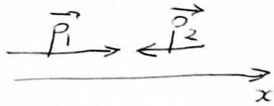
Parcial Integrador Final - 21/7/20 - Soluciones

a) $E_T = E_1 + E_2 = K_1 + m_e c^2 + K_2 + m_e c^2 = 3\, m_e c^2 + 2\, m_e c^2 = 5\, m_e c^2$

$\vec{P}_T = \vec{P}_1 + \vec{P}_2$; $(cP_1)^2 + (m_e c^2)^2 = E_1^2 = 9\,(m_e c^2)^2 \Rightarrow cP_1 = \sqrt{8}\, m_e c^2$

$(cP_2)^2 + (m_e c^2)^2 = E_2^2 = 4\,(m_e c^2)^2 \Rightarrow cP_2 = \sqrt{3}\, m_e c^2$

$\rightarrow \vec{P}_T = (P_1 - P_2)\,\hat{x} = (\sqrt{8} - \sqrt{3})\, m_e c\, \hat{x} = 1{,}0964\, m_e c\, \hat{x}$



b) $E'_{TOT} = \gamma_{CM}(E_{TOT} - \beta_{CM} \cdot cP_T)$

$cP'_{TOT} = 0 = \gamma_{CM}(cP_T - \beta_{CM}\, E_T) \Rightarrow \beta_{CM} = \frac{cP_T}{E_T} = \frac{\sqrt{8}-\sqrt{3}}{5} = 0{,}219$

c) $\rightarrow \gamma_{CM} = \frac{1}{\sqrt{1-\beta_{CM}^2}} = 1{,}025 \rightarrow E'_{TOT} = 1{,}025\,(5 - 0{,}219 \times 1{,}0964)\, m_e c^2$

$= 4{,}879\, m_e c^2$

---

a) $F = -\frac{dV}{dr} = -kR = -m \cdot \frac{v^2}{R} \rightarrow v = \sqrt{\frac{k}{m}}\, R$; $L = mvR = \sqrt{km}\, R^2 = n\hbar$

$\rightarrow R_n = \frac{\sqrt{n\hbar}}{(km)^{1/4}}$; $v_n = \sqrt{\frac{k}{m}} \cdot \frac{\sqrt{n\hbar}}{(km)^{1/4}} = \left(\frac{k}{m^3}\right)^{1/4} \sqrt{n\hbar}$

b) $E_n = V(R_n) + \frac{1}{2} m v_n^2 = \frac{1}{2} k \cdot \frac{n\hbar}{(km)^{1/2}} + \frac{1}{2} m \left(\frac{k}{m^3}\right)^{1/2} n\hbar = \frac{1}{2}\sqrt{\frac{k}{m}}\, n\hbar + \frac{1}{2}\sqrt{\frac{k}{m}}\, n\hbar = \sqrt{\frac{k}{m}}\, n$

c) Oscilador armónico cuántico en 3D $E_n = \sqrt{\frac{k}{m}}\left(n + \frac{3}{2}\right)\hbar$

$\rightarrow E_0$ diferente, igual espaciamiento de los niveles

---

a) $\int_0^\infty |A|^2 x^2 e^{-\frac{2x}{a}}\, dx = |A|^2 \cdot \frac{2!}{\left(\frac{2}{a}\right)^3} = |A|^2 \cdot \frac{a^3}{4} \Rightarrow |A| = \frac{2}{a^{3/2}}$

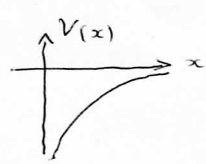
$\rightarrow A = \frac{2}{a^{3/2}}\, e^{i\varphi}$, donde la fase φ es arbitraria (real).

b) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\left[A x\, e^{-\frac{x}{a}}\right] + V(x)\left[A \cdot x \cdot e^{-\frac{x}{a}}\right] = E\left[A \cdot x\, e^{-\frac{x}{a}}\right]$

$-\frac{\hbar^2}{2m} A\, \frac{x-2a}{a^2}\, e^{-\frac{x}{a}} + V(x) \cdot A \cdot x \cdot e^{-\frac{x}{a}} = E \cdot A \cdot x\, e^{-\frac{x}{a}}$

$-\frac{\hbar^2}{2m}\, \frac{x-2a}{a^2} + V(x) \cdot x = E \cdot x$; $V(x) = E + \frac{\hbar^2}{2m}\, \frac{x-2a}{a^2 x}$

$V(\infty) = E + \frac{\hbar^2}{2ma^2} = 0 \rightarrow V(x) = -\frac{\hbar^2}{am} \cdot \frac{1}{x}$, $x > 0$



Como la derivada primera es discontinua $\Rightarrow V(0) = \infty$
Luego de la barrera infinita que impide la presencia de la partícula en $x<0$ el potencial en esa región no interesa, puede ser cualquier función arbitraria.

c) De la condición $V(\infty) = 0 \rightarrow E = -\frac{\hbar^2}{2ma^2}$

d) $\langle V(x)\rangle = \int_0^\infty |A|^2 x^2 e^{-\frac{2x}{a}} \cdot \left(-\frac{\hbar^2}{max}\right) dx = -\frac{\hbar^2 |A|^2}{ma} \int_0^\infty x \cdot e^{-\frac{2x}{a}}\, dx$

$= -\frac{\hbar^2}{ma}\, \frac{4}{a^3} \cdot \frac{1!}{\left(\frac{2}{a}\right)^2} = -\frac{\hbar^2}{ma^2}$

$\langle K(x)\rangle = E - \langle V(x)\rangle = -\frac{\hbar^2}{2ma^2} + \frac{\hbar^2}{ma^2} = \frac{\hbar^2}{2ma^2}$