Universidad de la República
Facultad de Ingeniería
Instituto de Matemática y Estadística

Matemática Discreta 1

Primer semestre 2024

PRÁCTICO 4: COMBINATORIA III

Ejercicio 1. En una playa se juntan 13 chicos y deciden hacer 4 equipos para jugar al voleibol, para ello hacer tres equipos de 3 jugadores y uno de 4. Entre los chicos se encuentra uno sumamente habilidoso y otro que es de madera, los restantes 11 jugadores son intermedios. Para equiparar, al habilidoso lo colocan en uno de los equipos de 3 jugadores y al de madera en el equipo de 4 jugadores.
¿De cuántas formas se pueden armar los equipos?

Ejercicio 2. Para una selección de fútbol, fueron convocados 2 goleros, 6 zagueros, 7 mediocampistas y 4 atacantes. ¿De cuántos modos es posible formar una selección con un golero, 4 zagueros, 4 mediocampistas y 2 atacantes?

Ejercicio 3.

(a) Hallar la cantidad de soluciones naturales de la ecuación $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 4$.

(b) ¿Cuántas soluciones hay si se reemplaza el signo $=$ por el signo $<$?
O sea: hallar la cantidad de soluciones naturales de la inecuación $x_1+x_2+x_3+x_4+x_5+x_6+x_7 < 4$.

(c) Hallar la cantidad de soluciones naturales de la ecuación $x_1 + x_2 + x_3 + x_4 = 15$ tal que se cumplen simultáneamente las siguientes condiciones $x_1 \geq 3$ y $x_4 \geq 3$.

Ejercicio 4.

a. Hallar la cantidad de subconjuntos de un conjunto con n elementos razonando con la fórmula del binomio.

b. Probar que: $\displaystyle\sum_{j=0}^{n} (-1)^j \, C_j^n = 0$.

c. (Ej. 4 del 1^{er} parcial del 2000) Hallar el valor de la siguiente suma: $\displaystyle\sum_{k=0}^{203} C_k^{203} \, (-4)^k$.

Ejercicio 5.

(a) Hallar el coeficiente en x^5 en el desarrollo de $(x^5 + x - 1)^{10}$.

(b) Hallar el coeficiente en xy^3z^5 del polinomio $(2x + 4y + 2z + 5)^{14}$.

Ejercicio 6. (Ej. 4 del 2^{do} examen del curso 2001)
Hallar la cantidad n de palabras de 1 a 5 letras que se pueden formar usando las letras de la palabra *CASAS*. (Por ejemplo *C* y *AA* son dos palabras posibles, pero *CAC* no lo es).

Ejercicio 7. (Ej. 2 del 1^{er} parcial del curso 2009)
¿Cuántas palabras de longitud 6 existen que no tengan dos consonantes o dos vocales juntas?

Ejercicio 8. Dados $A = \{1, 2, \ldots, n\}$ y $B = \{1, 2, \ldots, m\}$, hallar la cantidad de funciones $f : A \to B$ tales que:

(a) No hay restricciones.

(b) f es inyectiva.

(c) f es biyectiva

(d) f es monótona creciente estrictamente.

(e) f es monótona creciente.

(f) Cada elemento $i \in B$ es alcanzado r_i veces, donde $r_1 + \ldots + r_m = n$.

Ejercicio 9.

a. Para n y t positivos, repasar la demostración (hay al menos dos posibles demostraciones) que el coeficiente de $x_1^{n_1}\ x_2^{n_2} \ldots x_t^{n_t}$ en $(x_1 + x_2 + \cdots + x_t)^n$ es

$$\frac{n!}{n_1!n_2! \ldots n_t!}$$

con $n_1 + n_2 + \cdots + n_t = n$.

b. (Ej. 3 del 1^{er} parcial del 2001) Determinar el coeficiente de x^4 en el desarrollo de $\left(x^3 - x^2 + x - 1\right)^6$.

c. (Ej. 1b del 1^{er} parcial 2018 sem. impar) Hallar el coeficiente de x^6 en $(2 + 2x + 2x^2 + 2x^3 + 2x^4 + x^5)^5$.

Ejercicio 3.

(a) Hallar la cantidad de soluciones naturales de la ecuación $x_1+x_2+x_3+x_4+x_5+x_6+x_7=4$.

(b) ¿Cuántas soluciones hay si se reemplaza el signo $=$ por el signo $<$?
O sea: hallar la cantidad de soluciones naturales de la inecuación $x_1+x_2+x_3+x_4+x_5+x_6+x_7<4$.

