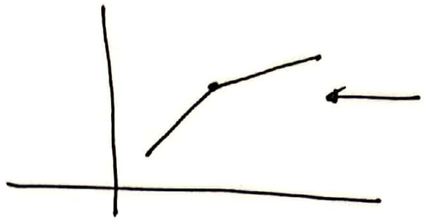
RESOLUCIÓN - VERSIÓN 1 - SEGUNDO PARCIAL CDiV 23-2S

VERDADERO O FALSO:

Af 1: FALSO. Contraejemplo:

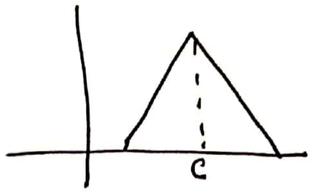


No es derivable

Af 2: VERDADERO. Weierstrass

Af 3: VERDADERO. TVM derivadas

Af 4: FALSO. Contraejemplo:

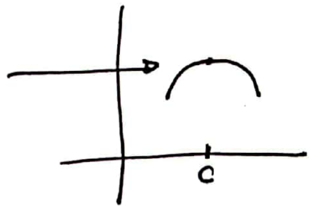


f no es derivable en c, pero hay un máx. relativo en c

Af 5: VERDADERO:

$f'(c)=0$ y $f''(c)<0$ $\Rightarrow$ en c:  $\Rightarrow$ tiene que ser:

máximo relativo



MÚLTIPLE OPCIÓN:

Ejercicio 1:

- Continuidad en $x=0$:

$$\left.\begin{array}{l}\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} 2x+1 = 1 \\ \\ \lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} a(x-1)^2+b = a+b\end{array}\right\} \Longrightarrow \boxed{a+b=1} \quad (*)$$

- Derivabilidad en $x=0$:

$$\lim_{h\to 0^-} \frac{f(0+h)-\overbrace{f(0)}^{=1}}{h} = \lim_{h\to 0^-} \frac{2h+1-1}{h} = 2$$

$$\lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+} \frac{a(h-1)^2+b-1}{h} = \lim_{h\to 0} \frac{a(h^2-2h+1)+b-1}{h}$$

$$= \lim_{h\to 0} \frac{a(h^2-2h)+\overbrace{a+b}^{=1 \text{ por } (*)}-1}{h} = \lim_{h\to 0} \frac{a(h^2-2h)}{h} =$$

$$= \lim_{h\to 0} a(h-2) = -2a$$

$\Longrightarrow$ tiene que ser $\boxed{-2a=2} \iff \boxed{a=-1}$ y $b=2$ por $(*)$

Escaneado con CamScanner

Ejercicio 2:

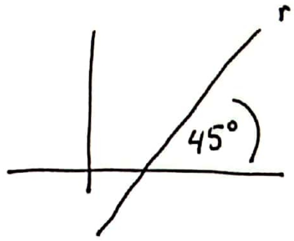
$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2} \overset{\frac{0}{0}}{=} \lim_{x \to 0} \frac{e^x - 1}{2x} = \lim_{x \to 0} \frac{e^x}{2} = \frac{1}{2}$$

L'Hopital

Ejercicio 3:

Sea $f(x) = 2x^2 - 3x - 1$. La ec. de la recta tangente a la curva $y = f(x)$ en el punto $(x_0, f(x_0))$ es:

$$r:\ y = f(x_0) + f'(x_0)(x - x_0)$$



r forma un ángulo de 45° con el eje Ox: si $f'(x_0) = 1$ (pendiente de r)

Luego, buscamos x_0 tal que $f'(x_0) = 1$:

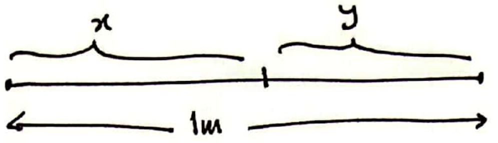
$$f'(x) = 4x - 3$$

$$f'(x_0) = 1 \iff 4x_0 - 3 = 1 \iff x_0 = \frac{4}{4} = 1$$

Las coords. son:

$$(x_0, f(x_0)) = (1, f(1)) = (1, -2).$$

Ejercicio 4:



