

$$\log\left(\frac{75}{16}\right) = \log(75) - \log(16) = \log(25 \times 3) - \log(2^4) = \log(5^2) + \log(3) - 4 \log(2) = 2 \log(5) + \log(3) - 4 \log(2).$$

$$-2 \log\left(\frac{5}{9}\right) = -2 \log 5 + 2 \log(9) = -2 \log 5 + 4 \log(3)$$

$$\log\left(\frac{32}{243}\right) = \log(2^5) - \log(3^5) = 5 \log(2) - \log(3^5) = 5 \log(2) - 5 \log(3).$$

Entonces

$$\log\left(\frac{75}{16}\right) - 2 \log\left(\frac{5}{9}\right) + \log\left(\frac{32}{243}\right) = 2 \log(5) + \log(3) - 4 \log(2) - 2 \log 5 + 4 \log(3) + 5 \log(2) - 5 \log(3) = \log(2).$$