Clase de Consulta.

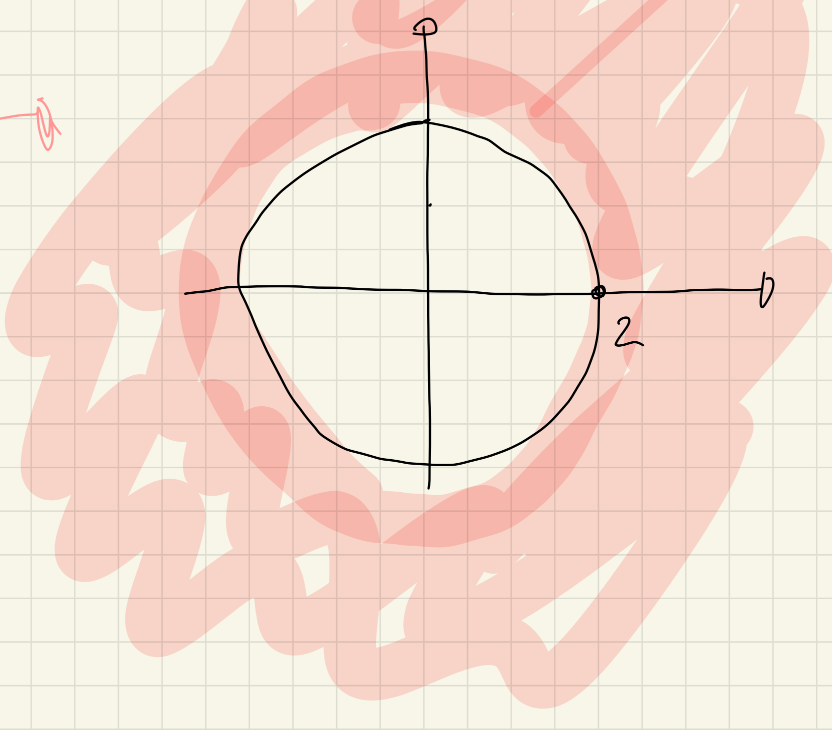
CDIVV - 2023 - 2sem

Eugenia Ellis

eellis@fing.edu.uy

Hallar el área de D.

$$D=\left\{(x,y)\in\mathbb{R}^2 : x^2+y^2\geq 4,\ \frac{x^2}{12}+\frac{y^2}{4}\leq 1,\ y\leq x,\ y\geq 0\right\}$$

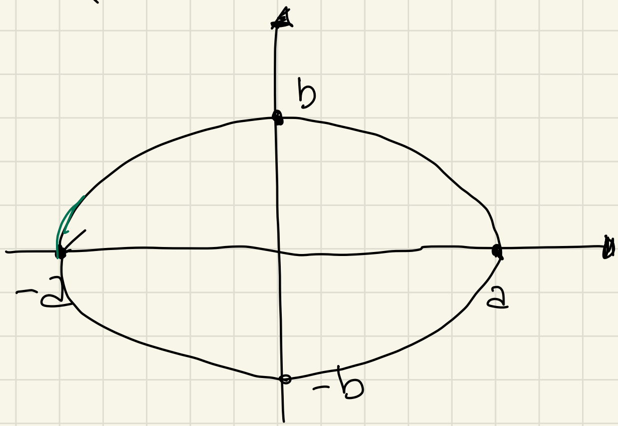


$$\frac{x^2}{12}+\frac{y^2}{4}\leq 1$$

$$x^2+y^2=r^2$$

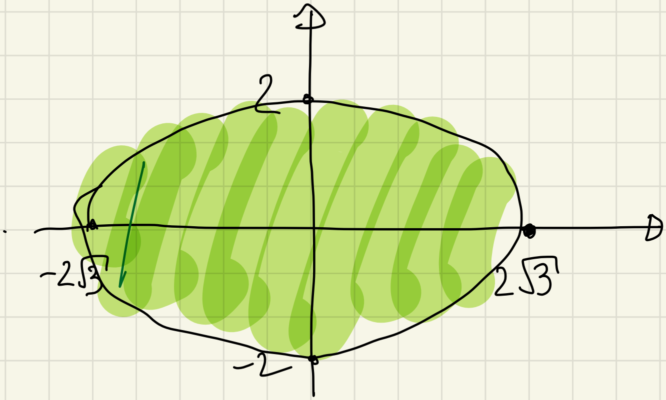
$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=1$$