$\boxed{x+y=1}$

Con x: ◯ $\quad A(\circ) = \pi r^2 \quad$ pero $\quad 2\pi r = x \iff r = \dfrac{x}{2\pi}$

Con y: $\square\, \ell \quad A(\square) = \ell^2 = \left(\dfrac{y}{4}\right)^2 \underset{\substack{\uparrow \\ x+y=1}}{=} \left(\dfrac{1-x}{4}\right)^2 = \dfrac{x^2-2x+1}{16}$

Queremos optimizar:

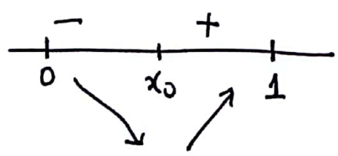
$$f(x) = A(\circ) + A(\square) = \pi\left(\frac{x}{2\pi}\right)^2 + \frac{x^2-2x+1}{16} = \cancel{\pi}\,\frac{x^2}{4\pi^{\cancel{2}}} + \frac{x^2-2x+1}{16}$$

$$= \frac{4x^2 + \pi(x^2-2x+1)}{16\pi} = \frac{(4+\pi)x^2 - 2\pi x + \pi}{16\pi}$$

$$f'(x) = \frac{2(4+\pi)x - 2\pi}{16\pi} = \frac{(4+\pi)x - \pi}{4\pi} = 0 \iff (4+\pi)x = \pi$$

$$\iff \boxed{x_0 = \frac{\pi}{4+\pi}}$$

$\mathrm{sgn}(f')$



En $x_0 = \dfrac{\pi}{4+\pi}$ hay un mínimo relativo (y absoluto) de f en $[0,1]$.

Ejercicio 5:

Queremos saber cuántas raíces tiene $f'(x)$. Por el Teor. Fundamental,

$$f'(x) = \frac{[(x+1)^2-1][(x+1)^2-5]}{[(x+1)^2]^2+1} \cdot ((x+1)^2)'$$

$$= \frac{[x^2+2x+1-1][x^2+2x+1-5]}{(x+1)^4+1} \cdot 2(x+1)$$

$$= \frac{(x^2+2x)(x^2+2x-4) \cdot 2(x+1)}{(x+1)^4+1} = 0 \quad \text{si:}$$

* $x^2+2x=0 \iff x(x+2)=0 \iff x=0$ ó $x=-2$

* $x^2+2x-4=0 \iff x = \frac{-2\pm\sqrt{4-4(-4)}}{2} = \frac{-2\pm\sqrt{4.5}}{2} = -1\pm\sqrt{5}$ $\begin{cases} -1+\sqrt{5} \\ -1-\sqrt{5} \end{cases}$

* $(x+1)=0 \iff x=-1$

Luego, las raíces de $f'(x)$ son:
- 0
- -2
- $-1+\sqrt{5}$
- $-1-\sqrt{5}$
- -1

Respuesta: 5 p.c.

Ejercicio 6:

$$\frac{1}{2}\int_0^{\frac{\pi}{2}} e^{2x}\cdot\cos(2x)\,\overset{2}{d}x = \left[\begin{array}{l} C.V \\ u=2x \\ du = 2dx \end{array}\right] \overset{(*)}{=} \frac{1}{2}\int_0^{\pi} e^u\cos(u)\,du$$

Ahora calculemos $\int_0^{\pi} e^u\cos(u)\,du$:

$$\underbrace{\int_0^{\pi} e^u\cos(u)\,du}_{I} = \left[\begin{array}{l} f'=e^u \to f=e^u \\ g=\cos(u) \to g'=-\mathrm{sen}(u) \end{array}\right] = e^u\cos(u) + \int e^u\,\mathrm{sen}(u)\,du$$

$$= \left[\begin{array}{l} f'=e^u \to f=e^u \\ g=\mathrm{sen}(u) \to g'=\cos(u) \end{array}\right] = e^u\cos u + \left[e^u\,\mathrm{sen}\,u - \int e^u\cos(u)\right]$$

$$= e^u(\cos u + \mathrm{sen}(u))\Big|_0^{\pi} - \underbrace{\int_0^{\pi} e^u\cos(u)\,du}_{I}$$

