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Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 5: **Total stations**

Optique et instruments d'optique — Méthodes d'essai sur site des instruments géodésiques et d'observation —

Partie 5: Stations totales

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see the following URL: www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 172, *Optics and photonics*, Subcommittee SC 6, *Geodetic and surveying instruments*.

This third edition cancels and replaces the second edition (ISO 17123-5:2012), which has been technically revised.

A list of all parts in the ISO 17123 series can be found on the ISO website.

Introduction

This document specifies field procedures for adoption when determining and evaluating the uncertainty of measurement results obtained by geodetic instruments and their ancillary equipment, when used in building and surveying measuring tasks. Primarily, these tests are intended to be field verifications of suitability of a particular instrument for the immediate task. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

The definition and concept of uncertainty as a quantitative attribute to the final result of measurement was developed mainly in the last two decades, even though error analysis has already long been a part of all measurement sciences. After several stages, the CIPM (Comité Internationale des Poids et Mesures) referred the task of developing a detailed guide to ISO. Under the responsibility of the ISO Technical Advisory Group on Metrology (TAG 4), and in conjunction with six worldwide metrology organizations, a guidance document on the expression of measurement uncertainty was compiled with the objective of providing rules for use within standardization, calibration, laboratory, accreditation and metrology services. ISO/IEC Guide 98–3 was first published in 1995.

With the introduction of uncertainty in measurement in ISO 17123 (all parts), it is intended to finally provide a uniform, quantitative expression of measurement uncertainty in geodetic metrology with the aim of meeting the requirements of customers.

ISO 17123 (all parts) provides not only a means of evaluating the precision (experimental standard deviation) of an instrument, but also a tool for defining an uncertainty budget, which allows for the summation of all uncertainty components, whether they are random or systematic, to a representative measure of accuracy, i.e. the combined standard uncertainty.

ISO 17123 (all parts) therefore provides, for defining for each instrument investigated by the procedures, a proposal for additional, typical influence quantities, which can be expected during practical use. The customer can estimate, for a specific application, the relevant standard uncertainty components in order to derive and state the uncertainty of the measuring result.

Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 5: Total stations

1 Scope

This document specifies field procedures to be adopted when determining and evaluating the precision (repeatability) of coordinate measurement of total stations and their ancillary equipment when used in building and surveying measurements. Primarily, these tests are intended to be field verifications of the suitability of a particular instrument for the immediate task at hand and to satisfy the requirements of other standards. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

These field procedures have been developed specifically for in situ applications without the need for special ancillary equipment and are purposely designed to minimize atmospheric influences.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 4463-1, *Measurement methods for building — Setting-out and measurement — Part 1: Planning and organization, measuring procedures, acceptance criteria*

ISO 7077, *Measuring methods for building — General principles and procedures for the verification of dimensional compliance*

ISO 7078, *Building construction — Procedures for setting out, measurement and surveying — Vocabulary and guidance notes*

ISO 9849, *Optics and optical instruments — Geodetic and surveying instruments — Vocabulary*

ISO 17123-1, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 1: Theory*

ISO 17123-3, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 3: Theodolites*

ISO 17123-4, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 4: Electro-optical distance meters (EDM measurements to reflectors)*

ISO/IEC Guide 98-3:2008, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

ISO/IEC Guide 99:2007, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purposes of this document, the terms and definition given in ISO 3534-1, ISO 4463-1, ISO 7077, ISO 7078, ISO 9849, ISO 17123-1, ISO/IEC Guide 98-3 (GUM) and ISO/IEC Guide 99 (VIM) apply.

4 Symbols and subscripts

4.1 Symbols

Symbol	Quantity	Unit
a	mean value of height differences	m
d	deviation, differences	m
k	coverage factor	—
L	mean value of horizontal distance between two target points	m
l	horizontal distance between two target points	m
M	vertex of the triangle of the mathematical model	—
p	parameter to calculate the rotation angle	—
q	parameter to calculate the rotation angle	—
r	difference to the mean value	m
S	instrument station	—
s	experimental standard deviation	various
$\tilde{s}$	experimental standard deviation of the same population	various
T	target point	—
U	expanded uncertainty	various
u	uncertainty	various
v	degree of freedom	—
X	mathematical model coordinate X	m
x	measured coordinate x	m
Y	mathematical model coordinate Y	m
y	measured coordinate y	m
Z	mathematical model coordinate Z	m
z	measured coordinate z	m
α	confidence level	—
σ	standard deviation of a population	various
$\tilde{\sigma}$	standard deviation of the same population	various
θ	horizontal rotation angle	°
Ψ	Vertical (elevation) angle	°
χ	Chi-Quadrat distribution	—

4.2 Subscripts

Subscript	Term
0,975	confidence level 0,975
1-α	confidence level
dist	distance
disp	minimum display digit of coordinates

Subscript	Term
dispx	minimum display digit of coordinate x
dispy	minimum display digit of coordinate y
dispz	minimum display digit of coordinate z
E	east axis
g	centre of gravity
H	height axis
hs	height stability of tripod
i	instrument station No.
ISO-TS	total station according to ISO 17123-5
ISO-TS-XY	coordinates XY measured once in both face positions of the telescope according to ISO 17123-5
ISO-TS-Z	coordinates Z measured once in both face positions of the telescope according to ISO 17123-5
j	target point No.
k	measured set number (single telescope face)
m	coordinate of the centre of gravity of the mathematical model after rotation
N	north axis
prs	pressure
rh	relative humidity
dist-TS	distance measurement in total station
θ-TS	horizontal angle on specification of total station
t	coordinate of the centre of gravity of the mathematical model after the shift
temp	temperature
trd	tripod torsion
θ	horizontal angle
Ψ	vertical angle or elevation angle
Ψ-TS	vertical angle on specification of total station
x	coordinate x (up)
xy	coordinates xy (horizontal)
XY	coordinates XY (horizontal) of the mathematical model
y	coordinate y (right)
Z	coordinate Z (height) of the mathematical model
z	coordinate z (height)

5 General

5.1 Requirements

Before commencing the measurements, it is important that the operator ensures that the precision in use of the measuring equipment is appropriate for the intended measuring task.

The total station and its ancillary equipment shall be in known and acceptable states of permanent adjustment according to the methods specified in the manufacturer's reference manual, and used tripods with reflectors as recommended by the manufacturer.

The coordinates are considered as observables because on modern total stations they are selectable as output quantities.

All coordinates shall be measured on the same day. The instrument should always be levelled carefully. The correct zero-point correction of the reflector prism shall be used.

The results of these tests are influenced by meteorological conditions, especially by the gradient of temperature. An overcast sky and low wind speed guarantee the most favourable weather conditions. Actual meteorological data shall be measured in order to derive atmospheric corrections, which shall be added to the raw distances. The particular conditions to be taken into account can vary depending on where the tasks are to be undertaken. These conditions shall include variations in air temperature, wind speed, cloud cover and visibility. Note should also be taken of the actual weather conditions at the time of measurement and the type of surface above which the measurements are made. The conditions chosen for the tests should match those expected when the intended measuring task is actually carried out (see ISO 7077 and ISO 7078).

