Práctico 9: Funciones generatrices.

Repaso:

Sucesión $(a_n)_{n\in\mathbb{N}}$	Generatriz $\sum_{n\geq 0}^{\infty} a_n x^n$	Generatriz como cociente ...
$(1,1,1,1,1,\dots)$	$\sum_{n=0}^{\infty} x^n$	$\frac{1}{1-x}$
$(0,0,0,1,1,1,\dots)$	$\sum_{n=3}^{\infty} x^n = x^3 \sum_{n=0}^{\infty} x^n$	$\frac{x^3}{1-x}$
$(1,0,1,0,1,0\dots)$	$\sum_{n=0}^{\infty} (x)^{2n}$ $(y \overset{c.v}{=} x^2)$	$\frac{1}{1-x^2}$
$(1,a,a^2,a^3,a^4,\dots)$	$\sum_{n=0}^{\infty} \underbrace{a^n x^n}_{(ax)^n}$ $(y=ax)$	$\frac{1}{1-ax}$
$(1,0,a,0,a^2,0,a^3,0\dots)$	$\sum_{n=0}^{\infty} a^n x^{2n}$ $(y=ax^2)$	$\frac{1}{1-ax^2}$
$(0,2,0,2,0,2\dots)$ ↳ es $(2,0,2,0,\dots)$ corrida un lugar	$\sum_{n=0}^{\infty} 2x^{2n+1} = 2x\sum_{n=0}^{\infty} x^{2n}$ $(y=x^2)$	$\frac{2x}{1-x^2}$
$\overset{a_0\,a_1\,a_2}{(1,2,3,4,5,6,\dots)}$	$\sum_{n=1}^{\infty} n x^{n-1}$ → es la derivada de $\sum_{n=0}^{\infty} x^n$ $\left(\sum_{n=0}^{\infty}(n+1)x^n\right)$	$\frac{1}{(1-x)^2}$

pares: $2n$

impares: $2n+1$

Ejercicio 3. Determine la sucesión generada por cada una de las siguientes funciones generatrices.

a. $f(x) = (2x-3)^3$ **c.** $f(x) = \frac{x^3}{1-x^2}$ **e.** $f(x) = \frac{1}{2-x}$ -8

b. $f(x) = \frac{x^3}{1-x}$ **d.** $f(x) = \frac{1}{1+3x}$ **f.** $f(x) = \frac{3x^6-9+1}{1-x}$

(a) $f(x) = (2x-3)^3 \leftarrow$ T. Binomio

$$= \sum_{n=0}^{3} C_n^3 (2x)^n (-3)^{3-n} = \sum_{n=0}^{3} C_n^3 \underbrace{(-3)^{3-n}\, 2^n}_{a_n} x^n \quad (*)$$

$(*)$ $\underbrace{C_0^3 (-3)^3 2^0}_{a_0} x^0 + \underbrace{C_1^3 (-3)^2 2^1}_{a_1} x^1 + \underbrace{C_2^3 (-3)^1 2^2}_{a_2} x^2 + \underbrace{C_3^3 (-3)^0 2^3}_{a_3} x^3 + 0 \ldots$

$a_4 = 0 \quad a_5 = 0 \ldots \quad a_n = 0 \;\; \forall n \geq 4.$

$(2x-3)(2x-3)(2x-3)$

$(2x-3)(4x^2-12x+9)$

$(8x^3 - 24x^2 + 18x - 12x^2 + 36x - 27)$

$(8x^3 - 36x^2 + 54x - 27)$

$a_0 = -27 \quad a_1 = 54 \quad a_2 = -36 \quad a_3 = 8 \quad a_n = 0 \;\; \forall n \geq 3.$

(e) $f(x) = \frac{1}{2-x} = \frac{1}{2} \cdot \frac{1}{1-\frac{x}{2}} = \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n = \left(\frac{1}{2}\right) \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n x^n = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} x^n$

$\left(\frac{x}{2}\right)^n = \frac{x^n}{2^n}$

$\frac{1}{1-y}$

$a_n = \left(\frac{1}{2}\right)^{n+1} \;\; \forall n \in \mathbb{N}.$

$2 - x = 2\left(1 - \frac{x}{2}\right)$

$\frac{1}{1-y} = \sum_{n=0}^{\infty} y^n \quad y = \frac{x}{2} \quad \frac{1}{1-x/2} = \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$

(f) $f(x) = \dfrac{3x^6 - 8}{1-x}$ $\qquad f(x) = \sum_{n=0}^{\infty} a_n x^n$ ← nos piden esto

$$f(x) = \frac{3x^6}{1-x} - \frac{8}{1-x}$$

$(0,0,0,0,0,0,3,3,3,\ldots) - (8,8,8,8,\ldots) = (-8,-8,-8,-8,-8,-8,-5,-5,\ldots)$

$$f(x) = 3x^6 \sum_{n=0}^{\infty} x^n - 8 \sum_{n=0}^{\infty} x^n$$

$$= \sum_{n=0}^{\infty} 3x^{n+6} - 8 \sum_{n=0}^{\infty} x^n$$

$$= \sum_{n=6}^{\infty} 3x^n - \sum_{n=0}^{\infty} 8x^n$$

$$a_n = \begin{cases} -8 & n \leq 5 \\ -5 & n \geq 6 \end{cases}$$

$(1-x)^n \rightarrow$ F. Binomio

$\left(\sum x^n\right)^m \leftarrow (1-x)^{-m} \rightarrow$ F. Binomio generalizado

$(1-x)^{-1} = \sum_{n=0}^{\infty} x^n$

Ejercicio 6. Hallar el coeficiente de x^{15} en las funciones:

- (a) $x^3(1-2x)^{10}$. 0
- (b) $\frac{x^3-5x}{(1-x)^3}$.
- (c) $\frac{(1+x)^4}{(1-x)^4}$.

(a) $x^3 \underbrace{(1-2x)^{10}}$

Queremos el coeficiente de x^{12}

$(1-2x)^{10}$ es un polinomio de grado 10 $\Rightarrow$ el coef. de x^{12} es 0

$\Rightarrow$ el coef. de x^{15} en $x^3(1-2x)^{10}$ es 0.

(b) $\dfrac{x^3-5x}{(1-x)^3}$

$$\boxed{(1-x)^{-m} = \sum_{n=0}^{\infty} CR_n^m \, x^n}$$

$$\boxed{(1-x)^{-3} = \sum_{n=0}^{\infty} CR_n^3 \, x^n}$$

Queremos el coef de x^{12} $\qquad$ Queremos el coef de x^{14}

$$x^3 \, \overbrace{(1-x)^{-3}} - 5x \, \overbrace{(1-x)^{-3}}$$

coef. x^{12}: CR_{12}^3 $\qquad$ coef. de x^{14}: CR_{14}^3

El coeficiente de x^{15} en $\dfrac{x^3 - 5x}{(1-x)^3}$ es $\boxed{CR^3_{12} - 5 \cdot CR^3_{14}}$

$\binom{14}{12} - 5 \cdot \binom{16}{14}$

$91 - 5 \cdot 120$