1. *a*) Calcular la parte real e imaginaria de los complejos $\frac{1}{a+bi}$, $(a+bi)^2$.

$$\frac{1}{a+bi} = \frac{1}{a+bi} \cdot \frac{a-bi}{a-bi} = \frac{a-bi}{a^2-(bi)^2} = \frac{a-bi}{a^2+b^2} = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2} i$$

$$\frac{1}{z} = \frac{1}{z} \cdot \frac{\bar{z}}{\bar{z}} = \frac{\bar{z}}{|z|^2}$$

c) Probar que $|z_1 - z_2| \geq ||z_1| - |z_2||$ para todo $z_1, z_2 \in \mathbb{C}$.

Si $|z_1| \geq |z_2| \Rightarrow ||z_1| - |z_2|| = |z_1| - |z_2| \geq 0$

Si no, $||z_1| - |z_2|| = |z_2| - |z_1|$

$|z_1 - z_2| = |z_2 - z_1|$ y se reduce al caso anterior

S.p.g. supong. que $|z_1| \geq |z_2|$

$|a+b| \leq |a| + |b|$

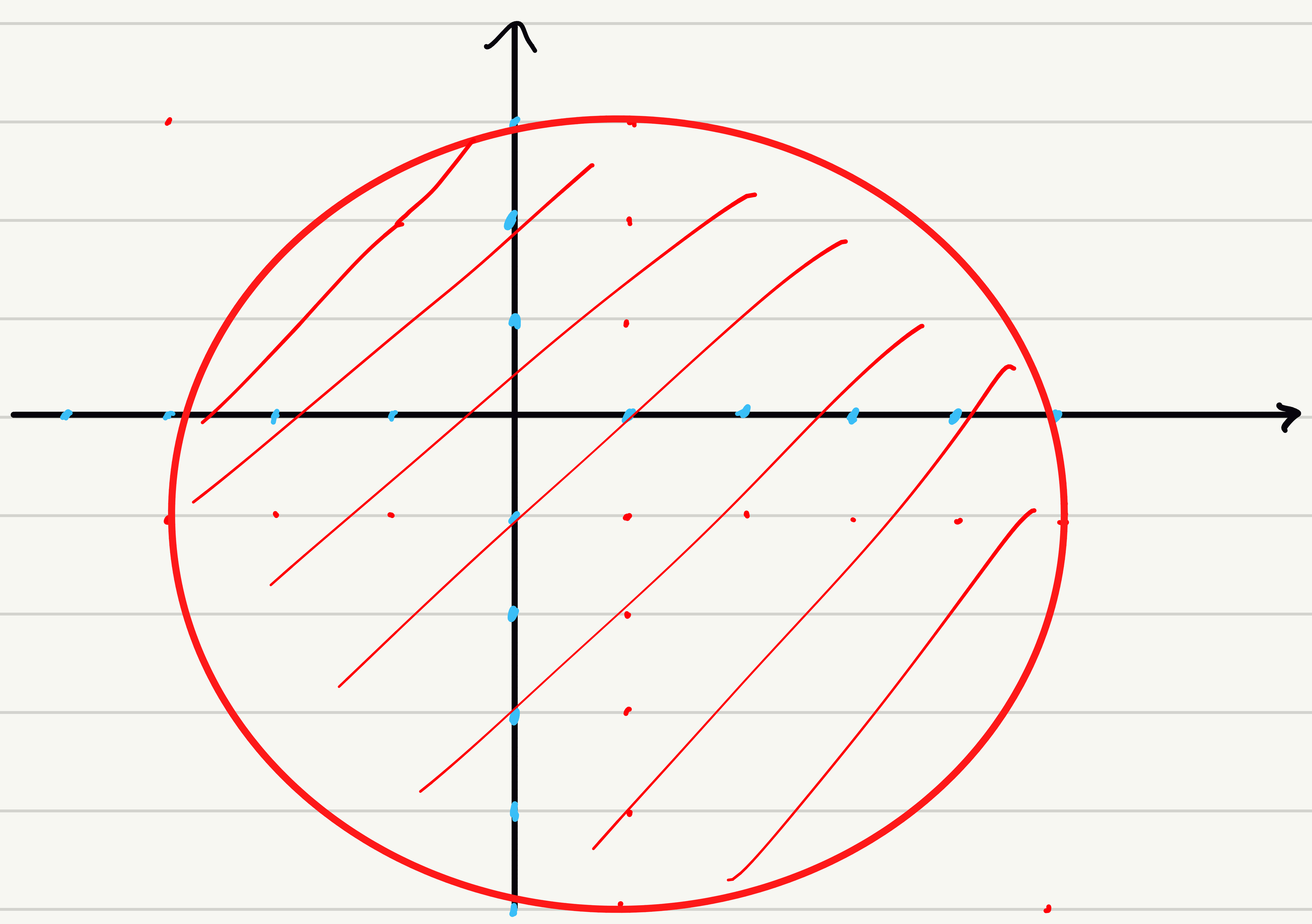
Q.p.q. $|z_1 - z_2| \geq |z_1| - |z_2| \Longleftrightarrow$

