$$f(m) * g(m) = \sum_n f(n) g(m-n)$$

def. Sea (a_n) una sucesion. La sucesion de sumas parciales es (s_n): $s_n = a_0 + a_1 + \dots + a_n$

prop. $\boxed{\text{si } F(x) = \sum_{n=0}^{\infty} a_n x^n \implies \sum_{n=0}^{\infty} s_n x^n = \frac{F(x)}{1-x}}$

dem. $(s_n) = (a_n) * (b_n)$ $\quad s_n = \sum_{i=0}^{n} a_i b_{n-i}$

$= 1 \ \forall n$

$$\implies \sum_{n=0}^{\infty} s_n x^n = \underbrace{\left(\sum_{n=0}^{\infty} a_n x^n\right)}_{F(x)} \cdot \underbrace{\left(\sum_{n=0}^{\infty} x^n\right)}_{1/1-x} = \frac{F(x)}{1-x}$$

Obtener una expresion $x + 2x^2 + 3x^3 + \dots + nx^n$ $\quad (a_n): a_n = n \ \forall n \in \mathbb{N}$

(s_n): $s_n = \sum_{i=0}^{n} i$ $\quad (0, 1, 2, 3, \dots)$ $\quad \boxed{s_n = \sum_{i=0}^{n} i = \frac{n(n+1)}{2}}$

$f(x) = x + 2x^2 + \dots = x(1 + 2x + 3x^2 + \dots)$

$f(x) = x(x + x^2 + x^3 + \dots)' = x(x[1 + x + x^2 + \dots])'$

$f(x) = x\left(x \cdot \left[\frac{1}{1-x}\right]\right)' = x\left(\frac{x}{1-x}\right)' = x\left(\frac{1}{(1-x)^2}\right)$

$\implies f(x) = \frac{x}{(1-x)^2}$

$$\left.\begin{aligned} A(x) &= \sum_{n=0}^{\infty} n x^n = \frac{x}{(1-x)^2} \\ S(x) &= \frac{A(x)}{1-x} \end{aligned}\right\} S(x) = \frac{x}{(1-x)^3}$$

$$\boxed{(1-ax)^{-n} = \sum_{i=0}^{\infty} C_i^{n+i-1} (ax)^i}$$

$$S(x) = \frac{x}{(1-x)^3} = \frac{A}{(1-x)^3} + \frac{B}{(1-x)^2} + \frac{C}{(1-x)} = \frac{1}{(1-x)^3} - \frac{1}{(1-x)^2}$$

$$\frac{x}{(1-x)^3} = \frac{A + B(1-x) + C(1-x)^2}{(1-x)^3} \implies x = Cx^2 + (-2C-B)x + C + B + A$$

$(1-x)^2 \to x^2 - 2x + 1$

$$\begin{cases} C = 0 \\ 1 = -2C - B \to B = -1 \\ 0 = C + B + A \to A = 1 \end{cases}$$

$$S(x) = (1-x)^{-3} - (1-x)^{-2} = \sum_{i=0}^{\infty} C_i^{3+i-1} x^i - \sum_{i=0}^{\infty} C_i^{2+i-1} x^i$$

$$s_n = C_n^{n+2} - C_n^{n+1} = \frac{(n+2)(n+1)}{2} - n + 1 = \frac{(n+2)(n+1) - 2n - 2}{2}$$

$$s_n = \frac{n^2 + 3n + 2 - 2n - 2}{2} = \frac{n^2 + n}{2} = \boxed{\frac{n(n+1)}{2} = s_n}$$

Ej 11

$$\begin{cases} a_n = -a_{n-1} - b_n \\ b_{n+1} = b_n - 3a_{n-1} \\ a_0 = 0, \ b_0 = 2, \ b_1 = 1 \end{cases}$$

$$\boxed{\begin{aligned} A(x) &= \sum_{i=0}^{\infty} a_i x^i \\ B(x) &= \sum_{i=0}^{\infty} b_i x^i \end{aligned}}$$

$$\begin{cases} \sum_{n=1}^{\infty} a_n x^n = -\sum_{n=1}^{\infty} a_{n-1} x^n - \sum_{n=1}^{\infty} b_n x^n \\ \sum_{n=1}^{\infty} b_{n+1} x^{n+1} = \sum_{n=1}^{\infty} b_n x^{n+1} - 3\sum_{n=1} a_{n-1} x^{n+1} \end{cases}$$

$$\left.\begin{aligned} A(x) &= a_0 + a_1 x + a_2 x^2 = \sum_{i=0}^{\infty} a_i x^i \\ \sum_{n=1}^{\infty} a_n x^n &= a_1 x + a_2 x^2 + \dots \end{aligned}\right\} \sum_{n=1}^{\infty} a_n x^n = A(x) - a_0$$

$$\begin{cases} A(x) - a_0 = -xA(x) - (B(x) - b_0) \\ B(x) - b_0 - b_1 x = x(B(x) - b_0) - 3x^2 A(x) \end{cases}$$