Funciones generatrices

a_n sucesion $\longleftrightarrow$ serie ("pol. inf.")

$$(a+b)^n = \sum_{i=0}^{n} C_i^n a^i b^{n-i}$$

$a_0, a_1, a_2, \ldots$ $\quad a_0 + a_1 x + \ldots + a_n x^n + \ldots$

$$f(x) = \sum_{i=0}^{\infty} a_n x^n$$

$$f(x) = (x+1)^n = \sum_{i=0}^{n} C_i^n x^i = \sum_{i=0}^{\infty} C_i^n x^i = C_0^n + C_1^n x + C_2^n x^2 + \ldots + C_n^n x^n + 0 + \ldots$$

la funcion $(x+1)^n$ genera una sucesion $a_0 = C_0^n$

$a_1 = C_1^n$

Ejemplo Vamos a buscar la funcion generatiz de $a_n = 1 \ \forall n$

$$f(x) = 1 + x + x^2 + \ldots + x^n + \ldots$$

$$\underbrace{(1-x)(1+x+x^2+\ldots)}_{} = 1 + \cancel{x} + \cancel{x^2} + \ldots - \cancel{x} - \cancel{x^2} - \cancel{x^3} - \ldots = 1$$

$$1 + x + x^2 + \ldots + x^n + \ldots = \frac{1}{1-x} \qquad \boxed{\sum_{i=0}^{\infty} x^n = \frac{1}{1-x}}$$

EJ2 $(1,1,1,1,\ldots)$ $\quad f(x) = 1 + x + x^2 + \ldots = \sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$

A) $(C_0^6, C_1^6, C_2^6, \ldots, C_6^6, 0, 0, 0, \ldots)$

$$C_0^6 + C_1^6 x + C_2^6 x^2 + \ldots + C_6^6 x^6 + 0 + \ldots = \sum_{i=0}^{6} C_i^6 x^i = (x+1)^6$$

C) $(1, -1, 1, -1, \ldots)$

$$1 - x + x^2 - x^3 + \ldots = 1 - x + (-x)^2 + (-x)^3 + \ldots = f(x)$$

Cambio de Variable $y = -x$

$$\to 1 + y + y^2 + y^3 + \ldots = \frac{1}{1-y} \longrightarrow \boxed{f(x) = \frac{1}{1+x}}$$

D) $(0,0,0,0,1,1,\ldots)$

$$0 + 0x + 0x^2 + 0x^3 + x^4 + x^5 + \ldots = x^4 + x^5 + \ldots$$

$$f(x) = x^4(1 + x + x^2 + \ldots)$$

$$\boxed{f(x) = \frac{x^4}{1-x}}$$

E) $(0,0,0,3,-3,3,-3,\ldots)$

$$3x^3 - 3x^4 + 3x^5 - 3x^6 + \ldots = f(x)$$

$$3x^3 \underbrace{(1 - x + x^2 - x^3 + \ldots)}_{}$$

CdeV $y = -x$ $\qquad \boxed{f(x) = \frac{3x^3}{1+x}}$

$$3x^3(1 + y + y^2 + \ldots) = 3x^3 \cdot \frac{1}{1-y}$$

J) $(0, 0, 1, b, a, b^2, a^2, \ldots)$

$$x^2 + bx^3 + ax^4 + b^2x^5 + a^2x^6 + \ldots = f(x)$$

$$x^2[1 + bx + ax^2 + b^2x^3 + a^2x^4 + \ldots]$$

$$x^2\left[\underbrace{(bx + b^2x^3 + \ldots)}_{} + 1 + \underbrace{(ax^2 + a^2x^4 + \ldots)}_{}\right]$$

$bx(1 + bx^2 + b^2x^4 + \ldots)$ $\qquad ax^2(1 + ax^2 + a^2x^4 + \ldots)$

CdeV $y = bx^2$ $\qquad$ CdeV $z = ax^2$

$bx(1 + y + y^2 + \ldots)$ $\qquad ax^2(1 + z + z^2 + \ldots)$

$bx \cdot \frac{1}{1-y}$ $\qquad ax^2 \cdot \frac{1}{1-z}$

$$\boxed{f(x) = x^2\left[1 + \frac{bx}{1-bx^2} + \frac{ax^2}{1-ax^2}\right]}$$

EJ3 a) $f(x) = (2x-3)^3 = \sum_{i=0}^{3} C_i^3 (2x)^i (-3)^{n-i}$

$$C_0^3(2x)^0(-3)^3 + C_1^3(2x)(-3)^2 + C_2^3(2x)^2(-3) + C_3^3(2x)^3(-3)^0$$

$(-27C_0^3, 18C_1^3, -12C_2^3, 8C_3^3, 0, 0, \ldots)$ $\qquad \boxed{a_n = 2^n(-3)^{3-n} C_n^3}$