(c) Hallar la cantidad de soluciones naturales de la ecuación $x_1+x_2+x_3+x_4=15$ tal que se cumplen simultáneamente las siguientes condiciones $x_1\geq 3$ y $x_4\geq 3$.

a. $CR(7,4) = C(7+4-1,4) = C(10,4)$

$\downarrow$ cant de vars

$= \frac{10!}{6!\,4!} = \frac{7\cdot 8\cdot 9\cdot 10}{4!}$

$= \frac{7\cdot \cancel{8}\cdot \cancel{9}^{3}\cdot 10}{\cancel{2}\cdot\cancel{3}\cdot\cancel{4}} = 210$

b. Podemos aplicar regla de la suma y considerar

$x_1+x_2+\dots+x_7=0 \to CR(7,0)=C(7-1,0)=1$

$x_1+\dots+x_7=1 \to CR(7,1)=C(7,1)=7$

$x_1+\dots+x_7=2 \to CR(7,2)=C(8,2)=7\cdot 4$

$x_1+\dots+x_7=3 \to CR(7,3)=C(9,3)=\frac{7\cdot\cancel{8}^{4}\cdot\cancel{9}^{3}}{\cancel{6}}$
$=7\cdot 4\cdot 3$

En total hay $1+7+7\cdot 4+7\cdot 4\cdot 3$

Hallemos las sols de

$x_1+\dots+x_7=r$ con $r=0,1,2,3$

si consideremos $\underline{x_8} = 3-(x_1+x_2+\dots+x_7)$

$\to x_8\geq 0$

Es lo mismo hallar soluciones

de $x_1+\dots+x_7+x_8=3$

y la cantidad es $\boxed{CR(8,3)=\sum_{i=0}^{3} CR(7,i)}$

$n=7,\ r=3$

En general vale

$$\sum_{i=0}^{r} CR(n,i) = CR(n+1,r)$$

Ejercicio 5.

(a) Hallar el coeficiente en x^5 en el desarrollo de $(x^5+x-1)^{10}$.

(b) Hallar el coeficiente en xy^3z^5 del polinomio $(2x+4y+2z+5)^{14}$.

<u>Teo del multinomio:</u>

Para n y t positivos, repasar la demostración (hay al menos dos posibles demostraciones) que el coeficiente de $x_1^{n_1}\, x_2^{n_2} \ldots x_t^{n_t}$ en $(x_1+x_2+\cdots+x_t)^n$ es

$$\frac{n!}{n_1!n_2!\ldots n_t!}$$

con $n_1+n_2+\cdots+n_t=n$.

5.a. $\boxed{x_1 = x^5,\quad x_2 = x,\quad x_3 = -1}$ suponemos eso

$$(x^5+x-1)^{10} = (x_1+x_2+x_3)^{10}$$

$$= x_1^{10} + x_1^9 \cdot x_2 \cdot \frac{10!}{9!} + \cdots + x_1^2 x_2^3 \cdot x_3^5 \left(\frac{10!}{2!\cdot 3!\cdot 5!}\right) + \ldots$$

¿De qué maneras puede aparecer x^5 como

$\boxed{x_1^{n_1}\, x_2^{n_2} \cdot x_3^{n_3} \quad \text{con} \quad n_1+n_2+n_3=10}$?

```
? (x^5+x-1)^10
%2 = x^50 + 10*x^46 - 10*x^45 + 45*x^42 - 90*x^41 + 45*x^40 + 120*x^38 - 360*x^37 + 360*x^36 - 120*x^35 + 210*x^34 - 840*x^33 + 1260*x^32 - 840*x^31 + 462*x^30 - 1260*x^29 + 2520*x^
28 - 2520*x^27 + 1470*x^26 - 1512*x^25 + 3150*x^24 - 4200*x^23 + 3270*x^22 - 2100*x^21 + 2730*x^20 - 4200*x^19 + 4245*x^18 - 2880*x^17 + 2100*x^16 - 2640*x^15 + 3160*x^14 - 2610*x^1
3 + 1620*x^12 - 1200*x^11 + 1306*x^10 - 1270*x^9 + 885*x^8 - 480*x^7 + 300*x^6 - 262*x^5 + 210*x^4 - 120*x^3 + 45*x^2 - 10*x + 1
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? (x1+x2+x3)^10
%1 = x1^10 + (10*x2 + 10*x3)*x1^9 + (45*x2^2 + 90*x3*x2 + 45*x3^2)*x1^8 + (120*x2^3 + 360*x3*x2^2 + 360*x3^2*x2 + 120*x3^3)*x1^7 + (210*x2^4 + 840*x3*x2^3 + 1260*x3^2*x2^2 + 840*x3^
3*x2 + 210*x3^4)*x1^6 + (252*x2^5 + 1260*x3*x2^4 + 2520*x3^2*x2^3 + 2520*x3^3*x2^2 + 1260*x3^4*x2 + 252*x3^5)*x1^5 + (210*x2^6 + 1260*x3*x2^5 + 3150*x3^2*x2^4 + 4200*x3^3*x2^3 + 315
0*x3^4*x2^2 + 1260*x3^5*x2 + 210*x3^6)*x1^4 + (120*x2^7 + 840*x3*x2^6 + 2520*x3^2*x2^5 + 4200*x3^3*x2^4 + 4200*x3^4*x2^3 + 2520*x3^5*x2^2 + 840*x3^6*x2 + 120*x3^7)*x1^3 + (45*x2^8 +
360*x3*x2^7 + 1260*x3^2*x2^6 + 2520*x3^3*x2^5 + 3150*x3^4*x2^4 + 2520*x3^5*x2^3 + 1260*x3^6*x2^2 + 360*x3^7*x2 + 45*x3^8)*x1^2 + (10*x2^9 + 90*x3*x2^8 + 360*x3^2*x2^7 + 840*x3^3*x2^
6 + 1260*x3^4*x2^5 + 1260*x3^5*x2^4 + 840*x3^6*x2^3 + 360*x3^7*x2^2 + 90*x3^8*x2 + 10*x3^9)*x1 + (x2^10 + 10*x3*x2^9 + 45*x3^2*x2^8 + 120*x3^3*x2^7 + 210*x3^4*x2^6 + 252*x3^5*x2^5 +
210*x3^6*x2^4 + 120*x3^7*x2^3 + 45*x3^8*x2^2 + 10*x3^9*x2 + x3^10)
```

