8.1.4. c)

$$\lim_{x \to a^+} \frac{\underbrace{\sqrt{x}}_{f} - \sqrt{a} + \sqrt{x-a}}{\sqrt{x^2-a^2}}$$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$$P_1(f,a) = f(a) + f'(a)(x-a) = \sqrt{a} + \frac{(x-a)}{2\sqrt{a}}$$

$$\lim_{x \to a^+} \frac{\cancel{\sqrt{a}} + \frac{(x-a)}{2\sqrt{a}} - \cancel{\sqrt{a}} + \sqrt{x-a}}{\sqrt{x^2-a^2}} = \lim_{x \to a^+} \frac{(x-a) - 2\sqrt{a}\sqrt{x-a}}{\sqrt{x^2-a^2}}$$

$$= \lim_{x \to a^+} \frac{x-a}{\sqrt{\underbrace{x^2-a^2}_{(x+a)(x-a)}}} - \frac{2\sqrt{a}\sqrt{x-a}}{\sqrt{\underbrace{x^2-a^2}_{(x-a)(x+a)}}} = \lim_{x \to a^+} \frac{\overbrace{\cancel{\sqrt{x-a}}\sqrt{x-a}}^{x-a}}{\cancel{\sqrt{x-a}}\sqrt{x+a}} - \frac{2\sqrt{a}\cancel{\sqrt{x-a}}}{\sqrt{x+a}\cancel{\sqrt{x-a}}}$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$\lim_{x \to a^+} \frac{\sqrt{x-a}}{\sqrt{x+a}} - \frac{2\sqrt{a}}{\sqrt{x+a}} = 0 - \frac{2\sqrt{a}}{\sqrt{2a}} = -\frac{\sqrt{2}\sqrt{2}\sqrt{a}}{\sqrt{2}\sqrt{a}} = -\sqrt{2}$$

8.2.3

$$x^2 = \text{sen}(x)$$

$$P_3(f,0) = 0 + \overset{\cos(0)}{1}x + \overset{-\text{sen}(0)}{0} - \overset{-\cos(0)}{1}\frac{x^3}{6} = x - \frac{x^3}{6}$$

$r = x$

$$r^2 = r - \frac{r^3}{6}$$

$r = 0$

$$\frac{-1 \pm \sqrt{1 + 4/6}}{1/3} = -3 \pm \sqrt{9\underbrace{(1+4/6)}_{10/6 = 5/3}} = -3 \pm \sqrt{15}$$

$$r \in [0,1]$$

$$r = -3 + \sqrt{15}$$

$$r\left(\frac{r^2}{6} + r - 1\right)$$

$r \simeq 0{,}8729$

$r^2 \simeq 0{,}7620$

$\text{sen}(r) \simeq 0{,}7662$

demostrar que $\left|r^2 - \underbrace{\text{sen}(r)}_{P_3(f,0) + R_3}\right| < 0{,}05$

$|a+b| \leq |a| + |b|$

$$|r^2 - P_3(f,0) - R_3| \leq \underbrace{|r^2 - P_3(f,0)|}_{=0} + |-R_3| = |R_3|$$

$$|R_3| = f''''(\varepsilon)\frac{x^4}{4!} \implies |r^2 - \text{sen}(r)| \leq \frac{f''''(\varepsilon)\, x^4}{4!}$$

$$P = f(0) + f'(0)x + f''(0)\frac{x^2}{2} + f'''(0)\frac{x^3}{6} + \dots$$

$$|r^2 - \mathrm{sen}(r)| \leq \overbrace{\frac{f''''(\varepsilon)x^4}{4!}}^{\text{acotada}} < \frac{1}{4!}$$

$$R_K(x) = \frac{f^{(K+1)}(\varepsilon)(x-a)^{K+1}}{(K+1)!}$$

$$|r^2 - \mathrm{sen}(r)| < \frac{1}{24} < 0{,}05$$