EJ3 (b) $f(x) = \frac{x^3}{1-x} = x^3 \cdot \left(\frac{1}{1-x}\right) =$

$= x^3(1 + x + x^2 + \dots) = x^3 + x^4 + x^5 + \dots$

$(0,0,0,1,1,\dots)$

CdeV $y = -3x$

(d) $f(x) = \frac{1}{1+3x} \Rightarrow f(y) = \frac{1}{1-y} =$

$= 1 + y + y^2 + \dots \Rightarrow f(x) = 1 - 3x + 9x^2 - 27x^3 + \dots$

$(1, -3, 9, -27, \dots)$

(e) $f(x) = \frac{1}{2-x} = \frac{2}{2} \cdot \frac{1}{2-x} = \frac{1}{2} \cdot \frac{1}{\frac{2}{2} - \frac{x}{2}}$

$f(x) = \frac{1}{2-x} = \frac{1}{2} \cdot \frac{1}{1-x/2}$ CdeV $y = \frac{x}{2}$

$f(y) = \frac{1}{2} \cdot \frac{1}{1-y} = \frac{1}{2}(1 + y + y^2 + \dots)$

$f(x) = \frac{1}{2}\left(1 + \frac{x}{2} + \left(\frac{x}{2}\right)^2 + \dots\right) = \frac{1}{2} + \frac{x}{4} + \frac{x^2}{8} + \frac{x^3}{16} + \dots$

$\left(\frac{x}{2}\right)^2 = \frac{x^2}{4}$

$\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots\right)$

(f) $f(x) = \frac{3x^6 - 9 + 1}{1-x} = \frac{3x^6 - 8}{1-x}$

$(3x^6 - 8) \cdot \left(\frac{1}{1-x}\right) = 3x^6 - 8\,(1 + x + x^2 + \dots) =$

$f(x) = 3x^6 + 3x^7 + 3x^8 + \dots - 8 - 8x - 8x^2 - \dots - 8x^6 - \dots$

$(-8, -8, -8, -8, -8, -8, -5, -5, \dots)$

Combinaciones Negativas

$(a+b)^n = \sum_{i=0}^{n} C_i^n a^i b^{n-i}$

$(1+x)^n = \sum_{i=0}^{n} C_i^n x^i \rightarrow$ $\boxed{\frac{1}{(1+x)^n} = (1+x)^{-n} = \sum_{i=0}^{n} C_i^{-n} x^i}$

def. $\boxed{C_i^{-n} = (-1)^i CR_i^n = (-1)^i C_i^{n+i-1}}$

$\frac{1}{1-x} = (1-x)^{-1} = (1 + (-x))^{-1} = \sum_{i=0}^{\infty} C_i^{-1} (-x)^i =$

$= \sum_{i=0}^{\infty} (-1)^i CR_i^1 (-x)^i = \sum_{i=0}^{\infty} (-1)^i CR_i^1 (-1)^i x^i =$

$= \sum_{i=0}^{\infty} (-1)^{2i} CR_i^1 x^i = \sum_{i=0}^{\infty} CR_i^1 x^i = \sum_{i=0}^{\infty} C_i^{1+i-1} x^i = \sum_{i=0}^{\infty} x^i$

$(-1)^{2i} = 1$, $C_i^i = 1$

EJ5 (a) Hallar el coef. de x^8 en $(1 + x + x^2 + \dots)^{10}$

$\sum_{i=0}^{\infty} a_i x^i = a_0 + a_1 x + \dots + a_8 x^8 + \dots$

$f(x) = (1 + x + x^2 + \dots)^{10} = \left(\frac{1}{1-x}\right)^{10} = \frac{1}{(1-x)^{10}} = (1-x)^{-10}$

$f(x) = (1 + (-x))^{-10} = \sum_{i=0}^{\infty} C_i^{-10} (-x)^i = \sum_{i=0}^{\infty} (-1)^i CR_i^{10} (-1)^i x^i$

$= \sum_{i=0}^{\infty} (-1)^{2i} C_i^{10+i-1} x^i = \sum_{i=0}^{\infty} C_i^{9+i} x^i = C_0^9 + C_1^{10} x + \dots + C_8^{17} x^8 + \dots$

