

8.1.4. c)

$$\lim_{x \rightarrow a^+} \frac{\sqrt{x} - \sqrt{a} + \sqrt{x-a}}{\sqrt{x^2 - a^2}}$$

(Note: In the original image, the term \sqrt{x} in the numerator is circled, and an arrow points from the circle to the letter f above it.)

$$f(x) = \sqrt{x} \quad P_1(f, a) = f(a) + f'(a)(x-a) = \sqrt{a} + \frac{(x-a)}{2\sqrt{a}}$$
$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$\lim_{x \rightarrow a^+} \frac{\cancel{\sqrt{a}} + \frac{(x-a)}{2\sqrt{a}} - \cancel{\sqrt{a}} + \sqrt{x-a}}{\sqrt{x^2 - a^2}} = \lim_{x \rightarrow a^+} \frac{(x-a) - 2\sqrt{a}\sqrt{x-a}}{\sqrt{x^2 - a^2}}$$

$$= \lim_{x \rightarrow a^+} \frac{x-a}{\underbrace{\sqrt{x^2 - a^2}}_{(x+a)(x-a)}} - \frac{2\sqrt{a}\sqrt{x-a}}{\underbrace{\sqrt{x^2 - a^2}}_{(x-a)(x+a)}} = \lim_{x \rightarrow a^+} \frac{\overbrace{\sqrt{x-a}\sqrt{x-a}}^{x-a}}{\cancel{\sqrt{x-a}}\sqrt{x+a}} - \frac{2\sqrt{a}\sqrt{x-a}}{\sqrt{x+a}\cancel{\sqrt{x-a}}}$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$\lim_{x \rightarrow a^+} \frac{\sqrt{x-a}}{\sqrt{x+a}} - \frac{2\sqrt{a}}{\sqrt{x+a}} = 0 - \frac{2\sqrt{a}}{\sqrt{2a}} = -\frac{\sqrt{2}\sqrt{2}\sqrt{a}}{\sqrt{2}\sqrt{a}} = -\sqrt{2}$$

8.2.3

$$x^2 = \text{sen}(x)$$

$$P_3(f, 0) = 0 + \overset{\cos(0)}{\underset{1}{1}}x + \overset{-\text{sen}(0)}{\underset{0}{0}} - \overset{-\cos(0)}{\underset{1}{1}}\frac{x^3}{6} = x - \frac{x^3}{6}$$

$$r = x$$

$$r^2 = r - \frac{r^3}{6}$$

$$\left. \begin{array}{l} r=0 \\ \frac{-1 \pm \sqrt{1+4/6}}{1/3} = -3 \pm \sqrt{9(1+4/6)} = -3 \pm \sqrt{15} \end{array} \right\}$$

$$\frac{4/6}{1/6} = 5/3$$

$$r = -3 + \sqrt{15}$$

$$r \in (0, 1]$$

$$r \approx 0,8729$$

$$r^2 \approx 0,7620$$

$$\sin(r) \approx 0,7662$$

demostrar que $|r^2 - \sin(r)| < 0,05$

$$P_3(f, 0) + R_3$$

$$|a+b| \leq |a| + |b|$$

$$|r^2 - P_3(f, 0) - R_3| \leq \underbrace{|r^2 - P_3(f, 0)|}_{=0} + |-R_3| = |R_3|$$

$$|R_3| = \frac{f^{(4)}(\xi) x^4}{4!}$$

$$\Rightarrow |r^2 - \sin(r)| \leq \frac{f^{(4)}(\xi) x^4}{4!}$$

$$P = f(0) + f'(0)x + f''(0)\frac{x^2}{2} + \underbrace{f'''(0)\frac{x^3}{6} + \dots}_{\text{acotada}}$$

$$|r^2 - \text{sen}(r)| \leq \frac{\overbrace{f^{(4)}(\xi)}^{\text{acotada}} x^4}{4!} < \frac{1}{4!}$$

$$|r^2 - \text{sen}(r)| < \frac{1}{24} < 0,05$$

$$R_k(x) = \frac{f^{(k+1)}(\xi)(x-a)^{k+1}}{(k+1)!}$$