Tests performed in laboratories would provide results which are almost unaffected by atmospheric influences, but the costs for such tests are very high, and therefore they are not practicable for most users. In addition, laboratory tests yield precisions much higher than those that can be obtained under field conditions.

This document describes two different field procedures as given in Clauses 6 and 7. The operator shall choose the procedure which is most relevant to the project's particular requirements.

To evaluate angle measurement and distance measurement separately, ISO 17123-3 and ISO 17123-4 shall be applied accordingly.

5.2 Procedure 1: Simplified test procedure

The simplified test procedure provides an estimate as to whether the precision of a given total station is within the specified permitted deviation in accordance with ISO 4463-1.

The simplified test procedure is based on a limited number of measurements. This test procedure relies on measurements of x-, y- and z-coordinates in a test field without nominal values. The maximum difference from mean value is calculated as an indicator for the precision.

A significant standard deviation cannot be obtained. If a more precise assessment of the total station under field conditions is required, it is recommended to adopt the more rigorous full test procedure as given in Clause 7.

An example of the simplified test procedure is given in Annex A.

5.3 Procedure 2: Full test procedure

The full test procedure shall be adopted to determine the best achievable measure of precision of a total station and its ancillary equipment under field conditions.

This procedure is based on measurements of coordinates in a test field without nominal values. The experimental standard deviation of the coordinate measurement of a single point is determined from least squares adjustments.

The full test procedure given in Clause 7 of this document is intended for determining the measure of precision in use of a particular total station. This measure of precision in use is expressed in terms of the experimental standard deviations $s_{\text{ISO-TS-XY}}$ and $s_{\text{ISO-TS-Z}}$ of a coordinate measured once in both face positions of the telescope.

Furthermore, this procedure can be used to determine:

— the measure of precision in use of total stations by a single survey team with a single instrument and its ancillary equipment at a given time;

— the measure of precision in use of a single instrument over time;

— the measure of precision in use of each of several total stations in order to enable a comparison of their respective achievable precisions to be obtained under similar field conditions.

Statistical tests should be applied to determine whether the experimental standard deviations obtained belong to the population of the instrumentation's theoretical standard deviations and whether two tested samples belong to the same population.

An example of the full test procedure is given in Annex B.

6 Simplified test procedure

6.1 Configuration of the test field

Two target points (T_1, T_2) shall be set out as indicated in Figure 1. The targets should be firmly fixed on to the ground. The distance between two target points should be set longer than the average distance (e.g. 60 m) according to the intended measuring task. Their heights should be as different as the surface of the ground allows.

Two instrument stations (S_1, S_2) shall be set out approximately in line with two target points. S_1 shall be set 5 m to 10 m away from T_1 and in the opposite direction to T_2. S_2 shall be set between two target points and 5 m to 10 m away from T_2.

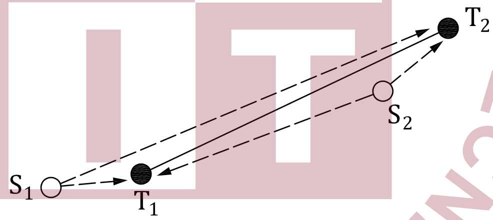

Figure 1 — Configuration of the test field

6.2 Measurement

One set consists of two measurements to each target point in one telescope face at one of the instrument stations.

The coordinates of the two target points shall be measured by 4 sets (telescope face: I – II – I – II) at the instrument station S_1. The instrument is shifted to station S_2 and the same sequence of measurements is carried out. Station coordinates and the reference orientation of the station are discretionary in each set.

On-board or stand-alone software shall be used for the observations. It is preferable to use the same software which will be used for the practical work.

The sequence of the measurements is shown in Table 1.

Table 1 — Sequence of the measurements for one series

Seq. No.	Instrument station *i*	Target point *j*	Set *k*	Telescope face	*x*	*y*	*z*
1	1	1	1	I	$x_{1,1,1}$	$y_{1,1,1}$	$z_{1,1,1}$
2		2			$x_{1,2,1}$	$y_{1,2,1}$	$z_{1,2,1}$
3		1	2	II	$x_{1,1,2}$	$y_{1,1,2}$	$z_{1,1,2}$
4		2			$x_{1,2,2}$	$y_{1,2,2}$	$z_{1,2,2}$
5		1	3	I	$x_{1,1,3}$	$y_{1,1,3}$	$z_{1,1,3}$
6		2			$x_{1,2,3}$	$y_{1,2,3}$	$z_{1,2,3}$
7		1	4	II	$x_{1,1,4}$	$y_{1,1,4}$	$z_{1,1,4}$
8		2			$x_{1,2,4}$	$y_{1,2,4}$	$z_{1,2,4}$
9	2	1	1	I	$x_{2,1,1}$	$y_{2,1,1}$	$z_{2,1,1}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
15	2	1	4	II	$x_{2,1,4}$	$y_{2,1,4}$	$z_{2,1,4}$
16		2			$x_{2,2,4}$	$y_{2,2,4}$	$z_{2,2,4}$

6.3 Calculation

6.3.1 x-, y-coordinates

The evaluation of the test results is given by the deviation of the horizontal distance of each set from the mean value of all measured horizontal distances.

Each horizontal distance between two target points $l_{i,k}$ is calculated as:

$$l_{i,k} = \sqrt{\left(x_{i,2,k} - x_{i,1,k}\right)^2 + \left(y_{i,2,k} - y_{i,1,k}\right)^2} \quad i = 1, 2;\ k = 1, 2, 3, 4 \tag{1}$$

Their mean value L is calculated as:

$$L = \frac{1}{8}\sum_{i=1}^{2}\sum_{k=1}^{4} l_{i,k} \tag{2}$$

The values of the deviation of each distance from its mean $r_{i,k}$ is calculated as:

$$r_{i,k} = l_{i,k} - L \quad i = 1, 2;\ k = 1, 2, 3, 4 \tag{3}$$

The maximum value d_{xy} of the $r_{i,k}$ is defined as:

$$d_{xy} = \max\left|r_{i,k}\right| \quad i = 1, 2;\ k = 1, 2, 3, 4 \tag{4}$$

6.3.2 z-coordinate

The height differences $d_{z,i,k}$ between target points are calculated using measured z-coordinate values in each set:

$$d_{z,i,k} = z_{i,2,k} - z_{i,1,k} \quad i = 1, 2; k = 1, 2, 3, 4 \tag{5}$$

The mean value a_z of height difference in all sets:

$$a_z = \frac{1}{8}\sum_{i=1}^{2}\sum_{k=1}^{4} d_{z,i,k} \tag{6}$$

The differences $r_{z,i,k}$ between height differences of two target points and the mean value a_z:

$$r_{z,i,k} = d_{z,i,k} - a_z \quad i = 1, 2; k = 1, 2, 3, 4 \tag{7}$$

The maximum difference value d_z is calculated as:

$$d_z = \max\left|r_{z,i,k}\right| \tag{8}$$

6.3.3 Evaluation

The differences d_{xy} and d_z shall be within the specified permitted deviation, p_{xy} and p_z respectively, (in accordance with ISO 4463-1 for the intended measuring task). If p_{xy} and p_z are not given, they shall be $d_{xy} \le 2{,}5 \times \sqrt{2} \times s_{\text{ISO-TS-XY}}$ and $d_z \le 2{,}5 \times \sqrt{2} \times s_{\text{ISO-TS-Z}}$ respectively, where $s_{\text{ISO-TS-XY}}$ and $s_{\text{ISO-TS-Z}}$ are the experimental standard deviations of the x-, y- and z-measurements respectively, determined according to the full test procedure with the same instrument.

