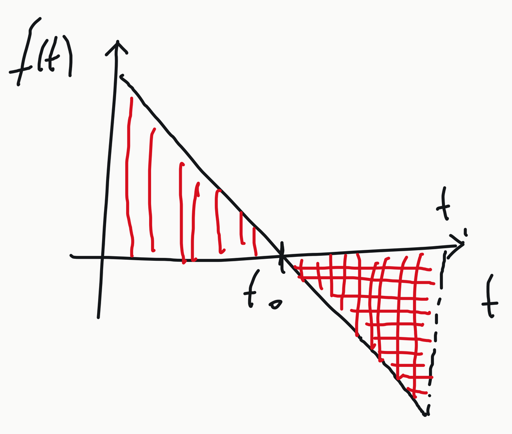
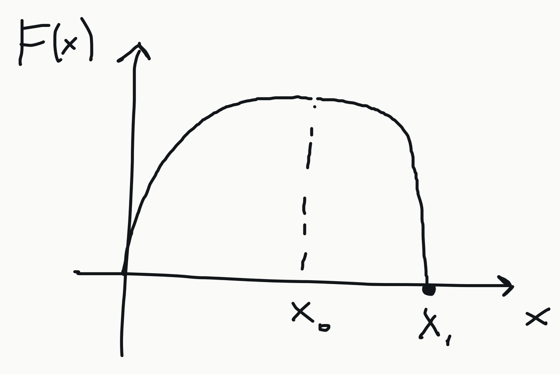
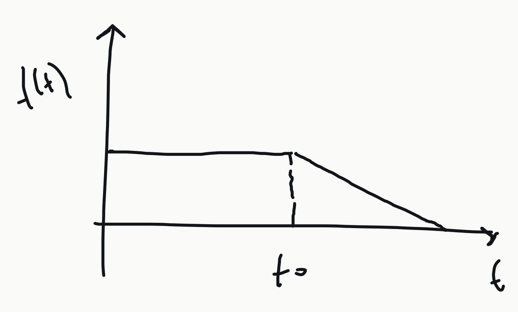
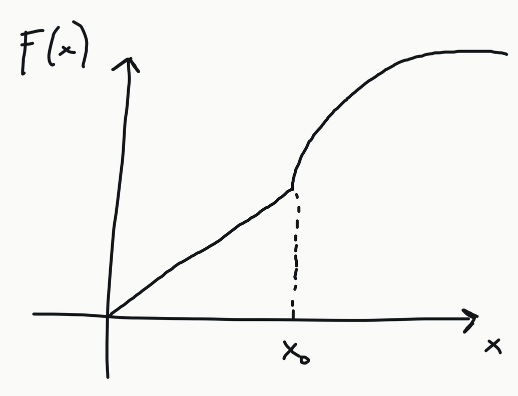
5.



