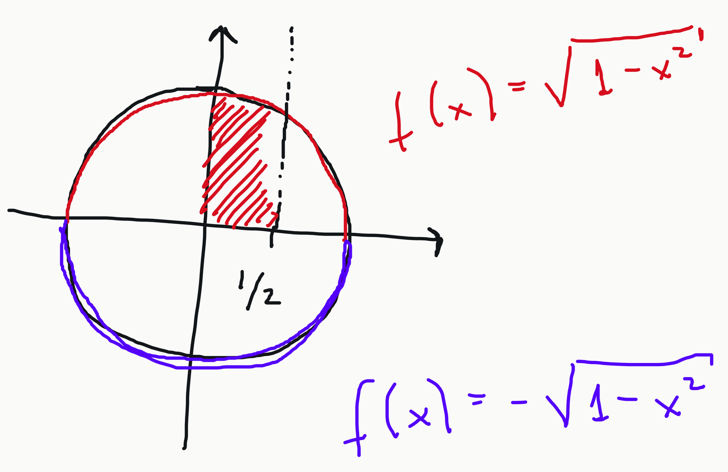
Acotaciones

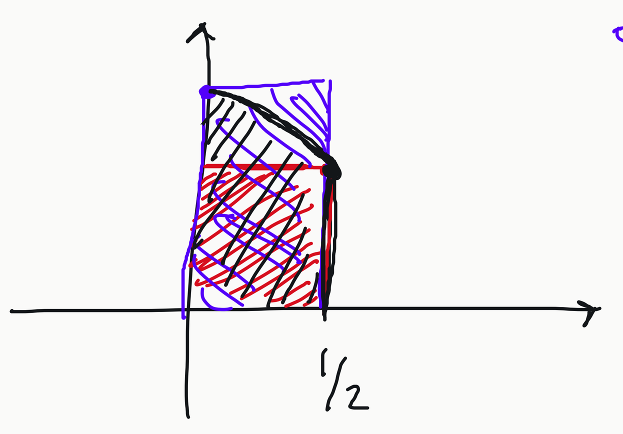
$(x-c_1)^2 + (y-c_2)^2 = r^2$

1. $\frac{\sqrt{3}}{4} \leq \int_0^{1/2} \underbrace{\sqrt{1-x^2}}_{f(x)}\,dx \leq \frac{1}{2}$



$x^2+y^2=1 \implies f(x)=\pm\sqrt{1-x^2}$

$y^2 = 1-x^2$



area azul $= b \times h = 1/2$

$b = 1/2 \quad h = f(0) = \sqrt{1-0} = 1$

area roja $= b \times h = \sqrt{3}/4$

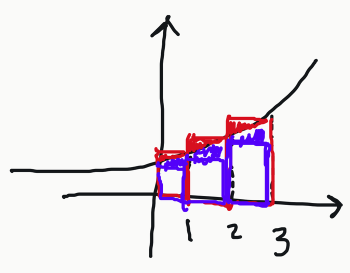
$b = 1/2 \quad h = f(1/2) = \sqrt{1-(1/2)^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$1/2 \geq \int_0^{1/2} \sqrt{1-x^2}\,dx \geq \sqrt{3}/2$

Particiones

2. a) $f(x) = 2^x$

$P = \{0,1,2,3\}$



$f(0)=1$

$f(1)=2$

$f(2)=4$

$f(3)=8$

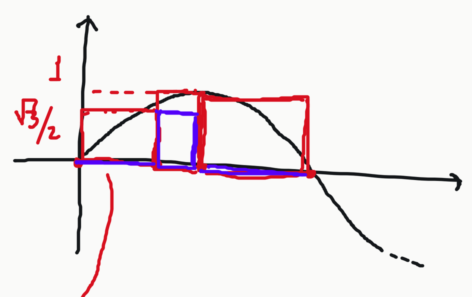
	0°	30°	45°	60°	90°
	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sen	$\sqrt{0}/2$	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$
cos	$\sqrt{4}/2$	$\sqrt{3}/2$	$\sqrt{2}/2$	$\sqrt{1}/2$	$\sqrt{0}/2$

$S^*(f,P) = (1-0)f(1) + (2-1)f(2) + (3-2)f(3) = 2+4+8 = 14$

$S_*(f,P) = (1-0)f(0) + (2-1)f(1) + (3-2)f(2) = 1+2+4 = 7$

c) $f(x) = \operatorname{sen}(x)$

$P = \{\underbrace{0, \pi/3}, \pi/2, \pi\}$



$f(0) = 0$

$f(\pi/3) = \sqrt{3}/2$

$f(\pi/2) = 1$

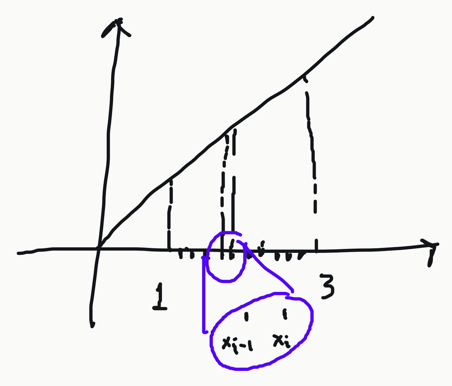
$f(\pi) = 0$

$S^*(f,P) = \left(\frac{\pi}{3}-0\right) f(\pi/3) + \left(\frac{\pi}{2}-\frac{\pi}{3}\right) f(\pi/2) + \left(\pi - \frac{\pi}{2}\right) f(\pi/2) =$