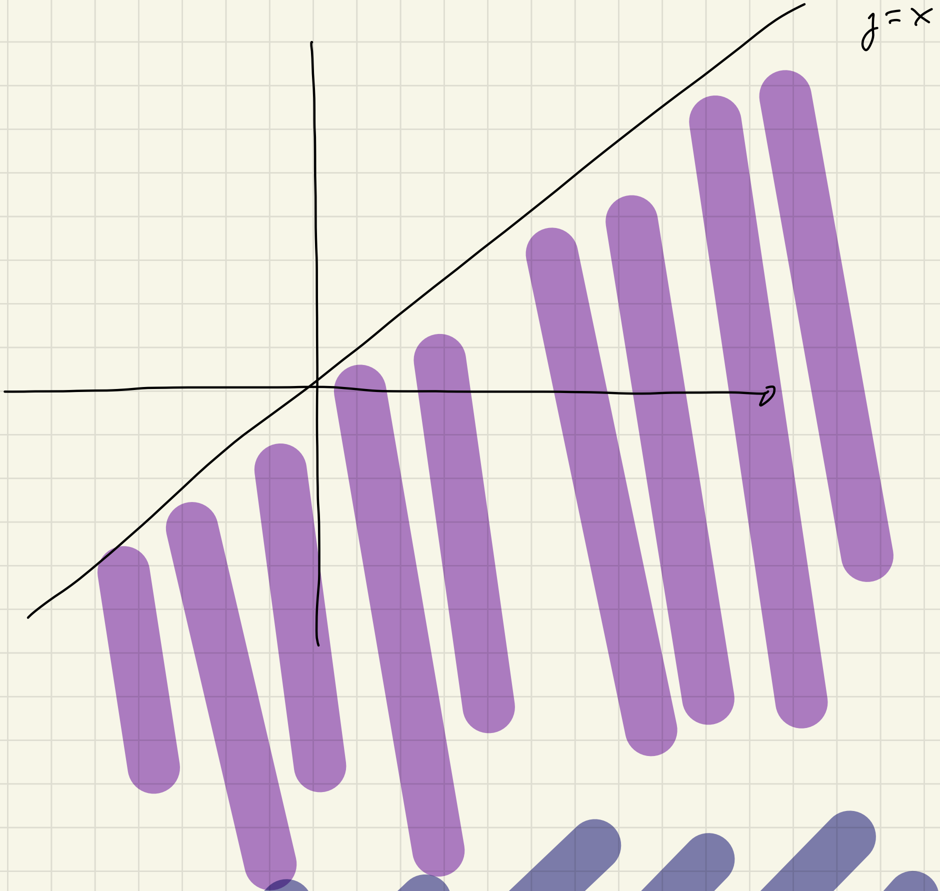
$$\left(\frac{x}{r}\right)^2+\left(\frac{y}{r}\right)^2=1$$