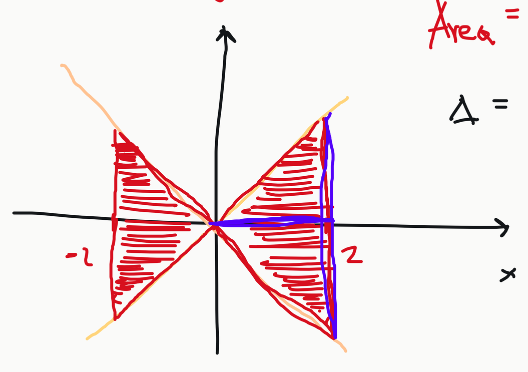






7. Calcular el area en la region S

ⓐ $f(x)=x$
$g(x)=-x$
$[-2,2]$

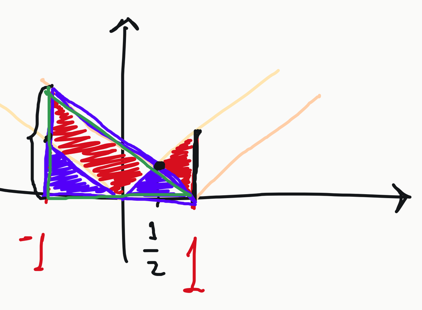


$\text{Area} = 2\Delta = 2.4 = 8$

$\Delta = \frac{b.h}{2} = \frac{4.2}{2} = 4$

ⓒ $f(x)=|x|$
$g(x)=|x-1|$
$[-1,1]$

$g(-1)=2$

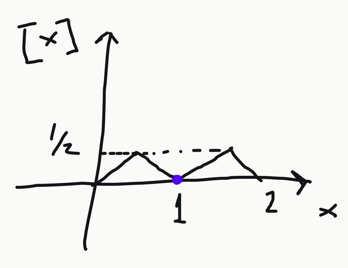


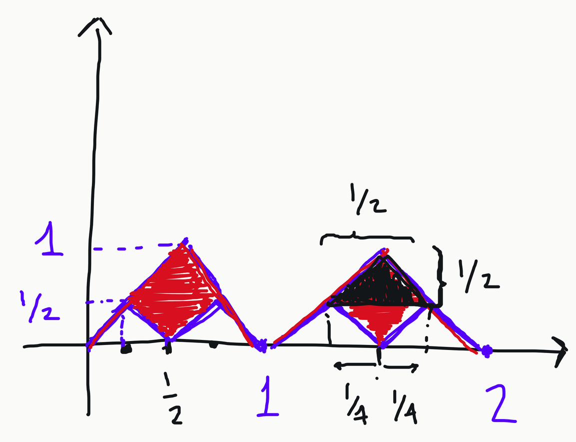
$\text{Area} = \text{triangulo} + \text{figura} = \frac{6}{4} + \frac{1}{4} = \frac{7}{4}$

$\text{triangulo} = \frac{b.h}{2} = \frac{1.(1/2)}{2} = \frac{1}{4}$

$\text{figura} = \frac{2.2}{2} - \left[\frac{1.1}{2} + \frac{1\,(1/2)}{2}\right] = \frac{6}{4}$

ⓔ $f(x)=[2x]$
$g(x)=2[x]$
$[0,2]$

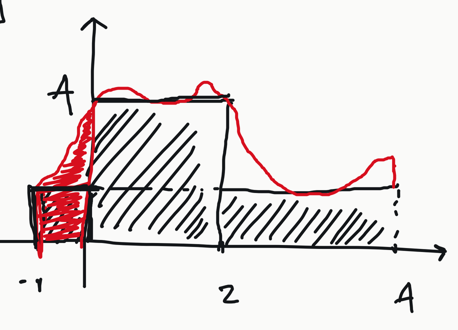




$\text{Area} = 4.\frac{(1/2)(1/2)}{2} = \frac{1}{2}$

Acotaciones

2. ⓐ $f(x) \geq 2 \quad \forall x\ [-1,0] \cup [2,4]$
$f(x) \geq 4 \quad \forall x\ [0,2]$



$\int_{-1}^{4} f(x)dx \geq 14$

$$\int_{-1}^{0} f(x)dx \geq 1.2$$

$$+ \int_{0}^{2} f(x)\,dx \geq 4.2$$

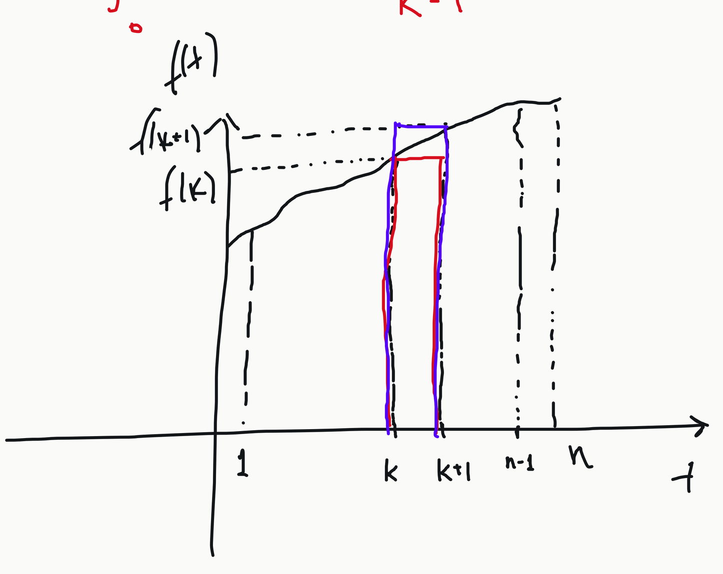
$$\int_{2}^{4} f(x)\,dx \geq 2.2$$

$$\int_{-1}^{4} f(x)\,dx \geq 14$$

3. f monotona creciente $\quad \sum_{k=0}^{n-1} f(k) \leq \int_0^n f(t)dt \leq \sum_{k=1}^{n} f(k)$

$\Downarrow$

$\forall k \in \mathbb{N} \quad f(k) \leq f(k+1)$



$$\underbrace{(k+1-k)f(k+1)}_{f(k+1)} \geq \int_k^{k+1} f(t)\,dt \geq \underbrace{(k+1-k)f(k)}_{f(k)}$$