$x_1 = x^5,\ x_2 = x,\ x_3 = -1$

$- x^5 = x_1^1 \cdot x_2^0 \cdot x_3^9$

$- x^5 = x_1^0 \cdot x_2^5 \cdot x_3^5$

el coeficiente que multiplica a $x_1^1 \cdot x_2^0 \cdot x_3^9$

$$\frac{10!}{1!\,0!\,9!} = 10$$

el coeficiente que multiplica a $x_1^0 \cdot x_2^5 \cdot x_3^5$

$$\frac{10!}{5!\,5!} = \frac{6 \cdot 7 \cdot 8 \cdot 9 \cdot 10}{5!} = \frac{\cancel{6} \cdot 7 \cdot \cancel{8}^{2} \cdot 9 \cdot \cancel{10}^{2}}{\cancel{6} \cdot \cancel{4} \cdot \cancel{5}} = 4 \cdot 7 \cdot 9 = 28 \cdot 9 = 252$$

Entonces el coeficiente de x^5 tiene que ser $-10 - 252 = -262$

$(x^5 + x - 1)^{10}$

$\square\, x^5 = (x^5)^{n_1} \cdot (x)^{n_2} \cdot (-1)^{n_3} = (-1)^{n_3} x^{(5 \cdot n_1 + n_2)}$

$$\boxed{n_1 + n_2 + n_3 = 10}$$

$\begin{cases} 5n_1 + n_2 = 5 \\ n_1 + n_2 + n_3 = 10 \end{cases}$ $\boxed{n_1, n_2, n_3 \in \mathbb{N}}$

Como $n_1, n_2, n_3 \geq 0$ $n_1 = 0$ o $n_1 = 1$

Si $n_1 = 0 \Rightarrow n_2 = 5$ $n_3 = 5$

Si $n_1 = 1 \Rightarrow n_2 = 0$ $n_3 = 9$

Si queremos el coeficiente de x^{14} en el desarrollo de $(x^5+x-1)^{10}$

$\square\, x^{14} = (x^5)^{n_1} \cdot (x)^{n_2} (-1)^{n_3} = (-1)^{n_3} \cdot x^{5n_1+n_2}$

$\begin{cases} 5n_1+n_2 = 14 \\ n_1+n_2+n_3 = 10 \end{cases}$ $\quad \begin{cases} n_1=2,\ n_2=4,\ n_3=4 \\ n_1=1,\ n_2=9,\ n_3=0 \end{cases}$

El coef de $(x^5)^2 (x)^4 (-1)^4$

es $\dfrac{10!}{2!\,4!\,4!} = \dfrac{5\cdot\cancel{6}\cdot 7\cdot\cancel{8}^{2}\cdot 9\cdot\cancel{10}^{5}}{\cancel{2!}\cdot\cancel{4!}\cdot\cancel{4}} = 2\cdot 3^2\cdot 5^2\cdot 7$ $= 3150$

El coef de $(x^5)^1 (x)^9 (-1)^0$

$\dfrac{10!}{9!\,1!\,0!} = 10$

Entonces el coef de x^{14} es $3150+10 = \boxed{3160}$

(b) Hallar el coeficiente en xy^3z^5 del polinomio $(2x+4y+2z+5)^{14}$.

$(x_1+x_2+x_3+x_4)^{14}$

$x_1 = 2x,\ x_2 = 4y,\ x_3 = 2z,\ x_4 = 5$

$x_1^{n_1}\, x_2^{n_2} \cdot x_3^{n_3} \cdot x_4^{n_4} = (2x)^{n_1} \cdot (4y)^{n_2} (2z)^{n_3} (5)^{n_4}$ $\quad n_1+n_2+n_3+n_4=14$

$= \boxed{2^{n_1}\cdot 4^{n_2}\cdot 2^{n_3}\cdot 5^{n_4}}\ x^{n_1}\cdot y^{n_2}\cdot z^{n_3}$