7 Full test procedure

7.1 Configuration of the test field

Three target points (T_1, T_2, T_3) shall be set out at the corner of the triangle (see Figure 2). The targets should be firmly fixed on to the ground. The distances of target points should be different and at least one distance should be longer than the average distance (e.g. 60 m) according to the intended measuring task. Their heights should be as different as the surface of the ground allows.

Three instrument stations (S_1, S_2, S_3) shall be set out close to each triangular side approximately 5 m to 10 m away from each target point.

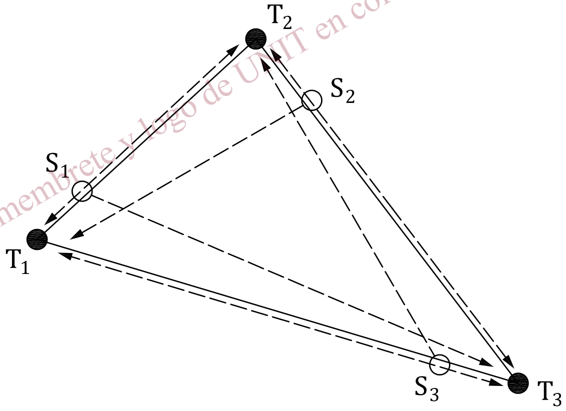

Figure 2 — Example of field configuration for full test

7.2 Measurement

One set consists of three measurements to each target point with a single telescope face at each instrument station.

From the instrument stations S_1, S_2, S_3, the coordinates of the three target points shall be measured by four sets of observation sequences (telescope face: I – II – I – II).

The station coordinates and the orientation are discretionary for each station set up. These configurations should not be changed while measuring four sets of observations from the same station point.

On-board or stand-alone software shall be used for the observations. It is preferable to use the same software which will be used for the practical work.

The sequence of the measurements is shown in Table 2.

Table 2 — Sequence of the measurements for one series

Seq. No.	Instrument station *i*	Target point *j*	Set *k*	Telescope face	*x*	*y*	*z*
1		1			$x_{1,1,1}$	$y_{1,1,1}$	$z_{1,1,1}$
2		2	1	I	$x_{1,2,1}$	$y_{1,2,1}$	$z_{1,2,1}$
3		3			$x_{1,3,1}$	$y_{1,3,1}$	$z_{1,3,1}$
4		1			$x_{1,1,2}$	$y_{1,1,2}$	$z_{1,1,2}$
5		2	2	II	$x_{1,2,2}$	$y_{1,2,2}$	$z_{1,2,2}$
6		3			$x_{1,3,2}$	$y_{1,3,2}$	$z_{1,3,2}$
7	1	1			$x_{1,1,3}$	$y_{1,1,3}$	$z_{1,1,3}$
8		2	3	I	$x_{1,2,3}$	$y_{1,2,3}$	$z_{1,2,3}$
9		3			$x_{1,3,3}$	$y_{1,3,3}$	$z_{1,3,3}$
10		1			$x_{1,1,4}$	$y_{1,1,4}$	$z_{1,1,4}$
11		2	4	II	$x_{1,2,4}$	$y_{1,2,4}$	$z_{1,2,4}$
12		3			$x_{1,3,4}$	$y_{1,3,4}$	$z_{1,3,4}$
13		1	1	I	$x_{2,1,1}$	$y_{2,1,1}$	$z_{2,1,1}$
⋮	2	⋮	⋮	⋮	⋮	⋮	⋮
24		3	4	II	$x_{2,3,4}$	$y_{2,3,4}$	$z_{2,3,4}$
25		1	1	I	$x_{3,1,1}$	$y_{3,1,1}$	$z_{3,1,1}$
⋮		⋮	⋮	⋮	⋮	⋮	⋮
34	3	1			$x_{3,1,4}$	$y_{3,1,4}$	$z_{3,1,4}$
35		2	4	II	$x_{3,2,4}$	$y_{3,2,4}$	$z_{3,2,4}$
36		3			$x_{3,3,4}$	$y_{3,3,4}$	$z_{3,3,4}$

7.3 Calculation

7.3.1 x-, y-coordinates

Construction of the mathematical model of the triangle is carried out as follows (See Figure 3).

Calculate the horizontal distances $l_{i,3,k}$ between T_1 and T_2; $l_{i,1,k}$ between T_2 and T_3; $l_{i,2,k}$ between T_3 and T_1 respectively by measured coordinates ($x_{i,j,k}$, $y_{i,j,k}$).

$$l_{i,j,k} = \sqrt{\left(x_{i,j-1,k} - x_{i,j+1,k}\right)^2 + \left(y_{i,j-1,k} - y_{i,j+1,k}\right)^2}$$ i = 1, 2, 3; j = 1, 2, 3 (if j-1 is 0 or j+1 is 4, then replace it by 3 or 1 respectively); k = 1, 2, 3, 4 (9)

The mean length of each side L_j:

$$L_j = \frac{1}{12}\sum_{i=1}^{3}\sum_{k=1}^{4} l_{i,j,k} \quad j = 1, 2, 3 \tag{10}$$

The vertex coordinates of the mathematical model of the triangle M_j (j = 1, 2, 3) is defined based on M_1 = (0,0) and the line from M_1 to M_2 as the x-axis.

Coordinates of M_1:

$$M_1\left(X_1, Y_1\right) = (0,0) \tag{11}$$

Coordinates of M_2:

$$M_2\left(X_2, Y_2\right) = \left(L_3, 0\right) \tag{12}$$

Coordinates of M_3:

$$M_3\left(X_3, Y_3\right) = \left[\frac{-\left(L_1^2\right) + L_2^2 + L_3^2}{2L_3}, \sqrt{L_2^2 - \left[\frac{-\left(L_1^2\right) + L_2^2 + L_3^2}{2L_3}\right]^2}\right] \tag{13}$$

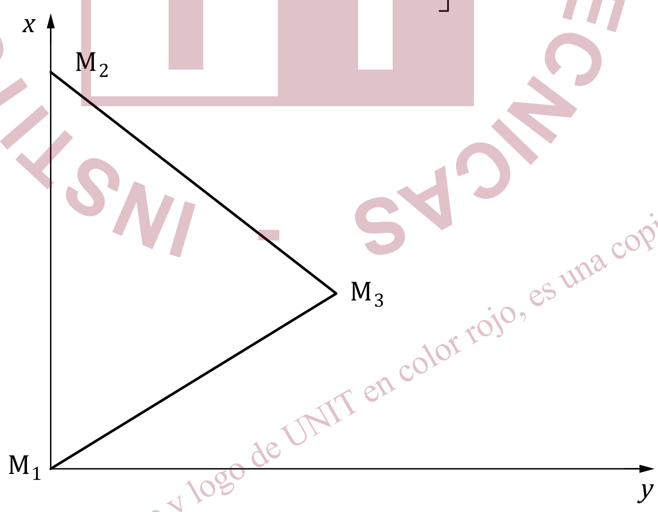

Figure 3 — Mathematical model of the triangle

The coordinates of the centre of gravity of the mathematical model, (X_g, Y_g):

$$\left(X_g, Y_g\right) = \left[\frac{\sum_{j=1}^{3} X_j}{3}, \frac{\sum_{j=1}^{3} Y_j}{3}\right] \quad (14)$$