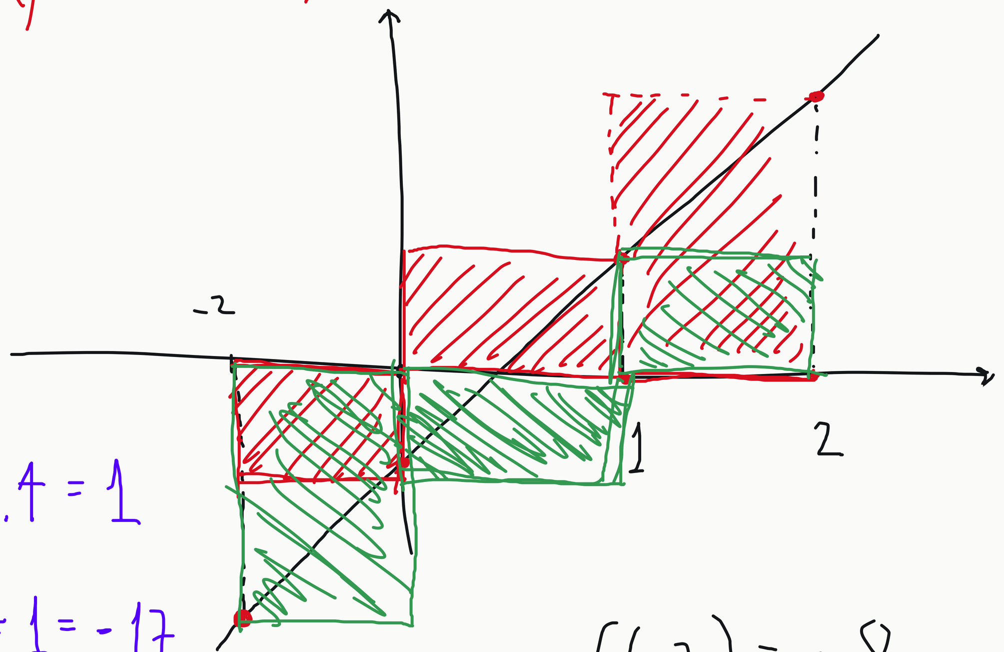
$$\underbrace{f(1)+\dots+f(n)}_{\sum_{k=1}^{n} f(k)} = \sum_{k=0}^{n-1} f(k+1) \geq \underbrace{\sum_{k=0}^{n-1} \int_k^{k+1} f(t)\,dt} \geq \sum_{k=0}^{n-1} f(k)$$

$$\sum_{k=1}^{n} f(k) \geq \int_0^n f(t)\,dt \geq \sum_{k=0}^{n-1} f(k)$$

Sumas superiores e inferiores

1. Calcular $S^*(f,P)$ y $S_*(f,P)$

ⓐ $f(x)=3x-2$
$P=\{-2,0,1,2\}$

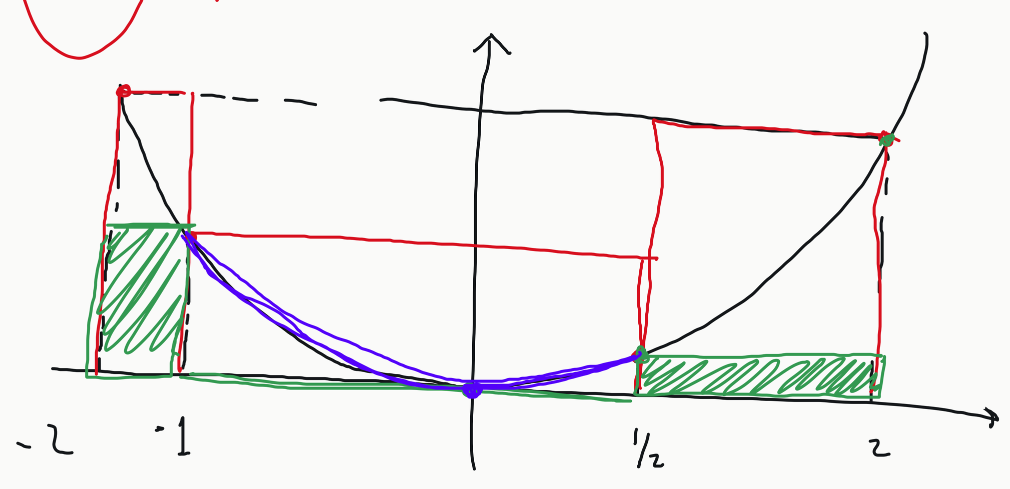


$S^*(f,P) = 2(-2)+1+1.4=1$

$S_*(f,P) = 2(-8)+1(-2)+1=-17$

$f(-2)=-8$
$f(0)=-2$
$f(1)=1$
$f(2)=4$

ⓑ $f(x)=x^2 \quad P=\{-2,-1,1/2,2\}$



$f(-2)=4$
$f(-1)=1$
$f(1/2)=1/4$
$f(2)=4$

$S^*(f,P) = 1.4 + \frac{3}{2}1 + \frac{3}{2}4 = 6 + \frac{3}{2}$

$S_*(f,P) = 1.1 + \frac{3}{2}.0 + \frac{3}{2}.\frac{1}{4} = 1 + \frac{3}{8}$