$$|z_1 - z_2| + |z_2| \geq |z_1| = |\underbrace{(z_1 - z_2)}_{a} + \underbrace{z_2}_{b}|$$

2. Bosquejar los $z \in \mathbb{C}$ tales que $|z - 1 + i| \leq 4$

$d(z, w) = |z - w|$

$$\| \quad d(z, 1-i) \leq 4 \leftarrow \overline{B(1-i, 4)}$$



3. Hallar los $z \in \mathbb{C}$ tales que $\underset{0\le}{|z+1|} \le 4 - |z-1|. \ge 0$

$$z = x+iy \Rightarrow |z \pm 1|^2 = (x\pm 1)^2 + y^2 = x^2+y^2 \pm 2x + 1$$
$$= |z|^2 \pm 2x + 1$$

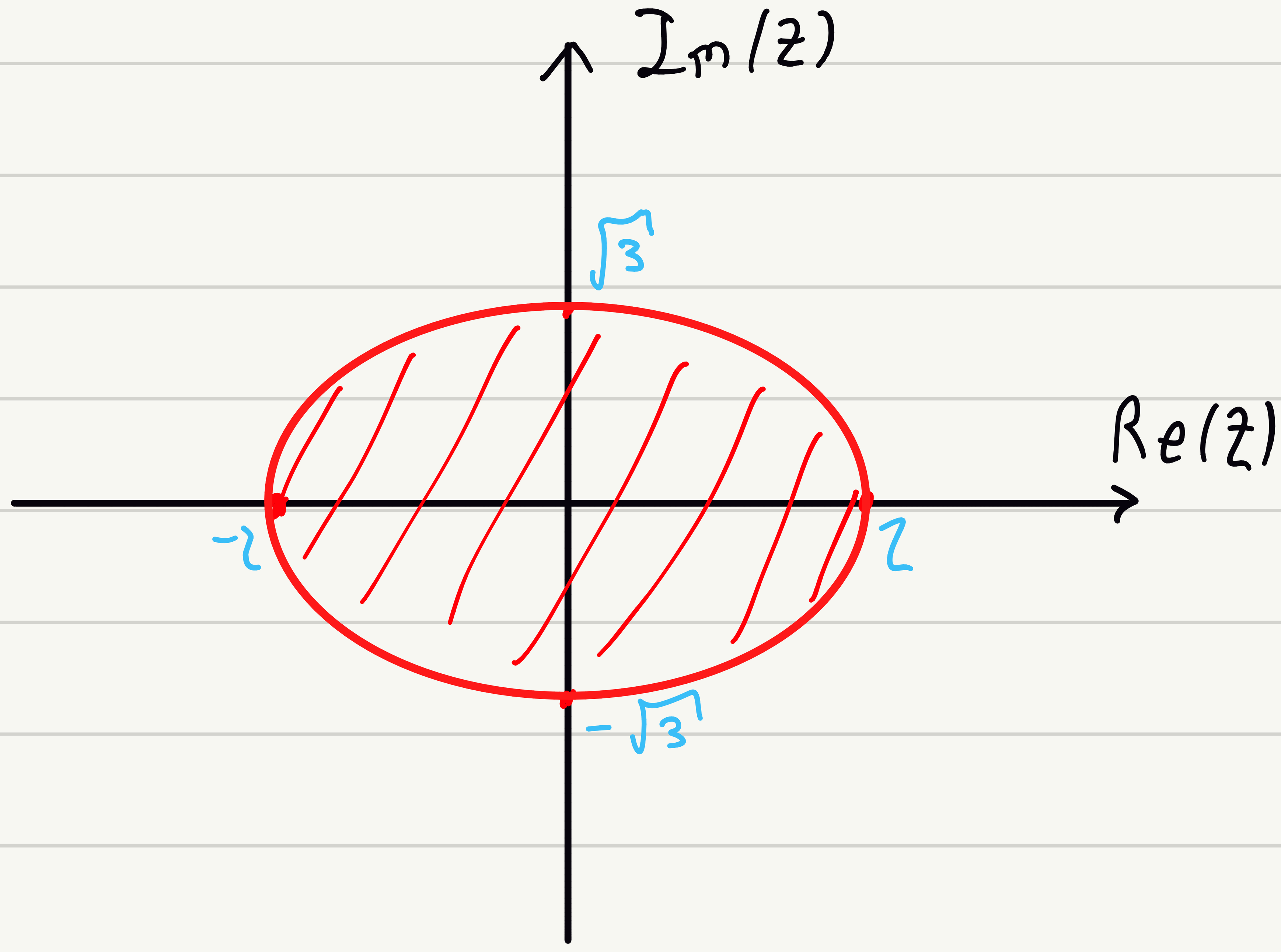
$$|z+1|^2 \le 16 - 8|z-1| + |z-1|^2 \Leftrightarrow$$

