Funciones

3. ⓐ 1) Si f no tiene raices reales $\Rightarrow f\circ g$ no tiene raices reales

. Supongo $f\circ g$ tiene raices reales

$\exists x_0 / f(g(x_0)) = 0$

Por (H) f no tiene raices reales $\not\Rightarrow \not\exists t / f(t) = 0$

si $t = g(x_0) \Rightarrow f(t) = 0$ ↯ $\Rightarrow f\circ g$ no tiene raices reales ∎

5. Par $f(x) = f(-x)$ Impar $f(x) = -f(-x)$

a-

ⓐ $f(x) = |x| = x$

$f(-x) = |-x| = x = f(x) \Longrightarrow$ funcion par

ⓑ $f(x) = x^2 + 1$

$f(-x) = (-x)^2 + 1 = x^2 + 1 = f(x) \Rightarrow$ funcion par

ⓒ $f(x) = x^3 + 1$

$f(-x) = (-x)^3 + 1 = -x^3 + 1$

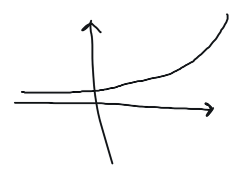
ⓕ $f(x) = \sqrt[3]{2x}$

acias

$f(-x) = \sqrt[3]{2(-x)} = -\sqrt[3]{2x} = -f(x) \Rightarrow$ funcion

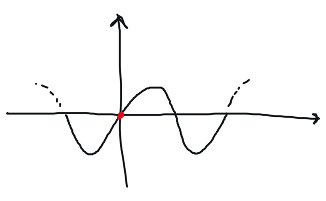
b-

ⓐ $f(x) = e^x$



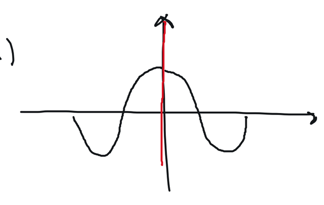
ⓑ $f(x) = sen(x)$

impar



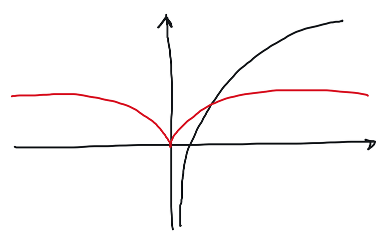
ⓒ $f(x) = cos(x)$

par



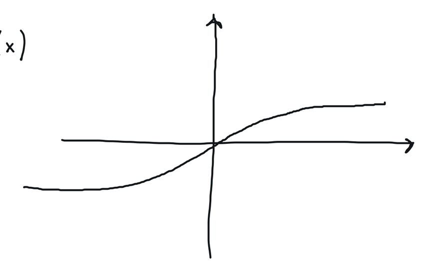
ⓓ $f(x) = log(x^2+1)$

par



ⓔ $f(x) = arctg(x)$

impar



c- ① suma de dos func. pares es par

$h(x) = \underbrace{f(x)}_{f(-x)} + \underbrace{g(x)}_{g(-x)} = f(-x) + g(-x) = h(-x)$

demostramos que h es par $\Rightarrow$ suma de pares es par

② $\lambda \in R$, si f par $\Rightarrow \lambda_f$ es par

$f(x) = f(-x)$, $h(x) = \lambda f(x)$

$h(-x) = \lambda f(-x) = \lambda f(x) = h(x) \Rightarrow h$ es par

③ $h'(x) = \underbrace{f'(x)}_{-f'(-x)} + \underbrace{g'(x)}_{-g'(-x)} = -f'(-x) - g'(-x) = -[\underbrace{f'(-x) + g'(-x)}_{h(-x)}] = -h(-x)$ ∎

$\lambda' \in R$, $f'(x) = -f'(-x)$, $h'(x) = \lambda f'(x)$

$h'(-x) = \lambda f'(-x) = -\underbrace{\lambda f'(x)}_{h'(x)} = -h'(x)$ h es impar

ⓓ el producto de par con impar es impar

f es par $f(x) = f(-x)$
g es impar $g(x) = -g(-x)$

$h(x) = f(x)g(x) = f(-x)(-1)g(-x) = -\underbrace{f(-x)g(-x)}_{h(-x)} = -h(-x)$

h es impar

ⓔ el producto de pares es par

$h(x) = \underbrace{f(x)}_{f(-x)} . \underbrace{g(x)}_{g(-x)} = h(-x)$

el producto de impares es par

$h(x) = \underbrace{f(x)}_{-f(-x)} \underbrace{g(x)}_{-g(-x)} = f(-x)g(-x) = h(-x)$