The coordinates of the centre of gravity of the triangle obtained at each instrument station, $(x_{g,i}, y_{g,i})$:

$$\left(x_{g,i}, y_{g,i}\right) = \left[\frac{\sum_{j=1}^{3}\sum_{k=1}^{4} x_{i,j,k}}{12}, \frac{\sum_{j=1}^{3}\sum_{k=1}^{4} y_{i,j,k}}{12}\right] \quad i = 1, 2, 3 \quad (15)$$

Shift the coordinates to coincide the centre of gravity of the mathematical model on the centre of gravity of the measured triangle.

The coordinates of the centre of gravity of the mathematical model $(X_{t,i,j,k}, Y_{t,i,j,k})$ after the shift are calculated as:

$$X_{t,i,j,k} = X_j + \left(x_{g,i} - X_g\right);\ Y_{t,i,j,k} = Y_j + \left(y_{g,i} - Y_g\right) \quad i = 1, 2, 3; j = 1, 2, 3; k = 1, 2, 3, 4 \quad (16)$$

Rotate the mathematical model around the centre of gravity to minimize residuals of the vertex coordinates between the mathematical model and respective measured triangles.

Rotation angle $\theta_{i,k}$:

$$\theta_{i,k} = \tan^{-1}\left(\frac{q_{i,k}}{p_{i,k}}\right) \quad i = 1, 2, 3; k = 1, 2, 3, 4 \quad (17)$$

$$q_{i,k} = \frac{\sum_{j=1}^{3}\left(\left(X_{t,i,j,k} - x_{g,i}\right)\cdot\left(y_{i,j,k} - y_{g,i}\right) - \left(Y_{t,i,j,k} - y_{g,i}\right)\cdot\left(x_{i,j,k} - x_{g,i}\right)\right)}{\sum_{j=1}^{3}\left(\left(X_{t,i,j,k} - x_{g,i}\right)^2 + \left(Y_{t,i,j,k} - y_{g,i}\right)^2\right)} \quad (18)$$

$$p_{i,k} = \frac{\sum_{j=1}^{3}\left(\left(X_{t,i,j,k} - x_{g,i}\right)\cdot\left(x_{i,j,k} - x_{g,i}\right) + \left(Y_{t,i,j,k} - y_{g,i}\right)\cdot\left(y_{i,j,k} - y_{g,i}\right)\right)}{\sum_{j=1}^{3}\left(\left(X_{t,i,j,k} - x_{g,i}\right)^2 + \left(Y_{t,i,j,k} - y_{g,i}\right)^2\right)} \quad (19)$$

Vertex coordinates of mathematical model $(X_{m,i,j,k}, Y_{m,i,j,k})$ after the rotation:

$$X_{m,i,j,k} = x_{g,i} + \cos\theta_{i,k}\cdot\left(X_{t,i,j,k} - x_{g,i}\right) - \sin\theta_{i,k}\cdot\left(Y_{t,i,j,k} - y_{g,i}\right) \quad i = 1, 2, 3; j = 1, 2, 3; k = 1, 2, 3, 4 \quad (20)$$

$$Y_{m,i,j,k} = y_{g,i} + \sin\theta_{i,k} \cdot \left(X_{t,i,j,k} - x_{g,i}\right) + \cos\theta_{i,k} \cdot \left(Y_{t,i,j,k} - y_{g,i}\right) \quad i = 1, 2, 3; j = 1, 2, 3; k = 1, 2, 3, 4 \tag{21}$$

Residuals ($r_{x,i,j,k}$, $r_{y,i,j,k}$) of the coordinates of the measured triangles from those of the rotated mathematical model are:

$$r_{x,i,j,k} = x_{i,j,k} - X_{m,i,j,k} \quad i = 1, 2, 3; j = 1, 2, 3; k = 1, 2, 3, 4 \tag{22}$$

$$r_{y,i,j,k} = y_{i,j,k} - Y_{m,i,j,k} \quad i = 1, 2, 3; j = 1, 2, 3; k = 1, 2, 3, 4 \tag{23}$$

The sum of squares of residuals:

$$\sum r_{xy}^2 = \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{4} \left(r_{x,i,j,k}^2 + r_{y,i,j,k}^2 \right) \tag{24}$$

Since there are 3 sides of the mathematical model, 6 [= 2 (vectors) × 3 (instrument stations)] centre of gravity points of the measured triangle and 12 [= 4 (sets) × 3 (instrument stations)] rotation parameters, the number of unknown parameters $v = 3 + 6 + 12 = 21$. Thus the number of degrees of freedom:

$$v_{XY} = 72 - 21 = 51 \tag{25}$$

The experimental standard deviation:

$$s_{XY} = \sqrt{\frac{\sum r_{xy}^2}{51}} \tag{26}$$

Finally, the standard uncertainty of x-, y-coordinates:

$$u_{ISO\text{-}TS\text{-}XY} = s_{XY} \tag{27}$$

7.3.2 z-coordinate

The height difference between T_1 and T_2 (and T_3) is calculated using measured z-values for each set.

$$d_{z,i,j,k} = z_{i,j,k} - z_{i,1,k} \quad i = 1, 2, 3; j = 2, 3; k = 1, 2, 3, 4 \tag{28}$$

The mean values of $d_{z,i,2,k}$ and $d_{z,i,3,k}$:

$$a_{z,j} = \frac{1}{12} \sum_{i=1}^{3} \sum_{k=1}^{4} d_{z,i,j,k} \quad j = 2, 3 \tag{29}$$

The residuals $r_{z,i,j,k}$ of the height differences $d_{z,i,2,k}$ and $d_{z,i,3,k}$ from obtained mean values for each set of measurements are calculated as:

$$r_{z,i,j,k} = d_{z,i,j,k} - a_{z,j} \quad i = 1, 2, 3; j = 2, 3; k = 1, 2, 3, 4 \tag{30}$$

The sum of the squares of the residuals is obtained by:

$$\sum r_Z^2 = \sum_{i=1}^{3} \sum_{j=2}^{3} \sum_{k=1}^{4} r_{z,i,j,k}^2 \tag{31}$$

The number of degrees of freedom is:

$$v_Z = 24 - 2 = 22 \tag{32}$$

Finally, the standard deviation of height difference or the standard deviation of the z-coordinate:

$$s_{dZ} = \sqrt{\frac{\sum r_Z^2}{22}}\ ; \ s_Z = \sqrt{\frac{\sum r_Z^2}{22 \times 2}} \tag{33}$$

Its standard uncertainty:

$$u_{ISO\text{-}TS\text{-}Z} = s_Z \tag{34}$$

7.4 Statistical tests

7.4.1 General

Statistical tests are applicable for the full test procedure only.