$= \square \cdot x\cdot y^3\cdot z^5$

Entonces $n_1=1,\ n_2=3,\ n_3=5,\ n_4=5$

El coef de $x_1^1\, x_2^3\, x_3^5\, x_4^5$ es $\dfrac{14!}{3!\,5!\,5!}$

```
? (x1+x2+x3+x4)^14
%3 = x1^14 + (14*x2 + (14*x3 + 14*x4))*x1^13 + (91*x2^2 + (182*x3 + 182*x4)*x2 + (91*x3^2 + 182*x4*x3 + 91*x4^2))*x1^12 + (364*x2^3 + (1092*x3 + 1092*x4)*x2^2 + (1092*x3^2 + 2184*x4*x3 + 1092*x4^2)*x2 + (364*x3^3 + 1092*x4*x3^2
+ 1092*x4^2*x3 + 364*x4^3))*x1^11 + (1001*x2^4 + (4004*x3 + 4004*x4)*x2^3 + (6006*x3^2 + 12012*x4*x3 + 6006*x4^2)*x2^2 + (4004*x3^3 + 12012*x4*x3^2 + 12012*x4^2*x3 + 4004*x4^3)*x2 + (1001*x3^4 + 4004*x4*x3^3 + 6006*x4^2*x3^2 + 4
004*x4^3*x3 + 1001*x4^4))*x1^10 + (2002*x2^5 + (10010*x3 + 10010*x4)*x2^4 + (20020*x3^2 + 40040*x4*x3 + 20020*x4^2)*x2^3 + (20020*x3^3 + 60060*x4*x3^2 + 60060*x4^2*x3 + 20020*x4^3)*x2^2 + (10010*x3^4 + 40040*x4*x3^3 + 60060*x4^2
*x3^2 + 40040*x4^3*x3 + 10010*x4^4)*x2 + (2002*x3^5 + 10010*x4*x3^4 + 20020*x4^2*x3^3 + 20020*x4^3*x3^2 + 10010*x4^4*x3 + 2002*x4^5))*x1^9 + (3003*x2^6 + (18018*x3 + 18018*x4)*x2^5 + (45045*x3^2 + 90090*x4*x3 + 45045*x4^2)*x2^4
+ (60060*x3^3 + 180180*x4*x3^2 + 180180*x4^2*x3 + 60060*x4^3)*x2^3 + (45045*x3^4 + 180180*x4*x3^3 + 270270*x4^2*x3^2 + 180180*x4^3*x3 + 45045*x4^4)*x2^2 + (18018*x3^5 + 90090*x4*x3^4 + 180180*x4^2*x3^3 + 180180*x4^3*x3^2 + 90090
*x4^4*x3 + 18018*x4^5)*x2 + (3003*x3^6 + 18018*x4*x3^5 + 45045*x4^2*x3^4 + 60060*x4^3*x3^3 + 45045*x4^4*x3^2 + 18018*x4^5*x3 + 3003*x4^6))*x1^8 + (3432*x2^7 + (24024*x3 + 24024*x4)*x2^6 + (72072*x3^2 + 144144*x4*x3 + 72072*x4^2)
*x2^5 + (120120*x3^3 + 360360*x4*x3^2 + 360360*x4^2*x3 + 120120*x4^3)*x2^4 + (120120*x3^4 + 480480*x4*x3^3 + 720720*x4^2*x3^2 + 480480*x4^3*x3 + 120120*x4^4)*x2^3 + (72072*x3^5 + 360360*x4*x3^4 + 720720*x4^2*x3^3 + 720720*x4^3*x
3^2 + 360360*x4^4*x3 + 72072*x4^5)*x2^2 + (24024*x3^6 + 144144*x4*x3^5 + 360360*x4^2*x3^4 + 480480*x4^3*x3^3 + 360360*x4^4*x3^2 + 144144*x4^5*x3 + 24024*x4^6)*x2 + (3432*x3^7 + 24024*x4*x3^6 + 72072*x4^2*x3^5 + 120120*x4^3*x3^4
+ 120120*x4^4*x3^3 + 72072*x4^5*x3^2 + 24024*x4^6*x3 + 3432*x4^7))*x1^7 + (3003*x2^8 + (24024*x3 + 24024*x4)*x2^7 + (84084*x3^2 + 168168*x4*x3 + 84084*x4^2)*x2^6 + (168168*x3^3 + 504504*x4*x3^2 + 504504*x4^2*x3 + 168168*x4^3)*x2
^5 + (210210*x3^4 + 840840*x4*x3^3 + 1261260*x4^2*x3^2 + 840840*x4^3*x3 + 210210*x4^4)*x2^4 + (168168*x3^5 + 840840*x4*x3^4 + 1681680*x4^2*x3^3 + 1681680*x4^3*x3^2 + 840840*x4^4*x3 + 168168*x4^5)*x2^3 + (84084*x3^6 + 504504*x4*x
3^5 + 1261260*x4^2*x3^4 + 1681680*x4^3*x3^3 + 1261260*x4^4*x3^2 + 504504*x4^5*x3 + 84084*x4^6)*x2^2 + (24024*x3^7 + 168168*x4*x3^6 + 504504*x4^2*x3^5 + 840840*x4^3*x3^4 + 840840*x4^4*x3^3 + 504504*x4^5*x3^2 + 168168*x4^6*x3 + 24
024*x4^7)*x2 + (3003*x3^8 + 24024*x4*x3^7 + 84084*x4^2*x3^6 + 168168*x4^3*x3^5 + 210210*x4^4*x3^4 + 168168*x4^5*x3^3 + 84084*x4^6*x3^2 + 24024*x4^7*x3 + 3003*x4^8))*x1^6 + (2002*x2^9 + (18018*x3 + 18018*x4)*x2^8 + (72072*x3^2 +