$$|z+1|^2 - |z-1|^2 \le 16 - 8|z-1|$$
$$\|$$
$$(\cancel{|z|^2}+2x+\cancel{1}) - (\cancel{|z|^2}-2x+\cancel{1}) = \cancel{4}x \le 16 - 8|z-1| = \cancel{4}(4-2|z-1|)$$

$$\Leftrightarrow x \le 4 - 2|z-1| \Leftrightarrow \underset{0\le}{2|z-1|} \le 4 - x \Leftrightarrow 4|z-1|^2 \le 16 - 8x + x^2$$

$$\Leftrightarrow 4(x^2+y^2-\cancel{2x}+1) \le 16 - \cancel{8x} + x^2 \Leftrightarrow$$
$$4x^2+4y^2+4 \le 16 + x^2 \Leftrightarrow 3x^2+4y^2 \le 12$$

$$3x^2+4y^2 = 12$$
$$x = 0 \Rightarrow 4y^2 = 12 \Leftrightarrow y^2 = 3 \Leftrightarrow y = \pm\sqrt{3}$$
$$y = 0 \Rightarrow 3x^2 = 12 \Leftrightarrow x^2 = 4 \Leftrightarrow x = \pm 2$$



7. Definimos $f : \mathbb{C} - \{0\} \to \mathbb{C}$, $f(z) = \log(z) = \log|z| + iArg(z)$, donde $Arg(z) \in [0, 2\pi)$ y le llamamos logaritmo principal de z. [redacted]

8. Hallar el error en la siguiente paradoja de J. Bernoulli, donde log denota el logaritmo principal.

$$(-z)^2 = z^2 \Rightarrow \log\left((-z)^2\right) = \log\left(z^2\right) \Rightarrow 2\log(-z) = 2\log(z) \Rightarrow \log(-z) = \log(z).$$

$$\log(i) = \log(1) + i\arg(i) = 0 + i\cdot\frac{\pi}{2} = \frac{\pi}{2}i$$

$$\neq$$

$$\log(-i) = \log(1) + i\arg(-i) = \frac{3\pi}{2}\cdot i$$

$$(-z)^2 = ((-1).z)^2 = (-1)^2\cdot z^2 = z^2$$

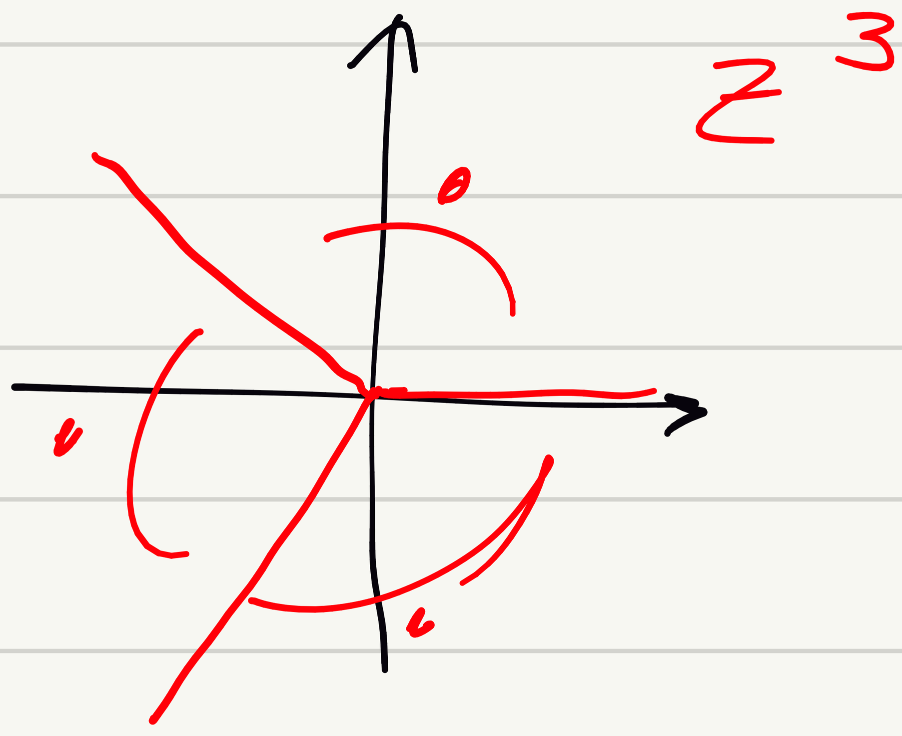
$$\log(i^2) = \log(-1) = \log 1 + i\arg(-1) = i\pi$$

