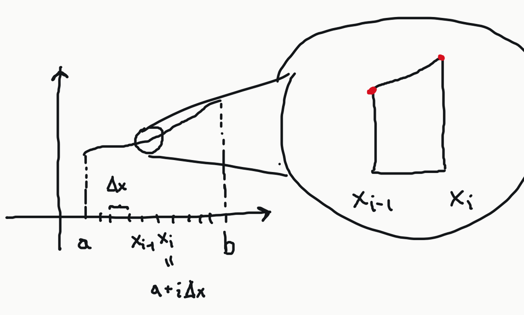
3.4) 2 _ Probar que una funcion monotona creciente y acotada es integrable

$\forall \varepsilon > 0, \exists P$ particion de $[a,b]$ / $S^*(f,P) - S_*(f,P) \leq \varepsilon$

$\Delta x = \frac{b-a}{n} \longrightarrow x_i = a + i\Delta x$




$S^*(f,P) = \sum_{i=1}^{n} \Delta x\, f(x_i)$

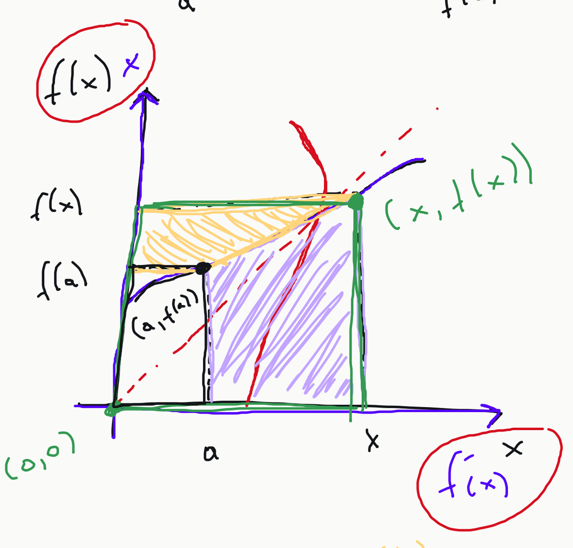
$S_*(f,P) = \sum_{i=1}^{n} \Delta x\, f(x_{i-1})$

$S^*(f,P) - S_*(f,P) = \Delta x \left[ \sum_{i=1}^{n} f(x_i) - \sum_{i=1}^{n} f(x_{i-1}) \right] = \Delta x \left( f(x_n) - f(x_0) \right)$

$f(x_1) + f(x_2) + \dots + f(x_n) - f(x_0) - f(x_1) - \dots - f(x_{n-1})$

$S^*(f,P) - S_*(f,P) = \Delta x \left( f(b) - f(a) \right) = \underbrace{(b-a)}_{n} \left( f(b) - f(a) \right) \leq \varepsilon$ ■

5. $\int_a^x f(t)dt + \int_{f(a)}^{f(x)} f^{-1}(t)dt - x f(x) + a f(a) = 0$




$\int_a^x f(t)dt$

$\int_{f(a)}^{f(x)} f^{-1}(t)dt$

$(f(a) - 0)(a - 0) = a f(a)$

$(f(x) - 0)(x - 0) = x \cdot f(x)$

$x f(x) = \int_{f(a)}^{f(x)} f^{-1}(t)dt + \int_a^x f(t)dt + a f(a)$

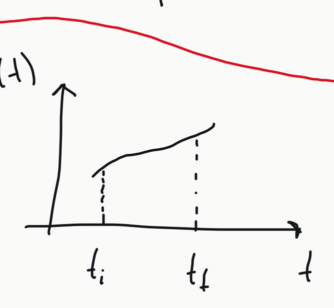
$\int_a^x f(t)dt + \int_{f(a)}^{f(x)} f^{-1}(t)dt + a f(a) - x f(x) = 0$ ■

## Cambio de variable

1. ⓐ $\int_a^b f(t)\,dt = \int_{a+p}^{b+p} f(t-p)\,dt = \int_{a+p=u_{ini}}^{b+p=u_{fin}} f(u-p)\,du$

