Practico 1 $f: I \to R$ es monotona creciente (estrictamente)

(11) $\forall x_0, x_1 \in I$ con $x_0 \leq x_1 \Rightarrow f(x_0) \leq f(x_1)$

(a) estrictamente creciente $\Rightarrow$ creciente

$\forall x_0, x_1 \in I$ si $x_0 < x_1 \Rightarrow f(x_0) < f(x_1)$

tomo x_0 y x_1 si $x_0 < x_1 \Rightarrow f(x_0) < f(x_1)$ por (H) $\Big\{$ si $x_0 \leq x_1$

$x_0 = x_1 \Rightarrow f(x_0) = f(x_1)$ $\Big\} \Rightarrow f(x_0) \leq f(x_1)$

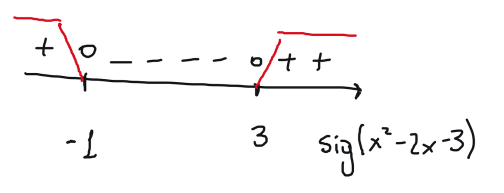
(b) Estrictamente monotona $\Rightarrow$ Inyectiva

Practico 2 Numero real

Axiomas de cuerpo ordenado

(2) a_ $x^2 - 2x > 3 \longrightarrow x^2 - 2x - 3 > 0$

$x = \dfrac{2 \pm \sqrt{(-2)^2 - 4.1.(-3)}}{2.1} = \dfrac{2 \pm \sqrt{16}}{2} = \begin{cases} 3 \\ -1 \end{cases}$



$x \in (-\infty, -1) \cup (3, +\infty)$

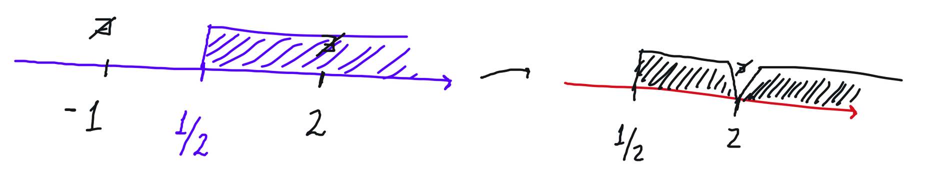
d_ $\dfrac{2-x}{1+x} \leq \dfrac{1+x}{2-x} \rightarrow \dfrac{(2-x)^2}{(2-x)(1+x)} \leq \dfrac{(1+x)^2}{(2-x)(1+x)}$

puntos de no existencia $(2-x)(1+x) = 0 \Rightarrow \begin{matrix} x = 2 \\ x = -1 \end{matrix}$

$(2-x)^2 \leq (1+x)^2$

$x^2 - 4x + 4 \leq x^2 + 2x + 1 \rightarrow 6x \geq 3 \rightarrow x \geq 1/2$

$x \in [1/2, +\infty) \cap R - \{2, -1\} = x \in [1/2, +\infty) - \{2\}$

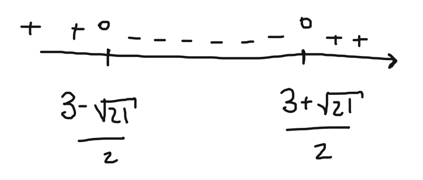


g- $\sqrt{x+4} < x - 1$

para la existencia necesitamos $x + 4 \geq 0 \rightarrow x \geq -4$

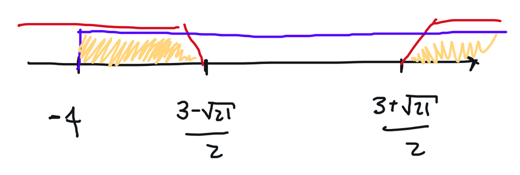
$(\sqrt{x+4})^2 < (x-1)^2 \longrightarrow x + 4 < x^2 - 2x + 1 \rightarrow x^2 - 3x - 3 > 0$

$x = \dfrac{3 \pm \sqrt{9+12}}{2} = \dfrac{3 \pm \sqrt{21}}{2}$



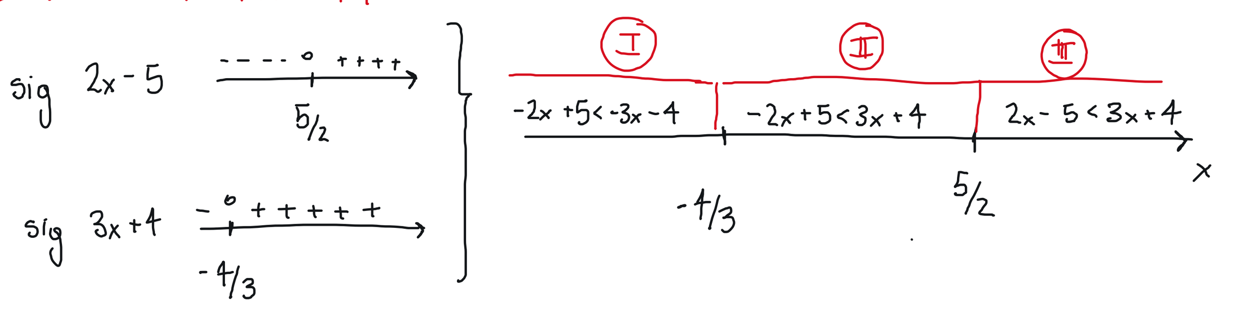
$x \in \left(-\infty, \frac{3-\sqrt{21}}{2}\right) \cup \left(\frac{3+\sqrt{21}}{2}, +\infty\right)$

$x \in \left[\left(-\infty, \frac{3-\sqrt{21}}{2}\right) \cup \left(\frac{3+\sqrt{21}}{2}, +\infty\right)\right] \cap [-4, +\infty) \Rightarrow x \in \left[-4, \frac{3-\sqrt{21}}{2}\right) \cup \left(\frac{3+\sqrt{21}}{2}, +\infty\right)$