ⓕ $h(x) = \dfrac{f(x) + f(-x)}{2} \Rightarrow h(-x) = \dfrac{f(-x) + f(x)}{2} = h(x)$ es par

$g(x) = \dfrac{f(x) - f(-x)}{2} \Rightarrow g(-x) = \dfrac{f(-x) - f(x)}{2} = (-1)\Big(\overbrace{\dfrac{f(x) - f(-x)}{2}}^{g(x)}\Big) = -g(x)$ es impar

$g(x) + h(x) = \dfrac{f(x)}{2} - \dfrac{f(-x)}{2} + \dfrac{f(x)}{2} + \dfrac{f(-x)}{2} = f(x)$

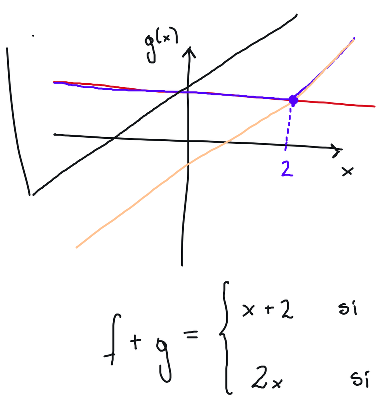
7. ⓐ $f(x) = 2x + 1$
$g(x) = 3x - 1$

$f + g = 2x + 1 + 3x - 1 = 5x$

$f(g(x)) = 2(3x - 1) + 1 = 6x - 1$

ⓖ $f(x) = x + 1$

$g(x) = max\{1, x-1\}$



$1 = x - 1 \Rightarrow x = 2$

$g(x) = \begin{cases} 1 & si\ x < 2 \\ x - 1 & si\ x \geq 2 \end{cases}$ $f + g = \begin{cases} x + 2 & si\ x < 2 \\ 2x & si\ x \geq 2 \end{cases}$

$f\circ g = \begin{cases} 1 + 1 & si\ x < 2 \\ x - 1 + 1 & si\ x \geq 2 \end{cases}$ $f\circ g = \begin{cases} 2 & si\ x < 2 \\ x & si\ x \geq 2 \end{cases}$

ⓙ $f(x) = \begin{cases} 0 & x \leq 0 \\ x/2 & 0 < x < 4 \\ 1 - x & x \geq 4 \end{cases}$ $g(x) = \begin{cases} x + 3 & x \leq 0 \\ 1/x & x > 0 \end{cases}$

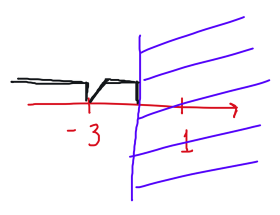
$0 + x + 3$ | $\frac{x}{2} + \frac{1}{x}$ | $1 - x + \frac{1}{x}$

(recta con marcas en 0 y 4, eje x)

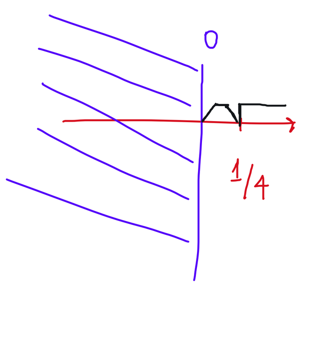
$f + g = \begin{cases} x + 3 & x \leq 0 \\ \frac{x^2+1}{2x} & 0 < x < 4 \\ \frac{-x^2+1}{x} + 1 & x \geq 4 \end{cases}$

$f(g(x)) = \begin{cases} 0 & g(x) \leq 0 \\ g(x)/2 & 0 < g(x) < 4 \\ 1 - g(x) & g(x) \geq 4 \end{cases}$

$x \leq 0$ $g(x) = x + 3$ $\begin{cases} x + 3 \leq 0 \rightarrow x \leq -3 \\ 0 < x + 3 < 4 \rightarrow x + 3 < 4 \rightarrow x < 1 \\ x + 3 \geq 4 \rightarrow x \geq 1 \end{cases}$



$x > 0$ $g(x) = \frac{1}{x}$ $\begin{cases} 1/x \leq 0 \\ 0 < 1/x < 4 \rightarrow 1/x < 4 \rightarrow x > 1/4 \\ 1/x \geq 4 \rightarrow x \leq 1/4 \end{cases}$



$f\circ g = \begin{cases} 0 & si\ x \leq -3 \\ \frac{x+3}{2} & si\ -3 < x \leq 0 \\ 1/2x & si\ 0 < x < 1/4 \\ 1 - \frac{1}{x} & si\ x \geq 1/4 \end{cases}$