$\frac{\pi}{3}\frac{\sqrt{3}}{2} + \frac{\pi}{6}\cdot 1 + \frac{\pi}{2}\cdot 1 = \frac{\pi}{6}(4+\sqrt{3})$

$S_*(f,P) = \left(\frac{\pi}{3}-0\right) 0 + \left(\frac{\pi}{2}-\frac{\pi}{3}\right) f(\pi/3) + \left(\pi-\frac{\pi}{2}\right) \cdot 0 = \frac{\pi}{6}\cdot\frac{\sqrt{3}}{2} = \frac{\pi}{4\sqrt{3}}$

5. Calcular $\int_1^3 x\,dx$ hallando sus sumas superiores en inferiores para particiones equiespaciadas



$\Delta x = \frac{3-1}{n} = \frac{2}{n}$

$x_0 = 1 \qquad x_i = 1 + i\Delta x$

$x_1 = 1 + \Delta x$

$x_2 = 1 + 2\Delta x$

$\vdots$

$x_n = 3 = 1 + n\Delta x = 1 + n\cdot\frac{2}{n} = 3$

Recordar $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$S^*(f,P) = \sum_{i=1}^{n} \Delta x\, f(x_i) = \sum_{i=1}^{n} \Delta x\cdot x_i = \sum_{i=1}^{n} \Delta x\cdot(1+i\Delta x) = \underbrace{\sum_{i=1}^{n} \Delta x}_{n\Delta x} + \underbrace{\sum_{i=1}^{n} i\,\Delta x^2}_{\Delta x^2 \sum_{i=1}^{n} i \,=\, \Delta x^2 \frac{n(n+1)}{2}}$

$S^*(f,P) = n\Delta x + \frac{n(n+1)}{2}\Delta x^2 = n\cdot\frac{2}{n} + \frac{n^2+n}{2}\cdot\frac{2^2}{n^2} = 2+2+\frac{2}{n} = 4+\frac{2}{n}$

$S_*(f,P) = \sum_{i=1}^{n} \Delta x\, f(x_{i-1}) = \sum_{i=1}^{n} \Delta x\cdot x_{i-1} = \sum_{i=1}^{n} \Delta x\left(1+(i-1)\Delta x\right) = \sum_{i=1}^{n} \Delta x + \sum_{i=1}^{n} i\,\Delta x^2 - \sum_{i=1}^{n} \Delta x^2$

$S_*(f,P) = n\cdot\frac{2}{n} + \left(\frac{n^2+n}{2}\right)\frac{4}{n^2} - n\frac{4}{n^2} = 2+2+\frac{2}{n}-\frac{4}{n} = 4-\frac{2}{n}$

b) $\int_0^3 x^2\,dx$ Recordar $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $\quad \Delta x = \frac{3-0}{n} \quad x_i = 0 + i\Delta x$

$S^*(f,P) = \sum_{i=1}^{n} \Delta x\, f(x_i) = \sum_{i=1}^{n} \Delta x\, x_i^2 = \sum_{i=1}^{n} \Delta x\, i^2\Delta x^2 = \sum_{i=1}^{n} \Delta x^3\, i^2$

$S^*(f,P) = \frac{27}{n^3}\cdot\frac{(n^2+n)(2n+1)}{6} = \frac{9}{2}\cdot\frac{2n^3+3n^2+n}{n^3} = 9 + \frac{27}{2n} + \frac{9}{2n^2}$

7. a) $f(x) = \begin{cases} 0 & \text{si } x\in\mathbb{Q} \\ 1 & \text{si } x\notin\mathbb{Q} \end{cases}$ $\quad P$ en $[0,1]$ ¿Es integrable?

$S_*(f,P) = \sum_{i=1}^{n} m_i\,\Delta x_i \qquad m_i = \inf\{f(x) : x\in[x_{i-1},x_i]\}$

$\Delta x_i = x_i - x_{i-1}$

En cada subintervalo $[x_{i-1}, x_i]$

$f(x) = 0 \;\forall x\in\mathbb{Q}$ y $\mathbb{Q}$ es denso en $\mathbb{R}$

siempre voy a tener un $x\in[x_{i-1},x_i] \,/\, f(x) = 0 = m_i$

$S_*(f,P) = \sum_{i=1}^{n} \underset{0}{m_i}\,\Delta x_i = 0$

Razonando de manera análoga $S^*(f,P) = \sum_{i=1}^{n} \underset{1}{M_i}\,\Delta x_i = \sum_{i=1}^{n} \Delta x_i = 1$

$S^*(f,P) - S_*(f,P) = 1 \Rightarrow f$ no es integrable

Integrabilidad de funciones

Ⓐ $\forall \varepsilon > 0, \exists P$ partición $[a,b] \,/\, S^*(f,P) - S_*(f,P) < \varepsilon$

$\underset{\text{mas fuerte}}{B} \to A \to \underset{\text{mas debil}}{C} \qquad B \to D$