$t = u - p$ $\longrightarrow$ $u = t + p$

$dt = du - \underbrace{dp}_{=0} = du$




$u_{ini} = t_{ini} + p$

$u_{fin} = t_{fin} + p$

$\int_{a=t_i}^{b=t_f} f(t)\,dt = \int_{u_i}^{u_f} f(u-p)\,du = \int_{a+p}^{b+p} f(u-p)\,du$

$\int_a^b f(t)\,dt = \int_{a+p}^{b+p} f(u-p)\,du$

ⓑ $r \int_a^b f(t)\,dt = \int_{ra}^{rb} f\left(\frac{t}{r}\right) dt = \int_{ra}^{rb} f(u/r)\,du$

$t = u/r \longrightarrow u = tr$ $\qquad t_f = u_f/r$

$du = r\,dt$ $\qquad t_i = u_i/r$

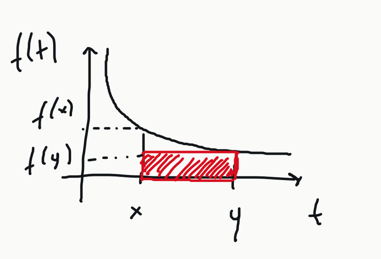
$\int_{ra=u_i}^{rb=u_f} f(\underbrace{u/r}_{t})\,\underbrace{du}_{r.dt} = \int_{t_i}^{t_f} f(t)\,r\,dt = r \int_{\frac{ra}{r}=a}^{\frac{rb}{r}=b} f(t)dt$

$\int_{ra}^{rb} f(u/r)\,du = r \int_a^b f(t)\,dt$

3. $\underbrace{\log(x)}_{\text{creciente}} = \int_1^x \underbrace{\frac{1}{t}}_{\text{decreciente}} dt$

ⓐ Probar que es creciente $\left[ g(x) = \log(x) \; ; \; f(t) = \frac{1}{t} \right]$

$\forall x, y$ si $x < y \Longrightarrow \underbrace{g(x) < g(y)}_{\log(x) < \log(y)}$




$g(y) - g(x) > 0$

$\log(y) - \log(x) = \int_1^y \frac{1}{t}\,dt - \int_1^x \frac{1}{t}\,dt = \int_x^y \frac{1}{t}\,dt \geq S_*(f,P) = \frac{\overbrace{y-x}^{>0}}{\underbrace{y}_{>1}} > 0$ ■

$P = \{x, y\} \rightarrow S_*(f,P) = (y-x) . \inf\{f, [x,y]\} = \frac{y-x}{y}$

$f = \frac{1}{t} \rightarrow \inf\{f, [x,y]\} = f(y) = \frac{1}{y}$

## Calculo de integrales

$\int_a^b f(t)dt = \int_a^c f(t)dt + \int_c^b f(t)dt$

$\int_a^a f(t)dt = 0$ $\qquad \int_a^b f(t)dt = -\int_b^a f(t)dt$

$\int_a^b [c f(t) + g(t)]dt = c \int_a^b f(t)dt + \int_a^b g(t)dt$

1. $\int_{-1}^{1} h(t)dt = 0 \; ; \; \int_1^3 h(t)dt = 6 \; ; \; \int_{-1}^{3} h(t)\,dt = ?$

$\int_{-1}^{3} h(t)dt = \int_{-1}^{1} h(t)dt + \int_1^3 h(t)\,dt = 6$

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Calcular $\int_2^4 f(t)dt$ sabiendo $\int_2^8 f(t)dt = 20$ y $\int_8^4 f(t)\,dt = 12$

$\int_4^8 f(t)dt + \int_2^4 f(t)dt = \int_2^8 f(t)dt \rightarrow \int_2^4 f(t)dt = \int_2^8 f(t)dt - \underbrace{\int_4^8 f(t)\,dt}_{-\int_8^4 f(t)dt} = 20 - (-12) = 32$

4. $\int_1^4 3x - 2\,dx = 3 \underbrace{\int_1^4 x\,dx}_{\frac{4^2 - 1^2}{2}} - 2 \underbrace{\int_1^4 1\,dx}_{4-1} = 3 . \frac{15}{2} - 2 . 3 = \frac{33}{2}$