144144*x4*x3 + 72072*x4^2)*x2^7 + (168168*x3^3 + 504504*x4*x3^2 + 504504*x4^2*x3 + 168168*x4^3)*x2^6 + (252252*x3^4 + 1009008*x4*x3^3 + 1513512*x4^2*x3^2 + 1009008*x4^3*x3 + 252252*x4^4)*x2^5 + (252252*x3^5 + 1261260*x4*x3^4 + 2
522520*x4^2*x3^3 + 2522520*x4^3*x3^2 + 1261260*x4^4*x3 + 252252*x4^5)*x2^4 + (168168*x3^6 + 1009008*x4*x3^5 + 2522520*x4^2*x3^4 + 3363360*x4^3*x3^3 + 2522520*x4^4*x3^2 + 1009008*x4^5*x3 + 168168*x4^6)*x2^3 + (72072*x3^7 + 504504
*x4*x3^6 + 1513512*x4^2*x3^5 + 2522520*x4^3*x3^4 + 2522520*x4^4*x3^3 + 1513512*x4^5*x3^2 + 504504*x4^6*x3 + 72072*x4^7)*x2^2 + (18018*x3^8 + 144144*x4*x3^7 + 504504*x4^2*x3^6 + 1009008*x4^3*x3^5 + 1261260*x4^4*x3^4 + 1009008*x4^
5*x3^3 + 504504*x4^6*x3^2 + 144144*x4^7*x3 + 18018*x4^8)*x2 + (2002*x3^9 + 18018*x4*x3^8 + 72072*x4^2*x3^7 + 168168*x4^3*x3^6 + 252252*x4^4*x3^5 + 252252*x4^5*x3^4 + 168168*x4^6*x3^3 + 72072*x4^7*x3^2 + 18018*x4^8*x3 + 2002*x4^
9))*x1^5 + (1001*x2^10 + (10010*x3 + 10010*x4)*x2^9 + (45045*x3^2 + 90090*x4*x3 + 45045*x4^2)*x2^8 + (120120*x3^3 + 360360*x4*x3^2 + 360360*x4^2*x3 + 120120*x4^3)*x2^7 + (210210*x3^4 + 840840*x4*x3^3 + 1261260*x4^2*x3^2 + 840840
*x4^3*x3 + 210210*x4^4)*x2^6 + (252252*x3^5 + 1261260*x4*x3^4 + 2522520*x4^2*x3^3 + 2522520*x4^3*x3^2 + 1261260*x4^4*x3 + 252252*x4^5)*x2^5 + (210210*x3^6 + 1261260*x4*x3^5 + 3153150*x4^2*x3^4 + 4204200*x4^3*x3^3 + 3153150*x4^4*
x3^2 + 1261260*x4^5*x3 + 210210*x4^6)*x2^4 + (120120*x3^7 + 840840*x4*x3^6 + 2522520*x4^2*x3^5 + 4204200*x4^3*x3^4 + 4204200*x4^4*x3^3 + 2522520*x4^5*x3^2 + 840840*x4^6*x3 + 120120*x4^7)*x2^3 + (45045*x3^8 + 360360*x4*x3^7 + 126
1260*x4^2*x3^6 + 2522520*x4^3*x3^5 + 3153150*x4^4*x3^4 + 2522520*x4^5*x3^3 + 1261260*x4^6*x3^2 + 360360*x4^7*x3 + 45045*x4^8)*x2^2 + (10010*x3^9 + 90090*x4*x3^8 + 360360*x4^2*x3^7 + 840840*x4^3*x3^6 + 1261260*x4^4*x3^5 + 1261260
*x4^5*x3^4 + 840840*x4^6*x3^3 + 360360*x4^7*x3^2 + 90090*x4^8*x3 + 10010*x4^9)*x2 + (1001*x3^10 + 10010*x4*x3^9 + 45045*x4^2*x3^8 + 120120*x4^3*x3^7 + 210210*x4^4*x3^6 + 252252*x4^5*x3^5 + 210210*x4^6*x3^4 + 120120*x4^7*x3^3 + 4
5045*x4^8*x3^2 + 10010*x4^9*x3 + 1001*x4^10))*x1^4 + (364*x2^11 + (4004*x3 + 4004*x4)*x2^10 + (20020*x3^2 + 40040*x4*x3 + 20020*x4^2)*x2^9 + (60060*x3^3 + 180180*x4*x3^2 + 180180*x4^2*x3 + 60060*x4^3)*x2^8 + (120120*x3^4 + 48048
0*x4*x3^3 + 720720*x4^2*x3^2 + 480480*x4^3*x3 + 120120*x4^4)*x2^7 + (168168*x3^5 + 840840*x4*x3^4 + 1681680*x4^2*x3^3 + 1681680*x4^3*x3^2 + 840840*x4^4*x3 + 168168*x4^5)*x2^6 + (168168*x3^6 + 1009008*x4*x3^5 + 2522520*x4^2*x3^4
+ 3363360*x4^3*x3^3 + 2522520*x4^4*x3^2 + 1009008*x4^5*x3 + 168168*x4^6)*x2^5 + (120120*x3^7 + 840840*x4*x3^6 + 2522520*x4^2*x3^5 + 4204200*x4^3*x3^4 + 4204200*x4^4*x3^3 + 2522520*x4^5*x3^2 + 840840*x4^6*x3 + 120120*x4^7)*x2^4 +
(60060*x3^8 + 480480*x4*x3^7 + 1681680*x4^2*x3^6 + 3363360*x4^3*x3^5 + 4204200*x4^4*x3^4 + 3363360*x4^5*x3^3 + 1681680*x4^6*x3^2 + 480480*x4^7*x3 + 60060*x4^8)*x2^3 + (20020*x3^9 + 180180*x4*x3^8 + 720720*x4^2*x3^7 + 1681680*x4^