$$2\log(i) = 2i\frac{\pi}{2} = i\pi$$

$$\log((-i)^2) = \log(-1) = i\pi$$

$$\neq$$

$$2\log(-i) = 2\cdot\frac{3\pi}{2}i = 3\pi i$$



12. Dados tres números complejos a, b y c de módulo 1 y tales que $a + b + c = 0$ muestre que los mismos (como puntos del plano) forman un triángulo equilátero.

Como $|a| = 1 \Rightarrow a \neq 0$: $a+b+c = 0 \Leftrightarrow 1 + \frac{b}{a} + \frac{c}{a} = 0$

$$\left|\frac{b}{a}\right| = \frac{|b|}{|a|} = \frac{1}{1} = 1 = \left|\frac{c}{a}\right|$$

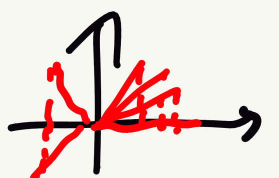
$z := b/a$; $w := c/a \longrightarrow 1 + z + w = 0 \Rightarrow z = -w-1$

$|w| = 1 \Rightarrow w = e^{i\theta}$. $\quad |z| = |-w-1| = 1 = |w+1|$

$= \cos\theta + i\sin\theta \quad = |(\cos\theta + 1) + i\sin\theta|$

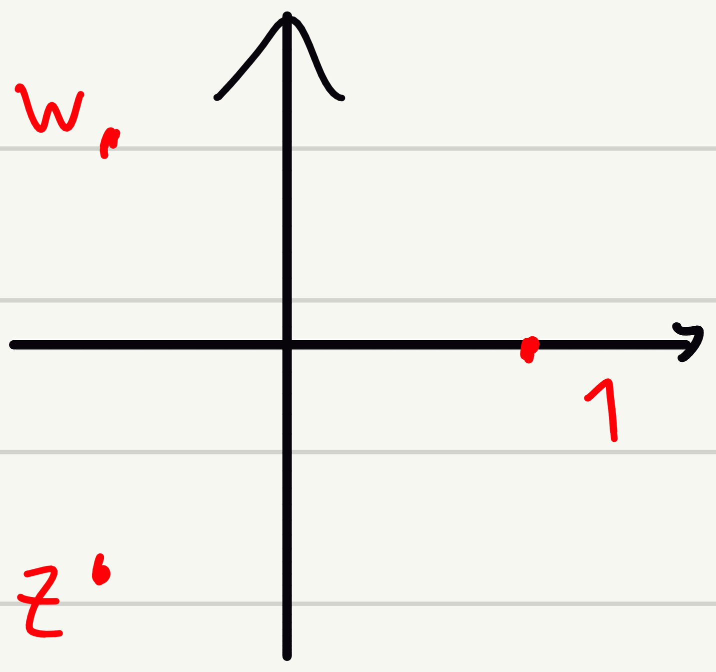
$$\Rightarrow 1^2 = \cancel{1} = |w+1|^2 = (\cos\theta+1)^2 + \sin^2\theta = \cancel{\cos^2\theta + \sin^2\theta} + 2\cos\theta + 1$$

$$\Rightarrow 2\cos\theta + 1 = 0 \Rightarrow \cos\theta = -1/2 \Leftrightarrow \theta \in \{4\pi/3, 2\pi/3\}$$



$\Rightarrow w = e^{i2\pi/3}$, $z = e^{i4\pi/3}$

$\leftarrow$ Calculando $-w-1$



$\xrightarrow{\times a}$