Ejercicio 2

a) En el choque se conservan la energía y la cantidad de movimiento

* Antes del Choque

* Después del Choque

→ Cons de E

$$hf + mc^2 = hf' + \sqrt{p^2c^2 + (mc^2)^2}$$

→ Cons de Cant. de mov.

$\hat{\imath}$) $\frac{hf}{c} = p\cos\theta$

$\hat{\jmath}$) $\frac{hf'}{c} = p\,\mathrm{sen}\,\theta$

→ De las ecuaciones anteriores se obtiene

a) $$f' = \frac{\frac{mc^2}{h} f}{\frac{mc^2}{h} + f}$$

b) $$p = \frac{h}{c}\sqrt{f^2 + f'^2}$$

$$\mathrm{tg}\,\theta = \frac{f'}{f} = \frac{\frac{mc^2}{h}}{\frac{mc^2}{h} + f}$$

→ Para una partícula relativista de masa m, la relación entre cant. de mov. y velocidad es:

$$p = \frac{v m}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$\Longrightarrow$ c) $$v = \frac{p}{m}\frac{1}{\sqrt{1 + \left(\frac{p}{mc}\right)^2}}$$

Problema 1

1) a) $x<0$ $\quad -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = +i\hbar\frac{\partial\psi}{\partial t} \longrightarrow \psi(x,t) = \varphi(x)\,\tau(t)$

$$\tau(t) = e^{-iE/\hbar\, t}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\varphi}{dx^2} = E \rightarrow \varphi_1(x) = A e^{ikx} + B e^{-ikx} \;/\; k = \frac{\sqrt{2mE}}{\hbar}$$

$x>0$ $\quad \varphi_2(x) = C e^{ikx}$

b) $\varphi_1(x=0) = \varphi_2(x=0) \rightarrow A + B = C \longrightarrow B = C - A$

$$\left.\frac{d\varphi_1}{dx}\right|_{x=0^-} = ik(A-B) \qquad \left.\frac{d\varphi_2}{dx}\right|_{x=0^+} = ikC$$

$$ik(C - A + B) = -\frac{2mS}{\hbar^2}C \longrightarrow \cancel{2}ik(C-A) = -\frac{\cancel{2}mS}{\hbar^2}C \rightarrow C = \frac{ikA}{mS/\hbar^2 + ik}$$

Llamamos $\alpha = \frac{mS}{\hbar^2} \rightarrow C = \frac{ikA(\alpha - ik)}{\alpha^2 + k^2} = \frac{Ak(k + i\alpha)}{\alpha^2 + k^2}$

$$T(E) = \frac{C^* C}{A^* A} = \frac{k^2(k^2+\alpha^2)}{(\alpha^2+k^2)^2} = \frac{k^2}{\alpha^2+k^2} = \frac{1}{\alpha^2/k^2 + 1} \;/\; \frac{\alpha^2}{k^2} = \frac{m^2S^2}{\hbar^4}\frac{\hbar^2}{2mE}$$

$$\boxed{T(E) = \left(1 + \frac{mS^2}{2\hbar^2 E}\right)^{-1}}$$

Problema 3

3) $E_n = -\frac{\mu\, ke^2}{2ma_0}\left(\frac{1}{n^2}\right) \qquad \mu = \frac{mM}{M+m}$

$$\mu = m/2 \rightarrow E_n = -\frac{ke^2}{4a_0}\left(\frac{1}{n^2}\right) = -\frac{13{,}6}{2n^2}$$

$$L = n\hbar = |\vec{r}\wedge\vec{p}| = mvr$$

$$F = ma_c = \frac{mv^2}{r} = +\frac{ke^2}{r^2} \rightarrow r = +\frac{ke^2}{mv^2} = +\frac{ke^2}{2T} = -\frac{ke^2}{2E}$$

$$E = T + V = T - \frac{ke^2}{r} = T - \frac{\cancel{ke^2}}{\cancel{ke^2}}2T = -T$$

$$\boxed{r_n = \frac{ma_0\, n^2}{\mu}}$$