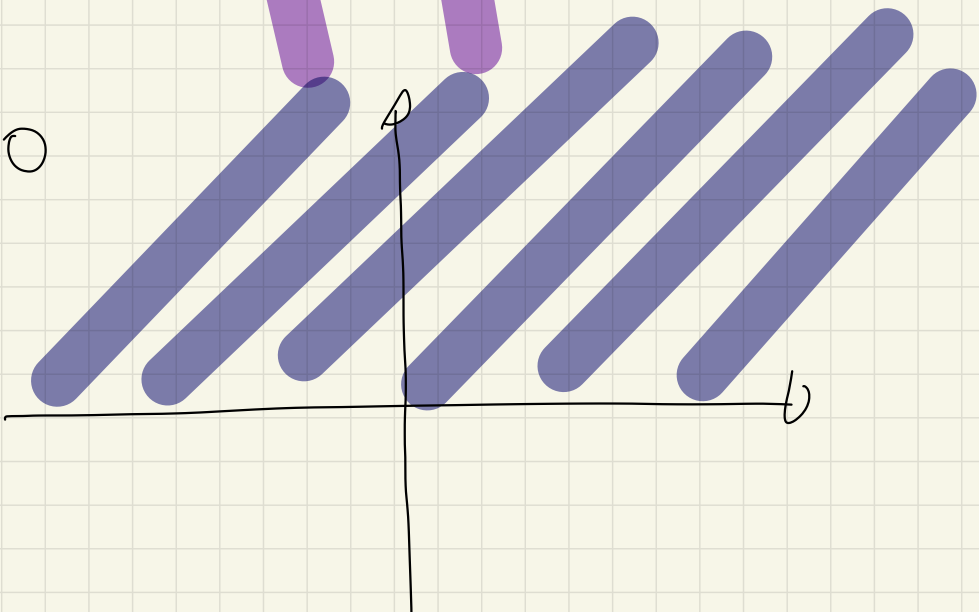


$$\frac{x^2}{(2\sqrt{3})^2}+\frac{y^2}{2^2}=1$$

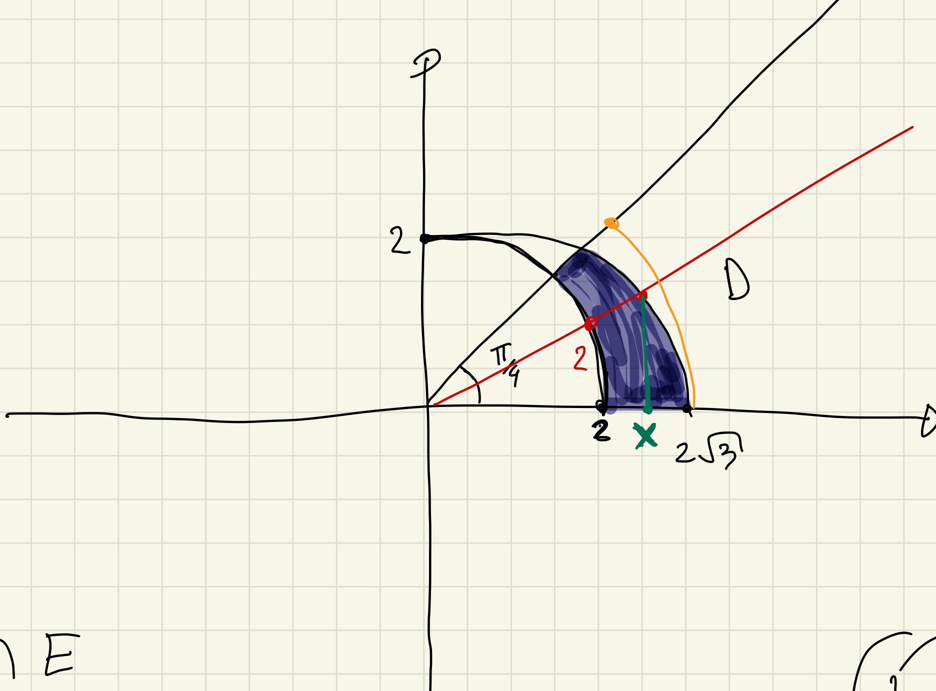
$$12=3\cdot 4=(\sqrt{3}\cdot 2)^2$$







$y \geq 0$



$C \cap E$

$= \{(0,2),(0,-2)\}$

$\boxed{x^2 + y^2 = 4}$ $\Rightarrow$ $y^2 = 4 - x^2$

$\boxed{\frac{x^2}{12} + \frac{y^2}{4} = 1}$

$\frac{x^2}{3} + y^2 = 4$

$\frac{x^2}{3} + \cancel{4} - x^2 = \cancel{4}$

$\frac{21}{3} x^2 = 0$

$A(D) = \iint_D 1$

$$A(D) = \iint_D 1 = \iint_{g^{-1}(D)} 1\,\rho\, d\rho\, d\theta$$

cv polares.

