Clase 27:

Derivadas parciales y direccionales.

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$f: D \longrightarrow \mathbb{R} \qquad p \in D$

$D \subseteq \mathbb{R}^2$

f es continua en $p \iff$ • $\exists \lim_{x \to p} f(x) = L$

• $L = f(p)$

Def: $f: D \longrightarrow \mathbb{R}$ función $p \in \mathring{D}$ $\quad p = (x_0, y_0)$

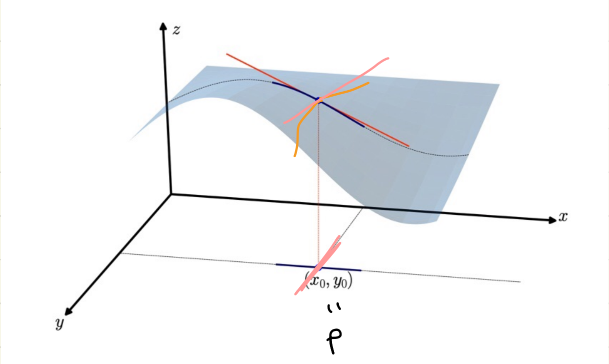
$D \subseteq \mathbb{R}^2$

La derivada parcial de f con respecto a x en $p = (x_0, y_0)$ es (si existe)

$$\boxed{\lim_{h \to 0} \frac{f(x_0 + h, y_0) - f(x_0, y_0)}{h}}$$

$$\frac{\partial f}{\partial x}(x_0, y_0) \qquad f_x(x_0, y_0)$$

Notación



Ejemplo: $f(x,y) = x^2 + xy$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \frac{f(0+h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{h^2 + h.0 - 0}{h}$$

$$= \lim_{h\to 0} h = 0$$

$$\frac{\partial f}{\partial x}(x_0,y_0) = \lim_{h\to 0} \frac{f(x_0+h, y_0) - f(x_0,y_0)}{h}$$

$$= \lim_{h \to 0} \frac{(x_0+h)^2 + (x_0+h)y_0 - x_0^2 - x_0 y_0}{h}$$

$$= \lim_{h \to 0} \frac{(x_0+h)^2 - x_0^2}{h} + \frac{(x_0+h)y_0 - x_0 y_0}{h}.$$

$$= \lim_{h \to 0} \frac{\cancel{x_0^2} + h^2 + 2hx_0 - \cancel{x_0^2}}{\cancel{h}} + \cancel{\frac{x_0 y_0 + hy_0 - x_0 y_0}{h}}$$

$$= \lim_{h \to 0} h + 2x_0 + y_0 = 2x_0 + y_0$$

$$\frac{\partial f}{\partial x}(x,y) = 2x + y$$

Def: La derivada parcial de f respecto a y en el punto (x_0, y_0) es (si existe)

$$\boxed{\lim_{h \to 0} \frac{f(x_0, y_0+h) - f(x_0, y_0)}{h}}$$

$$f(x,y) = x^2 + xy$$

$$\frac{\partial f}{\partial y}(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0, y_0+h) - f(x_0, y_0)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x_0^2} + \overbrace{x_0(y_0+h)}^{\cancel{x_0 y_0} + x_0 h} - \cancel{x_0^2} - \cancel{x_0 y_0}}{h}$$

$$= \lim_{h \to 0} \frac{x_0 \cancel{h}}{\cancel{h}} = x_0$$

$$\frac{\partial f}{\partial y}(x,y) = f_y(x,y) = x$$

Pregunta: ¿Si $f : D \longrightarrow \mathbb{R}$ tiene derivadas parciales respecto a x e y en (x_0, y_0) es f continua en (x_0, y_0)?

$D \subset \mathbb{R}^2$

~~NO~~

$$\boxed{f(x,y) = \begin{cases} 1 & \text{si } xy = 0 \\ 0 & \text{si } xy \neq 0 \end{cases}}$$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h \to 0} \frac{\overbrace{f(h,0)}^{1} - \overbrace{f(0,0)}^{1}}{h}$$

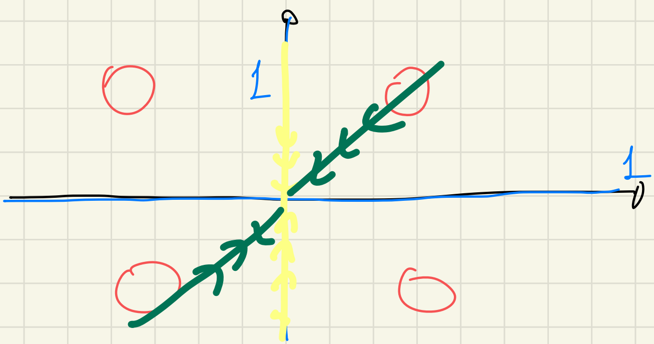
$$= \lim_{h \to 0} \frac{0}{h} = 0$$

$$\frac{\partial f(0,0)}{\partial y} = \lim_{h \to 0} \frac{f(0,h) - f(0,0)}{h}$$

$$= \lim_{h \to 0} \frac{1 - 1}{h} =$$

$$= \lim_{h \to 0} \frac{0}{h} = 0$$

$$\frac{\partial f}{\partial x}(0,0) = 0 \quad , \quad \frac{\partial f}{\partial y}(0,0) = 0$$



$$\nexists \lim_{(x,y)\to(0,0)} f(x,y)$$ porque tenemos límites direccionales distintos

$$\lim_{\substack{\boxed{x=0} \\ y\to 0}} f(x,y) = 1$$

$$\lim_{\substack{x=y \\ y\to 0}} f(x,y) = 0$$

Def: La derivada direccional de f respecto a un vector $v=(v_1,v_2)$. en un punto (x_0,y_0) es (si existe)

$$\boxed{\lim_{h\to 0} \frac{f(x_0+hv_1, y_0+hv_2)-f(x_0,y_0)}{h}}.$$

$$\frac{\partial f}{\partial v}(x_0, y_0) \qquad f_v(x_0, y_0)$$

Notacion

Si $\quad v = (1,0) \Rightarrow \dfrac{\partial f}{\partial v} = \dfrac{\partial f}{\partial x}$

$v = (0,1) \Rightarrow \dfrac{\partial f}{\partial v} = \dfrac{\partial f}{\partial y}$.

Ejemplo: Sea $f(x,y) = \begin{cases} 1 & xy = 0 \\ 0 & xy \neq 0 \end{cases}$

$v = (1,1)$

$$\frac{\partial f}{\partial v}(0,0) = \lim_{h \to 0} \frac{f(0+h.1,\ 0+h.1) - f(0,0)}{h}$$

$$= \lim_{h \to 0} \frac{\overbrace{f(h,h)}^{=0} - \overbrace{f(0,0)}^{1}}{h} \quad \nexists \quad \mp\infty$$

Pregunta: ¿Si $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ tiene todas las derivadas direccionales respecto a $v = (v_1, v_2)$ en (x_0, y_0) es f contínua en (x_0, y_0)?

NO

Ejemplo:

$$f(x,y) = \begin{cases} 1 & \text{si } 0 < y < x^2 \\ 0 & \text{en otro caso} \end{cases}$$