3*x3^6 + 2522520*x4^4*x3^5 + 2522520*x4^5*x3^4 + 1681680*x4^6*x3^3 + 720720*x4^7*x3^2 + 180180*x4^8*x3 + 20020*x4^9)*x2^2 + (4004*x3^10 + 40040*x4*x3^9 + 180180*x4^2*x3^8 + 480480*x4^3*x3^7 + 840840*x4^4*x3^6 + 1009008*x4^5*x3^5
+ 840840*x4^6*x3^4 + 480480*x4^7*x3^3 + 180180*x4^8*x3^2 + 40040*x4^9*x3 + 4004*x4^10)*x2 + (364*x3^11 + 4004*x4*x3^10 + 20020*x4^2*x3^9 + 60060*x4^3*x3^8 + 120120*x4^4*x3^7 + 168168*x4^5*x3^6 + 168168*x4^6*x3^5 + 120120*x4^7*x3
^4 + 60060*x4^8*x3^3 + 20020*x4^9*x3^2 + 4004*x4^10*x3 + 364*x4^11))*x1^3 + (91*x2^12 + (1092*x3 + 1092*x4)*x2^11 + (6006*x3^2 + 12012*x4*x3 + 6006*x4^2)*x2^10 + (20020*x3^3 + 60060*x4*x3^2 + 60060*x4^2*x3 + 20020*x4^3)*x2^9 +
(45045*x3^4 + 180180*x4*x3^3 + 270270*x4^2*x3^2 + 180180*x4^3*x3 + 45045*x4^4)*x2^8 + (72072*x3^5 + 360360*x4*x3^4 + 720720*x4^2*x3^3 + 720720*x4^3*x3^2 + 360360*x4^4*x3 + 72072*x4^5)*x2^7 + (84084*x3^6 + 504504*x4*x3^5 + 126126
0*x4^2*x3^4 + 1681680*x4^3*x3^3 + 1261260*x4^4*x3^2 + 504504*x4^5*x3 + 84084*x4^6)*x2^6 + (72072*x3^7 + 504504*x4*x3^6 + 1513512*x4^2*x3^5 + 2522520*x4^3*x3^4 + 2522520*x4^4*x3^3 + 1513512*x4^5*x3^2 + 504504*x4^6*x3 + 72072*x4^
7)*x2^5 + (45045*x3^8 + 360360*x4*x3^7 + 1261260*x4^2*x3^6 + 2522520*x4^3*x3^5 + 3153150*x4^4*x3^4 + 2522520*x4^5*x3^3 + 1261260*x4^6*x3^2 + 360360*x4^7*x3 + 45045*x4^8)*x2^4 + (20020*x3^9 + 180180*x4*x3^8 + 720720*x4^2*x3^7 + 1
681680*x4^3*x3^6 + 2522520*x4^4*x3^5 + 2522520*x4^5*x3^4 + 1681680*x4^6*x3^3 + 720720*x4^7*x3^2 + 180180*x4^8*x3 + 20020*x4^9)*x2^3 + (6006*x3^10 + 60060*x4*x3^9 + 270270*x4^2*x3^8 + 720720*x4^3*x3^7 + 1261260*x4^4*x3^6 + 151351
2*x4^5*x3^5 + 1261260*x4^6*x3^4 + 720720*x4^7*x3^3 + 270270*x4^8*x3^2 + 60060*x4^9*x3 + 6006*x4^10)*x2^2 + (1092*x3^11 + 12012*x4*x3^10 + 60060*x4^2*x3^9 + 180180*x4^3*x3^8 + 360360*x4^4*x3^7 + 504504*x4^5*x3^6 + 504504*x4^6*x3^
5 + 360360*x4^7*x3^4 + 180180*x4^8*x3^3 + 60060*x4^9*x3^2 + 12012*x4^10*x3 + 1092*x4^11)*x2 + (91*x3^12 + 1092*x4*x3^11 + 6006*x4^2*x3^10 + 20020*x4^3*x3^9 + 45045*x4^4*x3^8 + 72072*x4^5*x3^7 + 84084*x4^6*x3^6 + 72072*x4^7*x3^5
+ 45045*x4^8*x3^4 + 20020*x4^9*x3^3 + 6006*x4^10*x3^2 + 1092*x4^11*x3 + 91*x4^12))*x1^2 + (14*x2^13 + (182*x3 + 182*x4)*x2^12 + (1092*x3^2 + 2184*x4*x3 + 1092*x4^2)*x2^11 + (4004*x3^3 + 12012*x4*x3^2 + 12012*x4^2*x3 + 4004*x4^3)
*x2^10 + (10010*x3^4 + 40040*x4*x3^3 + 60060*x4^2*x3^2 + 40040*x4^3*x3 + 10010*x4^4)*x2^9 + (18018*x3^5 + 90090*x4*x3^4 + 180180*x4^2*x3^3 + 180180*x4^3*x3^2 + 90090*x4^4*x3 + 18018*x4^5)*x2^8 + (24024*x3^6 + 144144*x4*x3^5 + 36
0360*x4^2*x3^4 + 480480*x4^3*x3^3 + 360360*x4^4*x3^2 + 144144*x4^5*x3 + 24024*x4^6)*x2^7 + (24024*x3^7 + 168168*x4*x3^6 + 504504*x4^2*x3^5 + 840840*x4^3*x3^4 + 840840*x4^4*x3^3 + 504504*x4^5*x3^2 + 168168*x4^6*x3 + 24024*x4^7)*x
2^6 + (18018*x3^8 + 144144*x4*x3^7 + 504504*x4^2*x3^6 + 1009008*x4^3*x3^5 + 1261260*x4^4*x3^4 + 1009008*x4^5*x3^3 + 504504*x4^6*x3^2 + 144144*x4^7*x3 + 18018*x4^8)*x2^5 + (10010*x3^9 + 90090*x4*x3^8 + 360360*x4^2*x3^7 + 840840*x