$g^{-1}(D)$

$x = \rho\cos\theta$

$y = \rho\,\text{sen}\,\theta$

$$\frac{x^2}{3} + y^2 = 4$$

$$\frac{\rho^2\cos^2\theta}{3} + \rho^2\text{sen}^2\theta = 4.$$

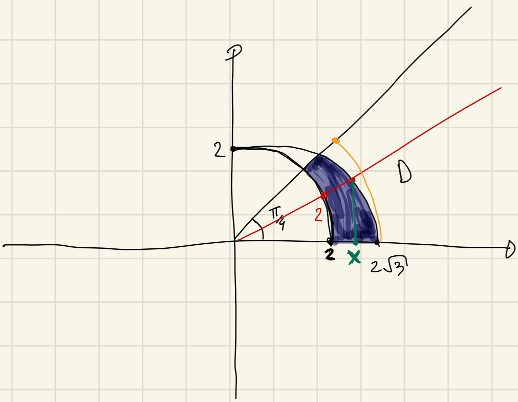
$$\rho^2\left(\frac{\cos^2\theta}{3} + \text{sen}^2\theta\right) = 4.$$

$$\rho^2\left(\frac{\cos^2\theta}{3} + \frac{\text{sen}^2\theta}{3} + \frac{2\,\text{sen}^2\theta}{3}\right) = 4$$

$$\rho^2\left(\frac{1}{3} + \frac{2\,\text{sen}^2\theta}{3}\right) = 4$$

$$\rho^2 = \frac{12}{1+2\,\text{sen}^2\theta}$$

$$\boxed{\rho = \sqrt{\frac{12}{1+2\,\text{sen}^2\theta}}}$$



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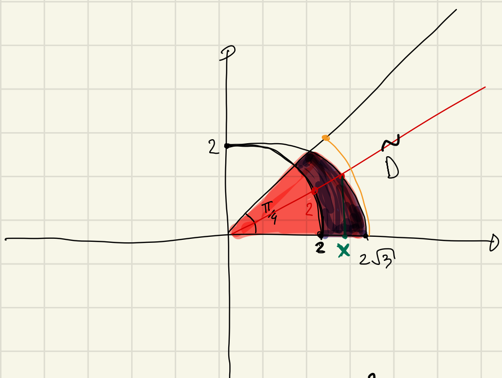

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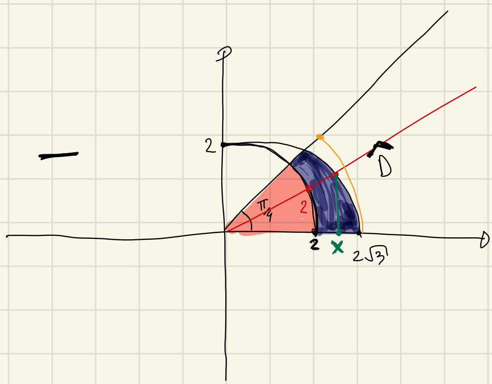


$$\tilde{D} = \left\{ (x,y) \in \mathbb{R}^2 : \ y \leq x, \ x \geq 0, \ \frac{x^2}{3} + y^2 \leq 4 \right\}$$

$$\hat{D} = \left\{ (x,y) \in \mathbb{R}^2 : \ y \leq x, \ x \geq 0 \ \ x^2 + y^2 \leq 4 \right\}$$

$$\hat{D} \cup D = \tilde{D}$$

$$\iint_D 1 = \iint_{\tilde{D}} 1 - \iint_{\hat{D}} 1$$

$$\iint_{\hat{D}} 1 \underset{\text{(v polares)}}{=} \iint_{g^{-1}(\hat{D})} \rho \, d\theta \, d\rho = \int_0^{\pi/4} \int_0^2 \rho \, d\theta \, d\rho$$

$$= \int_0^{\pi/4} \frac{\rho^2}{2} \Big|_0^2 \, d\theta$$

$$= \int_0^{\pi/4} 2 \, d\theta = 2 \cdot \theta \Big|_0^{\pi/4}$$

$$= \frac{\pi}{2}$$

$$\iint_{\tilde{D}} 1 \underset{\text{cv elíptico}}{=} \iint_{g^{-1}(\tilde{D})} 1 \cdot \underset{2\sqrt{3}}{a}\,\underset{2}{b}\,\rho \, d\theta \, d\rho$$