$|x| = \begin{cases} x & \text{si } x \geq 0 \\ -x & \text{si } x < 0 \end{cases}$

(i) $|2x - 5| < |3x + 4|$



(I) $-2x + 5 < -3x - 4 \longrightarrow x < -9$

$x \in (-\infty, -9) \cap (-\infty, -4/3] \rightarrow x \in (-\infty, -9)$

(II) $-2x + 5 < 3x + 4 \longrightarrow 5x > 1 \rightarrow x > 1/5$

$x\ (1/5, +\infty) \cap (-4/3, 5/2) \rightarrow x \in (1/5, 5/2)$

(III) $2x - 5 < 3x + 4 \longrightarrow x > -9$

$x \in (-9, +\infty) \cap [5/2, +\infty) \rightarrow x \in [5/2, +\infty)$

$\Rightarrow$ $x \in (-\infty, -9) \cup \underbrace{(1/5, 5/2) \cup [5/2, +\infty)}_{(1/5, +\infty)}$

4_ $b > a > 0$ $\qquad a < H < G < A < b$

$A = \dfrac{a+b}{2}$ $\quad G = \sqrt{ab}$ $\quad H = \dfrac{2}{\frac{1}{a} + 1/b}$

$a < H$ $\quad a < \dfrac{2}{1/a + 1/b} = \dfrac{2}{\frac{b+a}{ab}} = \dfrac{2ab}{b+a}$

$a < \dfrac{2ab}{b+a} \rightarrow 1 < \dfrac{2b}{a+b} \rightarrow a + b < 2b \longrightarrow b > a$

$(\sqrt{ab})^2 = ab$

$H < G$ $\quad \dfrac{2ab}{b+a} < \sqrt{ab} \longrightarrow \dfrac{2\sqrt{ab}\sqrt{ab}}{b+a} < \dfrac{\sqrt{ab}}{1} \rightarrow \dfrac{2\sqrt{ab}}{a+b} < 1$

$a + b > 2\sqrt{ab} \longrightarrow (a+b)^2 > 4ab \rightarrow a^2 + 2ba + b^2 > 4ab$

$\underbrace{a^2 - 2ab + b^2}_{} > 0$

$(a-b)^2 > 0$

$G < A$ $\quad \sqrt{ab} < \dfrac{a+b}{2} \longrightarrow 2\sqrt{ab} < a + b$

$A < b$ $\quad \dfrac{a+b}{2} < b = \dfrac{b+b}{2} \longrightarrow \dfrac{a+b}{2} < \dfrac{b+b}{2} \rightarrow a < b$

$C = \sqrt{\dfrac{a^2+b^2}{2}}$ $\qquad a < C < b$

$a < \sqrt{\dfrac{a^2+b^2}{2}} \rightarrow a^2 < \dfrac{a^2+b^2}{2} \rightarrow 2a^2 < a^2 + b^2 \rightarrow a^2 < b^2$ si $0 < a < b$

$\sqrt{\dfrac{a^2+b^2}{2}} < b \rightarrow \dfrac{a^2+b^2}{2} < b^2 \rightarrow a^2 + b^2 < 2b^2 \rightarrow a^2 < b^2$ si $0 < a < b$

$C > A$ $\quad \sqrt{\dfrac{a^2+b^2}{2}} > \dfrac{a+b}{2} \rightarrow \dfrac{a^2+b^2}{2} > \dfrac{a^2+b^2+2ab}{4} \rightarrow 2a^2 + 2b^2 > a^2 + b^2 + 2ab$

$a^2 + b^2 > 2ab \rightarrow a^2 - 2ab + b^2 > 0 \rightarrow (a-b)^2 > 0$

6_ $A, B \subset R$ $\qquad \alpha A = \{\alpha a : a \in A\}$

$\alpha \in R$ $\qquad A + B = \{a + b : a \in A, b \in B\}$

(a) αA para $\alpha = 2$, $A = \{0, 2, 4\}$ $\Rightarrow \alpha A = \{0, 4, 8\}$

(b) $A + B$ para $A = \{1, 2, 3\}$, $B = \{1, \pi\}$ $\Rightarrow A + B = \{1 + \pi, 2 + \pi, 3 + \pi, 2, 3, 4\}$

(c) $Z + \{-1, 2, 5\} = Z$ $\qquad Z + \{1, 2, 5/2\} = Z + \{Z + \{0, 5\}\}$

(d) $p, a, b \in R$, $a < b$ $\Big\}$ $\{p\} + [a, b] = [a + p, b + p]$

$\{p\} + (a, b) = (a + p, b + p)$

Funciones reales 2.2

5_ $f(x) = x - 5$ $\qquad \forall x_0, x_1 / x_0 < x_1 \Rightarrow f(x_0) < f(x_1)$

$f(x_0) = x_0 - 5$ $\qquad x_0 - 5 < x_1 - 5$

$f(x_1) = x_1 - 5$ $\qquad x_0 < x_1$

estrictamente creciente

$f(x) = -2x$ $\qquad \forall x_0, x_1 / x_0 < x_1 \Rightarrow f(x_0) > f(x_1)$

$f(x_0) = -2x_0$ $\qquad -2x_0 < -2x_1$

$f(x_1) = -2x_1$ $\qquad x_0 > x_1$

estrictamente decreciente

7_ $I = [0, 5]$, $J = [3, 4]$ $\qquad f(x) = ax + b$ $\qquad \begin{cases} 3 = a.0 + b \rightarrow b = 3 \\ 4 = a.5 + b \qquad a = 1/5 \end{cases}$

$f(x) = \dfrac{x}{5} + b$