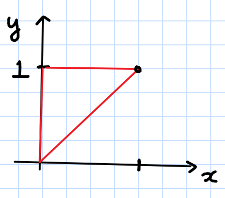
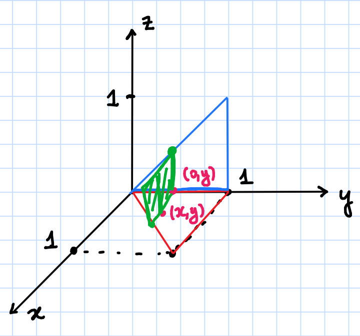
Julio 2019

$D = \{(x,y,z) \in \mathbb{R}^3 \,/\, 0 \le x \le y \le z \le 1\}$

Calcular el volumen.





$$\text{área}(D) = \int_0^1 \int_0^z \int_0^y 1 \; dx\, dy\, dz =$$

$$= \int_0^1 \int_x^1 \int_y^1 1 \; dz\, dy\, dx$$

$$\int_0^y 1\, dx = y$$

$$\int_0^z y\, dy = \frac{z^2}{2}$$

$$\int_0^1 \frac{z^2}{2} dz = \frac{z^3}{6}\Big|_0^1 = \frac{1}{6}$$

—— / ——

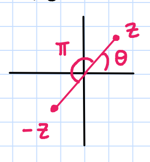
$A = \{ z \in \mathbb{C} \,/\, -z = z^2 \}$

Veamos quién es este conjunto.

$z = re^{i\theta} = r\cos\theta + ir\,\text{sen}\,\theta$

$-z = re^{i(\theta+\pi)}$

$z^2 = r^2 e^{i2\theta}$



$re^{i(\theta+\pi)} = r^2 e^{i2\theta}$

$\Rightarrow \begin{cases} r = r^2 \\ \theta + \pi = 2\theta \end{cases}$ y (como ángulos)

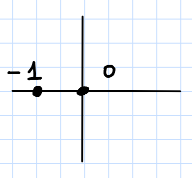
$r = r^2 \iff r = 0$ o $r = 1$

Cuando $r = 1$,

$\theta + \pi = 2\theta$

$\theta = \pi$ como ángulo

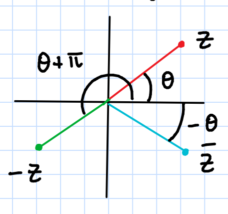
$\theta = \pi + 2k\pi$ para $k \in \mathbb{Z}$.



$0 = 0 + 0i$

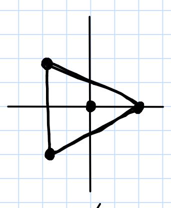
$-1 = -1 + 0i$

¿Y si el ejercicio dijera $\bar{z} = z^2$?



$re^{-i\theta} = r^2 e^{i2\theta}$

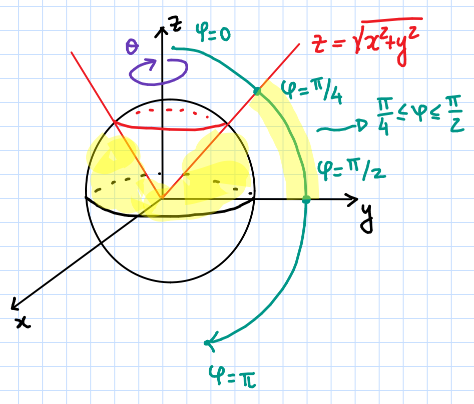
$\begin{cases} r = r^2 \\ -\theta = 2\theta \end{cases}$ i.e. $\begin{cases} r = r^2 \\ 3\theta = 0 \end{cases}$



—— / ——

Dic. 2023.

$D = \{(x,y,z) \in \mathbb{R}^3 \,/\, x^2+y^2+z^2 \le 1,$
$0 \le z \le \sqrt{x^2+y^2}\}$.



—— / ——

$(a_n)_{n\in\mathbb{N}}$ de términos positivos

Si $\sum_{n=0}^{\infty} a_n^2$ converge, entonces $\sum_{n=0}^{\infty} a_n$ converge.

¿$a_n \le a^2$?

$\sum a_n^2$ conv $\Rightarrow a_n^2 \xrightarrow[n\to\infty]{} 0 \rightarrow a_n \xrightarrow[n\to\infty]{} 0$.

$\left(\frac{1}{2}\right)^2 = \frac{1}{4} \qquad \frac{1}{2} \not< \left(\frac{1}{2}\right)^2$

Para $x > 1$, $\quad x < x^2 \quad (x^2 = x \cdot x)$

Para $0 < x < 1$, $\quad x > x^2$

$(a_n)_{n\in\mathbb{N}}$ de términos positivos.