4^3*x3^6 + 1261260*x4^4*x3^5 + 1261260*x4^5*x3^4 + 840840*x4^6*x3^3 + 360360*x4^7*x3^2 + 90090*x4^8*x3 + 10010*x4^9)*x2^4 + (4004*x3^10 + 40040*x4*x3^9 + 180180*x4^2*x3^8 + 480480*x4^3*x3^7 + 840840*x4^4*x3^6 + 1009008*x4^5*x3^5
+ 840840*x4^6*x3^4 + 480480*x4^7*x3^3 + 180180*x4^8*x3^2 + 40040*x4^9*x3 + 4004*x4^10)*x2^3 + (1092*x3^11 + 12012*x4*x3^10 + 60060*x4^2*x3^9 + 180180*x4^3*x3^8 + 360360*x4^4*x3^7 + 504504*x4^5*x3^6 + 504504*x4^6*x3^5 + 360360*x4
^7*x3^4 + 180180*x4^8*x3^3 + 60060*x4^9*x3^2 + 12012*x4^10*x3 + 1092*x4^11)*x2^2 + (182*x3^12 + 2184*x4*x3^11 + 12012*x4^2*x3^10 + 40040*x4^3*x3^9 + 90090*x4^4*x3^8 + 144144*x4^5*x3^7 + 168168*x4^6*x3^6 + 144144*x4^7*x3^5 + 9009
0*x4^8*x3^4 + 40040*x4^9*x3^3 + 12012*x4^10*x3^2 + 2184*x4^11*x3 + 182*x4^12)*x2 + (14*x3^13 + 182*x4*x3^12 + 1092*x4^2*x3^11 + 4004*x4^3*x3^10 + 10010*x4^4*x3^9 + 18018*x4^5*x3^8 + 24024*x4^6*x3^7 + 24024*x4^7*x3^6 + 18018*x4^8
*x3^5 + 10010*x4^9*x3^4 + 4004*x4^10*x3^3 + 1092*x4^11*x3^2 + 182*x4^12*x3 + 14*x4^13))*x1 + (x2^14 + (14*x3 + 14*x4)*x2^13 + (91*x3^2 + 182*x4*x3 + 91*x4^2)*x2^12 + (364*x3^3 + 1092*x4*x3^2 + 1092*x4^2*x3 + 364*x4^3)*x2^11 + (1
001*x3^4 + 4004*x4*x3^3 + 6006*x4^2*x3^2 + 4004*x4^3*x3 + 1001*x4^4)*x2^10 + (2002*x3^5 + 10010*x4*x3^4 + 20020*x4^2*x3^3 + 20020*x4^3*x3^2 + 10010*x4^4*x3 + 2002*x4^5)*x2^9 + (3003*x3^6 + 18018*x4*x3^5 + 45045*x4^2*x3^4 + 60060
*x4^3*x3^3 + 45045*x4^4*x3^2 + 18018*x4^5*x3 + 3003*x4^6)*x2^8 + (3432*x3^7 + 24024*x4*x3^6 + 72072*x4^2*x3^5 + 120120*x4^3*x3^4 + 120120*x4^4*x3^3 + 72072*x4^5*x3^2 + 24024*x4^6*x3 + 3432*x4^7)*x2^7 + (3003*x3^8 + 24024*x4*x3^7
+ 84084*x4^2*x3^6 + 168168*x4^3*x3^5 + 210210*x4^4*x3^4 + 168168*x4^5*x3^3 + 84084*x4^6*x3^2 + 24024*x4^7*x3 + 3003*x4^8)*x2^6 + (2002*x3^9 + 18018*x4*x3^8 + 72072*x4^2*x3^7 + 168168*x4^3*x3^6 + 252252*x4^4*x3^5 + 252252*x4^5*x3
^4 + 168168*x4^6*x3^3 + 72072*x4^7*x3^2 + 18018*x4^8*x3 + 2002*x4^9)*x2^5 + (1001*x3^10 + 10010*x4*x3^9 + 45045*x4^2*x3^8 + 120120*x4^3*x3^7 + 210210*x4^4*x3^6 + 252252*x4^5*x3^5 + 210210*x4^6*x3^4 + 120120*x4^7*x3^3 + 45045*x4^
8*x3^2 + 10010*x4^9*x3 + 1001*x4^10)*x2^4 + (364*x3^11 + 4004*x4*x3^10 + 20020*x4^2*x3^9 + 60060*x4^3*x3^8 + 120120*x4^4*x3^7 + 168168*x4^5*x3^6 + 168168*x4^6*x3^5 + 120120*x4^7*x3^4 + 60060*x4^8*x3^3 + 20020*x4^9*x3^2 + 4004*x4
^10*x3 + 364*x4^11)*x2^3 + (91*x3^12 + 1092*x4*x3^11 + 6006*x4^2*x3^10 + 20020*x4^3*x3^9 + 45045*x4^4*x3^8 + 72072*x4^5*x3^7 + 84084*x4^6*x3^6 + 72072*x4^7*x3^5 + 45045*x4^8*x3^4 + 20020*x4^9*x3^3 + 6006*x4^10*x3^2 + 1092*x4^11*
x3 + 91*x4^12)*x2^2 + (14*x3^13 + 182*x4*x3^12 + 1092*x4^2*x3^11 + 4004*x4^3*x3^10 + 10010*x4^4*x3^9 + 18018*x4^5*x3^8 + 24024*x4^6*x3^7 + 24024*x4^7*x3^6 + 18018*x4^8*x3^5 + 10010*x4^9*x3^4 + 4004*x4^10*x3^3 + 1092*x4^11*x3^2 +
182*x4^12*x3 + 14*x4^13)*x2 + (x3^14 + 14*x4*x3^13 + 91*x4^2*x3^12 + 364*x4^3*x3^11 + 1001*x4^4*x3^10 + 2002*x4^5*x3^9 + 3003*x4^6*x3^8 + 3432*x4^7*x3^7 + 3003*x4^8*x3^6 + 2002*x4^9*x3^5 + 1001*x4^10*x3^4 + 364*x4^11*x3^3 + 91*x
4^12*x3^2 + 14*x4^13*x3 + x4^14))
```