For the interpretation of the results, statistical tests shall be carried out using the experimental standard deviation of a coordinate measured on the test triangle in order to answer the following questions (see Table 3).

a) Is the calculated experimental standard deviation, s, smaller than or equal to a corresponding value, σ, stated by the manufacturer or smaller than another predetermined value, σ?

b) Do two experimental standard deviations, s and $\tilde{s}$, as determined from two different samples of measurements, belong to the same population, assuming that both samples have the same number of degrees of freedom, v?

The experimental standard deviations, s and $\tilde{s}$ can be obtained from

— two samples of measurements by the same instrument but different observers;

— two samples of measurements by the same instrument at different times; or

— two samples of measurements by different instruments.

For the following tests, a confidence level of 1-α = 0,95 and, according to the design of measurements, a number of degrees of freedom of ν_{XY} = 51 for the x- and y-coordinates and ν_Z = 22 for the z-coordinate are assumed.

Table 3 — Statistical tests

Question	Null hypothesis	Alternate hypothesis
a)	$s \le \sigma$	$s > \sigma$
b)	$s = \tilde{s}$	$s \ne \tilde{s}$

7.4.2 Response to Question a)

The null hypothesis stating that the experimental standard deviation, *s*, is smaller than or equal to a theoretical or a predetermined value, *σ*, is not rejected if the following condition is fulfilled:

For x and y:

$$s \le \sigma \times \sqrt{\frac{\chi^2_{1-\alpha}(\nu_{XY})}{\nu_{XY}}} \quad (35)$$

$$s \le \sigma \times \sqrt{\frac{\chi^2_{0,95}(51)}{51}} \quad (36)$$

$$\chi^2_{0,95}(51) = 68,67 \quad (37)$$

$$s \le \sigma \times \sqrt{\frac{68,67}{51}} \quad (38)$$

$$s \le \sigma \times 1,16 \quad (39)$$

For z:

$$s \le \sigma \times \sqrt{\frac{\chi^2_{1-\alpha}(\nu_Z)}{\nu_Z}} \quad (40)$$

$$s \le \sigma \times \sqrt{\frac{\chi^2_{0,95}(22)}{22}} \quad (41)$$

$$\chi^2_{0,95}(22) = 33,92 \quad (42)$$

$$s \le \sigma \times \sqrt{\frac{33,92}{22}} \quad (43)$$

$$s \le \sigma \times 1,24 \quad (44)$$

Otherwise, the null hypothesis is rejected.

7.4.3 Response to question b)

In the case of two different samples, a test indicates whether the experimental standard deviations, s and $\tilde{s}$ belong to the same population. The corresponding null hypothesis, $s = \tilde{s}$ is not rejected if the following condition is fulfilled:

For x and y:

$$\frac{1}{F_{1-\alpha/2}(\nu_{XY}, \nu_{XY})} \leq \frac{s^2}{\tilde{s}^2} \leq F_{1-\alpha/2}(\nu_{XY}, \nu_{XY}) \quad (45)$$

$$\frac{1}{F_{0,975}(51,51)} \leq \frac{s^2}{\tilde{s}^2} \leq F_{0,975}(51,51) \quad (46)$$

$$F_{0,975}(51,51) = 1,74 \quad (47)$$

$$0,57 \leq \frac{s^2}{\tilde{s}^2} \leq 1,74 \quad (48)$$

For z:

$$\frac{1}{F_{1-\alpha/2}(\nu_Z, \nu_Z)} \leq \frac{s^2}{\tilde{s}^2} \leq F_{1-\alpha/2}(\nu_Z, \nu_Z) \quad (49)$$

$$\frac{1}{F_{0,975}(22,22)} \leq \frac{s^2}{\tilde{s}^2} \leq F_{0,975}(22,22) \quad (50)$$

$$F_{0,975}(22,22) = 2,36 \quad (51)$$

$$0,42 \leq \frac{s^2}{\tilde{s}^2} \leq 2,36 \quad (52)$$

Otherwise, the null hypothesis is rejected.

The number of degrees of freedom and, thus, the corresponding test values $\chi^2_{1-\alpha/2}$ and $F_{1-\alpha/2}(\nu,\nu)$ (taken from reference books on statistics) change if a different number of measurements is analysed.

7.5 Combined uncertainty evaluation (Type A and Type B)

The sources of uncertainty (influence quantities) are described in Table 4 as an uncertainty budget.

Table 4 — Typical influence quantities of the total station

Sources of uncertainty	Symbol	Evaluation	Distribution
I. Result of measurement			
Standard deviation of x-, y- and z-coordinates	$u_{\text{ISO-TS}}$	Type A	normal
II. Relevant sources of the total station			
Distance uncertainty on the specification	$u_{\text{dist-TS}}$	Type B	normal, or specified by the manufacturer
Horizontal angle uncertainty on the specification	$u_{\theta\text{-TS}}$	Type B	normal, or specified by the manufacturer
Vertical angle uncertainty on the specification	$u_{\Psi\text{-TS}}$	Type B	normal, or specified by the manufacturer

Table 4 *(continued)*

Sources of uncertainty	Symbol	Evaluation	Distribution
Minimum display digit	u_{disp}	Type B	rectangular
III. Error patterns from the mechanical setup			
Torsion of a tripod (ISO 12858-2)	u_{trd}	Type B	rectangular
Stability of a tripod height (ISO 12858-2)	u_{hs}	Type B	rectangular
IV. Error sources of the atmospheres			
Temperature	u_{temp}	Type B	normal
Pressure	u_{prs}	Type B	normal
Relative humidity	u_{rh}	Type B	normal

Uncertainty on the polar coordinates system is described as:

$$u_{dist} = \sqrt{u_{dist\text{-}TS}^2 + u_{temp}^2 + u_{prs}^2 + u_{rh}^2} \quad (53)$$

$$u_\theta = \sqrt{u_{\theta\text{-}TS}^2 + u_{trd}^2} \quad (54)$$

$$u_\psi = \sqrt{u_{\psi\text{-}TS}^2 + u_{hs}^2} \quad (55)$$

The transfer formula to the rectangular coordinate from the polar coordinate:

$$u_N^2 + u_E^2 = \left(\cos\theta \cdot u_{dist}\right)^2 + \left(r \cdot \sin\theta \cdot u_\psi\right)^2 + \left(r \cdot \cos\theta \cdot u_\theta\right)^2 \quad (56)$$

$$u_H^2 = \left(\sin\theta \cdot u_{dist}\right)^2 + \left(r \cdot \cos\theta \cdot u_\psi\right)^2 \quad (57)$$

Combined uncertainty:

$$u_{xy} = \sqrt{u_{ISO\text{-}TS\text{-}XY}^2 + \left(u_N^2 + u_E^2\right) + u_{disp}^2} \quad (58)$$

$$u_z = \sqrt{u_{ISO\text{-}TS\text{-}Z}^2 + u_H^2 + u_{disp}^2} \quad (59)$$

Expanded uncertainty is, with coverage factor $k = 2$.

$$U_{xy} = 2 \times u_{xy} \quad (60)$$

$$U_z = 2 \times u_z \quad (61)$$

An example of the calculation of a combined uncertainty budget is given in Annex C.