Tenemos:

$$I = e^{\pi}(\underbrace{\cos(\pi)}_{-1} + \underbrace{\mathrm{sen}(\pi)}_{0}) - e^0(\underbrace{\cos(0)}_{1} + \underbrace{\mathrm{sen}(0)}_{0}) - I$$

$$\Rightarrow 2I = e^{\pi}(-1) - 1 \Rightarrow \boxed{I = \frac{-e^{\pi}-1}{2}}$$

Finalmente:

$$\int_0^{\frac{\pi}{2}} e^{2x}\cos(2x)\,dx = \underset{(*)}{\frac{1}{2}}\cdot I = \frac{-e^{\pi}-1}{4}$$

Ejercicio 7:

$$\int_3^8 \frac{x}{\sqrt{x+1}}\,dx = \begin{bmatrix} \text{C.V.} \\ u=\sqrt{x+1} \\ x=u^2-1 \\ dx = 2u\,du \end{bmatrix} = \int_{\sqrt{3+1}}^{\sqrt{8+1}} \frac{(u^2-1)}{\cancel{u}}\cdot 2\cancel{u}\,du$$

$$= \int_2^3 2(u^2-1)\,du = 2.\left(\frac{u^3}{3}-u\right)\Big|_2^3 =$$

$$= 2\left[\frac{3^3}{3}-3-\left(\frac{2^3}{3}-2\right)\right] = 2\left[9-3-\underbrace{\left(\frac{8}{3}-2\right)}_{\frac{8-6}{3}=\frac{2}{3}}\right]$$

$$= 2\left[6-\frac{2}{3}\right] = 2\left[\frac{18-2}{3}\right] = 2.\ \frac{16}{3} = \frac{32}{3}.$$

Ejercicio 8:

$$\frac{1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2} = \frac{A(x+2)+Bx}{x(x+2)} = \frac{(A+B)x+2A}{x(x+2)} \quad \forall x$$

$$\Longleftrightarrow \begin{cases} A+B=0 \\ 2A=1 \to A=\frac{1}{2} \quad y \quad B=-\frac{1}{2} \end{cases}$$

$$\int_2^3 \frac{1}{x(x+2)}\,dx = \int_2^3 \frac{1/2}{x} + \frac{-1/2}{x+2}\,dx = \frac{1}{2}\int_2^3 \frac{1}{x}\,dx - \frac{1}{2}\int_2^3 \frac{1}{x+2}\,dx$$

$$= \frac{1}{2}\log(x)\Big|_2^3 - \frac{1}{2}\log(x+2)\Big|_2^3 = \frac{1}{2}\left[\log(3)-\log(2)-\left(\log(5)-\log(4)\right)\right]$$

$$= \frac{1}{2}\left[\underbrace{\log(3)+\log(4)}_{\log(3\times 4)} - \underbrace{(\log(2)+\log(5))}_{\log(2\times 5)}\right] = \frac{1}{2}\log\left(\frac{3\times \cancel{4}^{2}}{\cancel{2}\times 5}\right) = \frac{1}{2}\log\left(\frac{6}{5}\right)$$

Ejercicio 9:

$$f(x) = x^2 - \int_0^{2x} e^{t^3}\,dt$$

$$P_2(f)(x) = \overbrace{f(0)}^{=0} + \overbrace{f'(0)}^{=-2}x + \overbrace{\frac{f''(0)}{2!}}^{=2}x^2 = -2x + x^2$$

- $f(0) = 0 - \int_0^0 e^{t^3}\,dt = 0$

- $f'(x) \underset{T.F.}{=} 2x - e^{(2x)^3}(2x)' = 2x - e^{8x^3}\cdot 2 \Longrightarrow f'(0) = -2$

- $f''(x) = (2x - e^{8x^3}\cdot 2)' = 2 - 2e^{8x^3}\underbrace{(8x^3)'}_{24x^2} = 2 - 48x^2 e^{8x^3}$

$$\Longrightarrow f''(0) = 2$$