$\Rightarrow$ el coef de $x\,y^3\,t^5$

es $\dfrac{14!}{3!\,5!\,5!} \cdot 2 \cdot 4^3 \cdot 2^5 \cdot 5^5 = \boxed{2^{12} \cdot 5^5 \cdot \dfrac{14!}{3!\,5!\,5!}}$

Ejercicio 6. (Ej. 4 del 2^{do} examen del curso 2001)
Hallar la cantidad n de palabras de 1 a 5 letras que se pueden formar usando las letras de la palabra *CASAS*. (Por ejemplo *C* y *AA* son dos palabras posibles, pero *CAC* no lo es).

Regla de la suma

$n=1$, 3 palabras C, A, S

$n=2$, 8 palabras CA, CS, AA, AC, AS
SS, SA, SC

$n=3$, ? " Permutaciones de
CAS, CAA, CSS, AAS, SSA

$$3! + \frac{3!}{2!} + \frac{3!}{2!} + \frac{3!}{2!} + \frac{3!}{2!}$$

$n=4$, ? " permutaciones de
CASA, CASS, AASS

$$\frac{4!}{2!} + \frac{4!}{2!} + \frac{4!}{2!\,2!}$$

$n=5$ $\frac{5!}{2!\,2!}$

$$3+8+6+4\cdot\frac{3!}{2!}+2\cdot\frac{4!}{2!}+\frac{4!}{2!\,2!}+\frac{5!}{2!\,2!}$$