Sources which are not included in the uncertainty evaluation are given in Annex D.

Annex A
(informative)

Example of a simplified test procedure

A.1 Measurements

In Table A.1 all measurements are compiled according to the observation scheme given in Table 1.

Table A.1 — Measurements

Dimensions in metres

Seq. No.	Instrument station *i*	Target point *j*	Set *k*	Telescope face	*x*	*y*	*z*
1	1	1	1	I	6,979	4,886	9,934
2		2			59,617	25,117	6,763
3		1	2	II	6,979	4,886	9,933
4		2			59,619	25,117	6,762
5		1	3	I	6,978	4,885	9,934
6		2			59,618	25,116	6,764
7		1	4	II	6,979	4,885	9,934
8		2			59,620	25,116	6,762
9	2	1	1	I	8,344	−47,323	12,767
10		2			1,214	8,619	9,596
11		1	2	II	8,346	−47,322	12,764
12		2			1,213	8,619	9,596
13		1	3	I	8,344	−47,323	12,767
14		2			1,213	8,619	9,596
15		1	4	II	8,345	−47,324	12,766
16		2			1,213	8,619	9,596

Observer: Y. Ohshima

Weather: sunny

Temperature: 29 °C

Air pressure: 1006 hPa

Instrument type and number: NT xxx 309090

Date: 2010–07–08

A.2 Calculation

A.2.1 x-, y-coordinates

According to Formula (1):

$l_{1,1} = 56{,}392\ 0\ \text{m};\ l_{2,1} = 56{,}394\ 5\ \text{m}$

$l_{1,2} = 56{,}393\ 8\ \text{m};\ l_{2,2} = 56{,}393\ 9\ \text{m}$

$l_{1,3} = 56{,}393\ 8\ \text{m};\ l_{2,3} = 56{,}394\ 7\ \text{m}$

$l_{1,4} = 56{,}394\ 8\ \text{m};\ l_{2,4} = 56{,}395\ 8\ \text{m}$

and according to Formula (2):

$L = 56{,}394\ 2\ \text{m}$

and according to Formula (3):

$r_{1,1} = -0{,}002\ 2\ \text{m};\ \text{r}_{2,1} = 0{,}000\ 4\ \text{m}$

$r_{1,2} = -0{,}000\ 2\ \text{m};\ \text{r}_{2,2} = -0{,}000\ 2\ \text{m}$

$r_{1,3} = -0{,}000\ 3\ \text{m};\ \text{r}_{2,3} = 0{,}000\ 5\ \text{m}$

$r_{1,4} = 0{,}000\ 6\ \text{m};\ \text{r}_{2,4} = 0{,}001\ 6\ \text{m}$

and according to Formula (4):

$d_{xy} = 0{,}002\ 2\ \text{m}$

A.2.2 z-coordinates

According to Formula (5):

$d_{z,1,1} = -3{,}171\ \text{m};\ d_{z,2,1} = -3{,}171\ \text{m}$

$d_{z,1,2} = -3{,}171\ \text{m};\ d_{z,2,2} = -3{,}168\ \text{m}$

$d_{z,1,3} = -3{,}170\ \text{m};\ d_{z,2,3} = -3{,}171\ \text{m}$

$d_{z,1,4} = -3{,}172\ \text{m};\ d_{z,2,4} = -3{,}170\ \text{m}$

and according to Formula (6):

$a_z = -3{,}170\ 5\ \text{m}$

and according to Formula (7):

$r_{z,1,1} = -0{,}000\ 5\ \text{m};\ \text{r}_{z,2,1} = -0{,}000\ 5\ \text{m}$

$r_{z,1,2} = -0{,}000\ 5\ \text{m};\ \text{r}_{z,2,2} = 0{,}002\ 5\ \text{m}$

$r_{z,1,3} = 0{,}000\ 5\ \text{m};\ \text{r}_{z,2,3} = -0{,}000\ 5\ \text{m}$

$r_{z,1,4} = -0{,}001\ 5\ \text{m};\ \text{r}_{z,2,4} = 0{,}000\ 5\ \text{m}$

and according to Formula (8):

$d_z = 0{,}002\ 5\ \text{m}$

Annex B
(informative)

Example of the full test procedure

B.1 Measurements of x- and y-coordinates

Table B.1 contains an example of observed data taken in accordance with the full test procedure.

Table B.1 — Measurements

Dimensions in metres

Seq. No.	Instrument station *i*	Target point *j*	Set *k*	Telescope face	*x*	*y*	*z*
1	1	1	1	I	57,053	50,000	10,902
2		2			1,469	39,157	13,120
3		3			39,429	−2,997	10,641
4		1	2	II	57,053	50,001	10,902
5		2			1,470	39,159	13,121
6		3			39,426	−2,998	10,640
7		1	3	I	57,054	50,001	10,902
8		2			1,468	39,156	13,120
9		3			39,427	−2,997	10,640
10		1	4	II	57,054	50,000	10,902
11		2			1,470	39,158	13,121
12		3			39,428	−2,998	10,640
13	2	1	1	I	23,040	96,697	8,837
14		2			45,141	44,555	11,056
15		3			78,535	90,411	8,576
16		1	2	II	23,043	96,698	8,834
17		2			45,139	44,555	11,056
18		3			78,535	90,412	8,576
19		1	3	I	23,042	96,697	8,835
20		2			45,142	44,555	11,056
21		3			78,534	90,412	8,574
22		1	4	II	23,040	96,696	8,834
23		2			45,140	44,555	11,056
24		3			78,534	90,412	8,574

Observer: Y. Ohshima
Weather: sunny
Temperature: 29 °C
Air pressure: 1006 hPa
Instrument type and number: NT xxx 309090
Date: 2010–07–08

Table B.1 *(continued)*

Seq. No.	Instrument station *i*	Target point *j*	Set *k*	Telescope face	*x*	*y*	*z*
25	3	1	1	I	74,685	92,755	11,703
26		2			18,066	93,974	13,922
27		3			46,198	44,716	11,442
28		1	2	II	74,686	92,752	11,703
29		2			18,068	93,975	13,922
30		3			46,198	44,715	11,442
31		1	3	I	74,687	92,752	11,703
32		2			18,068	93,976	13,922
33		3			46,199	44,715	11,442
34		1	4	II	74,689	92,751	11,701
35		2			18,068	93,975	13,923
36		3			46,199	44,715	11,442

Observer: Y. Ohshima
Weather: sunny
Temperature: 29 °C
Air pressure: 1006 hPa
Instrument type and number: NT xxx 309090
Date: 2010-07-08