Si $\sum^{\infty} a_n$ conv, entonces $\sum^{\infty} a_n^2$ también.

$\sum^{\infty} a_n$ conv $\Rightarrow \lim_{n\to\infty} a_n = 0 \Rightarrow$

$\rightarrow \exists n_0 \in \mathbb{N} \,/\, \forall n \ge n_0 \quad 0 \le a_n < 1$

Para $n \ge n_0$, $\quad a_n^2 < a_n$.

CRIT. COMP. $\Rightarrow \sum^{\infty} a_n^2$ conv.

—— / ——

Febrero 2023

$z = e^{i2\pi/5}$

Sabemos $|z| = 1$

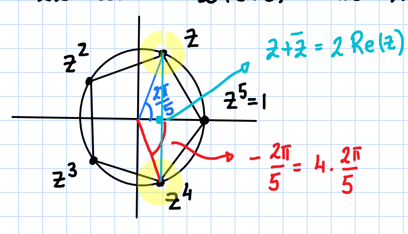
$z^5 = 1$

$z + z^4 + z^9 + z^{16} =$

$= z + z^4 + z^{5+4} + z^{15+1}$

$= z + z^4 + (z^5) \cdot z^4 + (z^5)^3 \cdot z$ (con $z^5 = 1$)

$= 2z + 2z^4 \overset{z^4 = \bar{z}}{=} 2(z + \bar{z}) = 4\,\text{Re}(z)$.



Recordar: si $|z| = 1$, $z^{-1} = \bar{z}$

dem.

$\left.\begin{array}{l} z = e^{i\theta} \\ \bar{z} = e^{-i\theta} \end{array}\right\} \Rightarrow z\bar{z} = e^{i\theta}e^{-i\theta} = e^{i(\theta-\theta)} = e^{i\cdot 0} = 1$

En el caso nuestro, $z^5 = 1$.

$z^4 \cdot z = 1 \Rightarrow z^4 = z^{-1} \Rightarrow z^4 = \bar{z}$.

—— / ——

Julio 2019

$$\lim_{(x,y)\to(0,0)} \frac{e^{x^2+y^2} + \ln(2(x^2+y^2)+1) - \frac{3}{2}(x^2+y^2) - 1}{x^2+y^2}$$

$r = \sqrt{x^2+y^2}$

$$\lim_{r\to 0} \frac{e^{r^2} + \ln(2r^2+1) - \frac{3}{2}r^2 - 1}{r^2} =$$

$$= \underbrace{\lim_{r\to 0} \frac{e^{r^2}-1}{r^2}}_{=1} + \lim_{r\to 0} \frac{\ln(2r^2+1) - \frac{3}{2}r^2}{r^2}$$

$$= 1 + \lim_{r\to 0} \frac{\ln(2r^2+1)}{r^2} - \frac{3}{2}$$

$$= 1 + \lim_{r\to 0} \frac{2r^2}{r^2} - \frac{3}{2} = 1 + 2 - \frac{3}{2} = \frac{3}{2}$$

$$\lim_{u\to 0} \frac{\ln(1+u)}{u} = 1$$

$u = r^2 \quad \lim_{u\to 0} \frac{\ln(2u+1)}{u} = \lim_{u\to 0} 2 \cdot \frac{\ln(2u+1)}{2u}$

$v = 2u \quad \lim_{v\to 0} 2 \cdot \frac{\ln(v+1)}{v} = 2.$ (con $\frac{\ln(v+1)}{v} \to 1$)

$$e^{x^2+y^2} + \ln(2(x^2+y^2)+1) - \frac{3}{2}(x^2+y^2) - 1$$

$u = x^2 + y^2$ ($e^u \sim 1+u$, $\ln(2u+1) \sim 2u$)

$$\lim_{u\to 0} \frac{e^u + \ln(2u+1) - \frac{3}{2}u - 1}{u} =$$

$$= \lim_{u\to 0} \frac{u + 2u - \frac{3}{2}u}{u} = 1 + 2 - \frac{3}{2} = \frac{3}{2}$$

Taylor:

$f(x) = P_n(x) + R_n(x)$

$\frac{f(x)}{x^n} = \frac{P_n(x)}{x^n} + \frac{R_n(x)}{x^n}$

Tma. de Taylor: $\lim_{x\to 0} \frac{R_n(x)}{x^n} = 0$

—— / ——

En $\mathbb{R}$, $\{x\}$ es cerrado.

$(0,1) = \bigcup_{x\in(0,1)} \{x\}$

↑ no es cerrado