$C_i^{9+i} = a_i$

el coef. de x^8 es C_8^{17} (C_8^{n+9})

EJ6 Hallar el coef. de x^{15} en $\frac{x^3 - 5x}{(1-x)^3}$

$f(x) = (x^3 - 5x) \cdot \frac{1}{(1-x)^3} \Rightarrow (1 + (-x))^{-3} = \sum_{i=0}^{\infty} C_i^{-3} (-x)^i =$

$= \sum_{i=0}^{\infty} (-1)^i CR_i^3 (-1)^i x^i = \sum_{i=0}^{\infty} C_i^{i+3-1} x^i = \sum_{i=0}^{\infty} C_i^{2+i} x^i$

$f(x) = (x^3 - 5x)\left(\sum_{i=0}^{\infty} C_i^{2+i} x^i\right) = \sum_{i=0}^{\infty} C_i^{2+i} x^{i+3} - \sum_{i=0}^{\infty} C_i^{2+i} \cdot 5 x^{i+1}$

$i = 12$ $\qquad$ $i = 14$

el coef. de x^{15} es $C_{12}^{2+12} - 5\,C_{14}^{2+14} = C_{12}^{14} - 5\,C_{14}^{16}$

EJ8 Hallar la funcion generatriz de "la cantidad de formas que tiene un cajero de darme n pesos, solo con billetes de 500, 1000 y 2000" $= a_n$

$a_1 = 0$

$a_2 = 0$

$\vdots$

$a_{500} = 1$

$a_{501} = 0$

$\vdots$

$a_{1000} = 2$

$\vdots$

$a_{1500} = 2$

$\vdots$

$\boxed{a_{2000} = 4}$

500: $(0, 500, 1000, 1500, 2000, \dots)$ 1000: $(0, 1000, 2000, \dots)$ 2000: $(0, 2000, \dots)$

2000 $\begin{cases} 500 \times 4 \\ 500 \times 2 + 1000 \\ 1000 \times 2 \\ 2000 \end{cases}$

$(1 + x^{500} + x^{1000} + x^{1500} + \dots)(1 + x^{1000} + x^{2000} + \dots)(1 + x^{2000} + \dots)$

Cambio de Variable

$y = x^{500}$ $\qquad$ $z = x^{1000}$ $\qquad$ $w = x^{2000}$

$(1 + y + y^2 + \dots)(1 + z + z^2 + \dots)(1 + w + w^2 + \dots)$

$f(x) = \frac{1}{1-y} \cdot \frac{1}{1-z} \cdot \frac{1}{1-w} = \boxed{\left(\frac{1}{1-x^{500}}\right)\left(\frac{1}{1-x^{1000}}\right)\left(\frac{1}{1-x^{2000}}\right)}$

EJ4 Examen 2010 $\quad (0, 0, a, 1, 0, a^2, 2, 0, a^3, 3, 0, \dots)$

$f(x) = ax^2 + x^3 + a^2x^5 + 2x^6 + a^3x^8 + 3x^9 + \dots$

$f(x) = ax^2 + a^2x^5 + a^3x^8 + \dots + x^3 + 2x^6 + 3x^9 + \dots$

$ax^2(1 + ax^3 + a^2x^6 + \dots)$ $\qquad$ $x^3(1 + 2x^3 + 3x^6 + \dots)$

CdeV $y = ax^3$ $\qquad$ CdeV $z = x^3$

$ax^2(1 + y + y^2 + \dots)$ $\qquad$ $z(1 + 2z + 3z^2 + 4z^3 + \dots)$

$z(z + z^2 + z^3 + \dots)'$

$\frac{ax^2}{1-y} = \frac{ax^2}{1-ax^3}$ $\qquad$ $z(z(1 + z^2 + z^3 + \dots))'$

$z\left(z\left(\frac{1}{1-z}\right)\right)' = \frac{z}{(1-z)^2}$

$\left(\frac{z}{1-z}\right)' = \frac{1-z+z}{(1-z)^2} = \frac{1}{(1-z)^2}$ $\qquad = \frac{x^3}{(1-x^3)^2}$

$\Longrightarrow \boxed{f(x) = \frac{ax^2}{1-ax^3} + \frac{x^3}{(1-x^3)^2}}$