B.2 Calculation

B.2.1 x-, y-coordinates

According to Formula (10):

$$L_1 = 56{,}726\ 7\ \text{m}$$

$$L_2 = 55{,}849\ 9\ \text{m}$$

$$L_3 = 56{,}632\ 1\ \text{m}$$

According to Formula (15):

$$\left(x_{g,1}, y_{g,1}\right) = \left(32{,}650\ 1\ \text{m},\ 28{,}720\ 2\ \text{m}\right)$$

$$\left(x_{g,2}, y_{g,2}\right) = \left(48{,}905\ 4\ \text{m},\ 77{,}221\ 3\ \text{m}\right)$$

$$\left(x_{g,3}, y_{g,3}\right) = \left(46{,}317\ 6\ \text{m},\ 77{,}147\ 6\ \text{m}\right)$$

Table B.2 according to Formula (16):

Table B.2 — Coordinates of the centre of gravity of the mathematical model

Dimensions in metres

Instrument station i	Set k	$X_{t,i,1,k}$	$Y_{t,i,1,k}$	$X_{t,i,2,k}$	$Y_{t,i,2,k}$	$X_{t,i,3,k}$	$Y_{t,i,3,k}$
1	1	4,624 5	12,506 3	61,256 6	12,506 3	32,069 1	61,147 9
1	2	4,624 5	12,506 3	61,256 6	12,506 3	32,069 1	61,147 9
1	3	4,624 5	12,506 3	61,256 6	12,506 3	32,069 1	61,147 9
1	4	4,624 5	12,506 3	61,256 6	12,506 3	32,069 1	61,147 9
2	1	20,879 9	61,007 4	77,512 0	61,007 4	48,324 4	109,649 0
2	2	20,879 9	61,007 4	77,512 0	61,007 4	48,324 4	109,649 0
2	3	20,879 9	61,007 4	77,512 0	61,007 4	48,324 4	109,649 0
2	4	20,879 9	61,007 4	77,512 0	61,007 4	48,324 4	109,649 0
3	1	18,292 0	60,933 7	74,924 1	60,933 7	45,736 6	109,575 3
3	2	18,292 0	60,933 7	74,924 1	60,933 7	45,736 6	109,575 3
3	3	18,292 0	60,933 7	74,924 1	60,933 7	45,736 6	109,575 3
3	4	18,292 0	60,933 7	74,924 1	60,933 7	45,736 6	109,575 3

Table B.3 according to Formulae (20) and (21):

Table B.3 — Vertex coordinates of the mathematical model

Dimensions in metres

Instrument station i	Set k	$X_{m,i,1,k}$	$Y_{m,i,1,k}$	$X_{m,i,2,k}$	$Y_{m,i,2,k}$	$X_{m,i,3,k}$	$Y_{m,i,3,k}$
1	1	57,052 9	49,999 8	1,468 5	39,157 1	39,428 9	−2,996 4
1	2	57,053 9	49,998 7	1,469 0	39,158 6	39,427 4	−2,996 8
1	3	57,053 0	49,999 7	1,468 5	39,157 3	39,428 7	−2,996 5
1	4	57,053 6	49,999 0	1,468 8	39,158 1	39,427 9	−2,996 7
2	1	23,040 1	96,697 0	45,141 0	44,555 5	78,535 1	90,411 2
2	2	23,040 8	96,698 0	45,139 8	44,555 6	78,535 6	90,410 1
2	3	23,039 9	96,696 8	45,141 4	44,555 4	78,535 0	90,411 6
2	4	23,039 9	96,696 8	45,141 4	44,555 4	78,535 0	90,411 6
3	1	74,685 9	92,753 9	18,067 0	93,974 0	46,199 8	44,714 9
3	2	74,686 7	92,752 5	18,067 8	93,975 4	46,198 2	44,714 9
3	3	74,686 8	92,752 4	18,067 9	93,975 5	46,198 1	44,714 9
3	4	74,686 9	92,752 1	18,068 1	93,975 8	46,197 8	44,714 9

According to Formula (24)

$$\sum r_{xy}^2 = 0{,}000\ 061\ 6\ \text{m}^2$$

According to Formulae (26) and (27):

$$s_{XY} = 0{,}001\ 10\ \text{m}$$

$u_{\text{ISO-TS-XY}} = 0{,}001\ 10\,\text{m}$

B.2.2 z-coordinate

Table B.4 according to Formulae (28), (29), (30), (31):

Table B.4 — Residuals of the height differences

Instrument station i	Set k	$d_{z,i,2,k}$ m	$d_{z,i,3,k}$ m	$r_{z,i,2,k}$ m	$r_{z,i,3,k}$ m	$r_z{}^2{}_{,i,2,k}$ m^2	$r_z{}^2{}_{,i,3,k}$ m^2
1	1	2,218	−0,261	−0,001 75	−0,000 25	0,000 003 1	0,000 000 1
	2	2,219	−0,262	−0,000 75	−0,001 25	0,000 000 6	0,000 001 6
	3	2,218	−0,262	−0,001 75	−0,001 25	0,000 003 1	0,000 001 6
	4	2,219	−0,262	−0,000 75	−0,001 25	0,000 000 6	0,000 001 6
2	1	2,219	−0,261	−0,000 75	−0,000 25	0,000 000 6	0,000 000 1
	2	2,222	−0,258	0,002 25	0,002 75	0,000 005 1	0,000 007 6
	3	2,221	−0,261	0,001 25	−0,000 25	0,000 001 6	0,000 000 1
	4	2,222	−0,260	0,002 25	0,000 75	0,000 005 1	0,000 000 6
3	1	2,219	−0,261	−0,000 75	−0,000 25	0,000 000 6	0,000 000 1
	2	2,219	−0,261	−0,000 75	−0,000 25	0,000 000 6	0,000 000 1
	3	2,219	−0,261	−0,000 75	−0,000 25	0,000 000 6	0,000 000 1
	4	2,222	−0,259	0,002 25	0,001 75	0,000 005 1	0,000 003 1
$\Sigma d_{z,i,j,k}$		26,637	−3,129				
		$a_{z,2}$	$a_{z,3}$				
Mean value		2,219 8	−0,260 7				

According to Formula (34)

$u_{\text{ISO-TS-Z}} = 0{,}000\ 98\,\text{m}$

B.3 Statistical tests

B.3.1 Statistical test according to Question a)

Test for x and y;

$\sigma = 5{,}0$ mm

$u_{\text{ISO-TS-XY}} = 1{,}10\,\text{mm}$

$\nu_{XY} = 51$

$1{,}10\ \text{mm} \leq 5{,}0\ \text{mm} \times 1{,}16$

$1,10 \text{ mm} \leq 5,8 \text{ mm}$

Since the above condition is fulfilled, the null hypothesis stating that the experimental standard deviation

$u_{\text{ISO-TS-XY}} = 1,10 \text{ mm}$

$\sigma = 5,0 \text{ mm}$

is smaller than or equal to the manufacturer's value is not rejected at the confidence level of 95 %.

Test for z:

$\sigma = 5,0 \text{ mm}$

$u_{\text{ISO-TS-Z}} = 0,98 \text{ mm}$

$\nu_Z = 22$

$0,98 \text{ mm} \leq 5,0 \text{ mm} \times 1,24$

$0,98 \text{ mm} \leq 6,2 \text{ mm}$

Since the above condition is fulfilled, the null hypothesis stating that the experimental standard deviation

$u_{\text{ISO-TS-Z}} = 0,98 \text{ mm}$

is smaller than or equal to the manufacuter's value σ = 5,0 mm is not rejected at the confidence level of 95 %.