$x = a\rho\cos\theta$

$y = b\rho\,\mathrm{sen}\,\theta$

$$= \iint_{g^{-1}(\tilde{D})} 4\sqrt{3}\,\rho \, d\theta \, d\rho$$

$x = 2\sqrt{3}\,\rho\cos\theta$

$y = 2\rho\,\mathrm{sen}\,\theta$

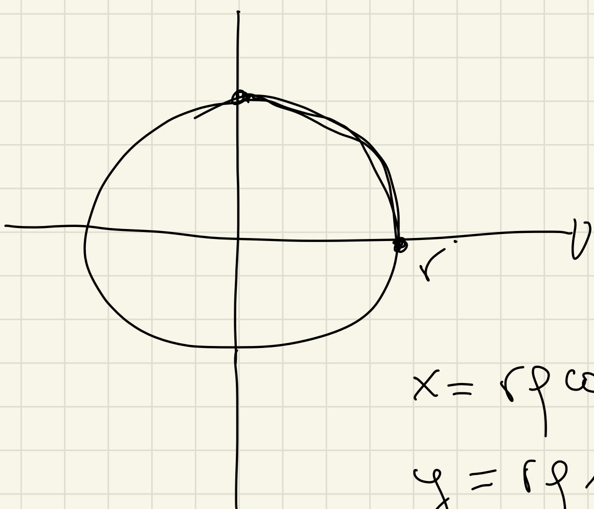
$$\underset{\text{TF}}{=} \int_0^1 \int_0^{\pi/4} 4\sqrt{3}\,\rho \, d\theta \, d\rho.$$

$$= 4\sqrt{3} \int_0^1 \int_0^{\pi/4} \rho \, d\theta \, d\rho$$

$$= \frac{4\sqrt{3}\,.\pi}{4}\int_0^1 \rho \, d\rho = \sqrt{3}\pi \left.\frac{\rho^2}{2}\right|_0^1$$

$$= \frac{\sqrt{3}\pi}{2}.$$

$$\frac{\sqrt{3}}{2}\pi - \frac{\pi}{2} = \left(\frac{\sqrt{3}-1}{2}\right)\pi$$



$x = r\rho\cos\theta$ $\quad \rho \in [0,1)$

$y = r\rho \,\mathrm{sen}\,\theta$ $\quad \theta \in [0,2\pi)$

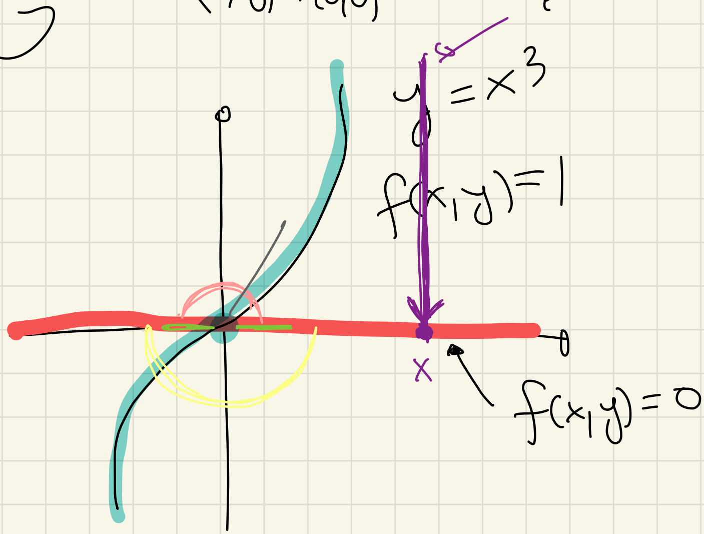
$x = \rho\cos\theta$ $\quad \rho \in [0,r)$

$y = \rho \,\mathrm{sen}\,\theta$ $\quad \theta \in [0,2\pi).$

$$f(x,y) = \begin{cases} x^3/y & y \neq 0 \\ 0 & y = 0 \end{cases}$$

Estudiar continuidad y existencia de derivadas direccionales

$$¿\exists \lim_{(x,y)\to(0,0)} f(x,y)?$$



$v = (v_1, v_2)$ $\quad v_2 \neq 0$

$$\lim_{h \to 0} \frac{f\left((a,0) + h(v_1, v_2)\right) - f(a,0)}{h}$$

$$= \lim_{h \to 0} \frac{f(a + hv_1, hv_2) - f(a,0)}{h}$$

$(a,0) \quad a \in \mathbb{R}$