B.3.2 Statistical test according to Question b)

Test for x and y:

$s = 1,10 \text{ mm}$

$\tilde{s} = 1,15 \text{ mm}$

$\nu_{XY} = 51$

$$0,57 \leq \frac{1,21 \text{ mm}^2}{1,32 \text{ mm}^2} \leq 1,74$$

$$0,57 \leq 0,92 \leq 1,74$$

Since the above condition is fulfilled, the null hypothesis stating that the experimental standard deviations s = 1,10 mm and $\tilde{s}$ = 1,15 mm belong to the same population is not rejected at the confidence level of 95 %.

Test for z:

$s = 0{,}98$ mm

$\tilde{s} = 1{,}15$ mm

$v_Z = 22$

$$0{,}42 \leq \frac{0{,}96\ \text{mm}^2}{1{,}32\ \text{mm}^2} \leq 2{,}36$$

$$0{,}42 \leq 0{,}73 \leq 2{,}36$$

Since the above condition is fulfilled, the null hypothesis stating that the experimental standard deviations $s = 0{,}98$ mm and $\tilde{s} = 1{,}15$ mm belong to the same population is not rejected at the confidence level of 95 %.

Annex C
(informative)

Example of the calculation of a combined uncertainty budget (Type A and Type B)

C.1 Uncertainty budget example

C.1.1 Sources of uncertainty

The analysis of measurements:

$u_{\text{ISO-TS}}$

are obtained from Annex B

$u_{\text{ISO-TS-XY}} = 0{,}001\ 10$ m

$u_{\text{ISO-TS-Z}} = 0{,}000\ 98$ m

Total station:

According to the specification by the manufacturer, the uncertainty of distance $u_{\text{dist-TS}}$ is obtained by applying the manufacturer's ± (3 + 2 ppm × D) and maximum measured distance = 57 m.

$$u_{\text{dist-TS}} = 3 + 2 \times 57\ 000 \times 10^{-6} = 3{,}1 \text{ mm}$$

The uncertainty of horizontal angle measurement $u_{\theta\text{-TS}}$ is obtained by applying the manufacturer's specification 5" (according to ISO 17123-3) as

$u_{\theta\text{-TS}}$=5"

The uncertainty of vertical angle measurement $u_{\Psi\text{-TS}}$ is obtained by applying the manufacturer's specification 5" (according to ISO 17123-3) as

$u_{\Psi\text{-TS}}$=5"

The uncertainty of minimum display digit u_{disp}

$$u_{\text{dispx}} = u_{\text{dispy}} = u_{\text{dispz}} = \frac{0{,}5}{\sqrt{3}} = 0{,}29 \text{ mm}$$

when minimum digit is 1 mm.

Tripod:

The influenced quantity of the tripod u_{trd}

$$u_{trd} = \frac{3}{\sqrt{3}} = 1{,}73''$$

with the estimated torsion according to ISO 12858-2 and rectangular distribution.

The stability of the tripod height u_{hs} is estimated within 0,05 mm according to ISO 12858-2, which can be omitted from the budget.

Atmospheric condition

The uncertainty of temperature u_{temp}:

$$u_{temp} = 1 \times 57\ 000 \times 10^{-6} = 0{,}057 \text{ mm, with } \pm 1\ °\text{C from experience}$$

The uncertainty of pressure u_{prs}:

$$u_{prs} = 0{,}3 \times 5 \times 57\ \ 000 \times 10^{-6} = 0{,}086 \text{ mm, with 5 hPa from experience}$$

The uncertainty of humidity can be omitted from the budget, as its influence is negligibly small for the maximum distance of 100 m in the test.

C.1.2 Uncertainty calculation

The uncertainty on polar coordinate is calculated according Formulae (53), (54), (55):

$$u_{dist} = \sqrt{u_{dist\text{-}TS}^2 + u_{temp}^2 + u_{prs}^2} = \sqrt{3{,}114^2 + 0{,}057^2 + 0{,}086^2} = 3{,}116 \text{ mm}$$

$$u_{\theta} = \sqrt{u_{\theta\text{-}TS}^2 + u_{trd}^2} = \sqrt{5^2 + 1{,}73^2} = 5{,}29''$$

$$u_{\psi} = \sqrt{u_{\psi\text{-}TS}^2 + u_{hs}^2} = 2{,}89''$$

The uncertainty on rectangular coordinate is calculated according to Formulae (56), (57):

$$u_N^2 + u_E^2 = 11{,}84 \text{ mm}^2$$

$$u_H^2 = 1{,}91 \text{ mm}^2$$

Combined uncertainty is calculated according to Table C.1 and Formulae (58), (59):

$$u_{xy} = \sqrt{1{,}10^2 + 11{,}84 + 0{,}29^2} = 3{,}62 \text{ mm}$$

$$u_z = \sqrt{0{,}98^2 + 1{,}91 + 0{,}29^2} = 1{,}72 \text{ mm}$$

Table C.1 — Uncertainty budget on rectangular coordinate

Dimensions in millimetres

Input quantity $u(x_i)$	Input estimate	Standard uncertainty $u(x_i)$	Distribution	Sensitivity coefficient	$u_i(c_{xy}) \equiv \lvert c_i \rvert \cdot u(x_i)$	Evaluation	Remark
$u_{ISO\text{-}TS\text{-}XY}$	—	1,10	normal	1	1,10	Type A	Formula (27)
$u_{ISO\text{-}TS\text{-}Z}$	—	0,98	normal	1	0,98	Type A	Formula (34)
$(u_x^2 + u_y^2)^{0,5}$	D_{max} = 57 m, Va = 1°	3,44	normal	1	3,44	Type B	
u_z	D_{max} = 57 m, Va = 1°	1,38	normal	1	1,38	Type B	
u_{disp}	0	0,29	rectangle	1	0,29	Type B	
		Final results		u_{xy}	3,62		
				u_z	1,72		

C.2 Expanded uncertainty

$$U_{xy} = 2 \times 3{,}62 \approx 7 \text{ mm}$$

$$U_z = 2 \times 1{,}72 \approx 3 \text{ mm}$$

Annex D
(informative)

Sources which are not included in uncertainty evaluation

The sources of uncertainty shown in Table D.1 are not to be evaluated individually, since those are already considered in the corresponding influence quantities listed in Table 4 or not relevant.

Table D.1 — Sources of uncertainty not to be evaluated individually

Source of uncertainty	Distance	Vertical angle	Horizontal angle
Resolving power of telescope	•	•	•
Cross hair error		•	•
Centring of total station	•	•	•
Sighting axis and vertical axis			•
Vertical-axis tilt of total station	•	•	•
Line-of-sight error			•
Tilting-axis error		•	•
Graduation error of H circle			•
Eccentric error of H circle			•
Vertical compensate error		•	
Horizontal compensate error			•
Additional constant	•		
Prism constant error	•		
Parameter of atmospheric factor	•		
Centring of prism	•		
Direction of prism face	•	•	•

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