$$\int \frac{1}{1+e^x}\,dx$$

7.6.3.a)

$$\int \frac{e^x}{e^x}\frac{1}{1+e^x}\,dx = \int \frac{e^x}{e^x+e^{2x}}\,dx$$

du

$$\left.\begin{aligned} e^x &= u \\ e^x dx &= du \end{aligned}\right\} \int \frac{du}{u+u^2} = \int \frac{du}{u(u+1)} = \int \frac{A}{u}\,du + \int \frac{B}{u+1}\,du$$

fracciones simples

$A = 1$

$B = -1$

$$\int \frac{du}{u} - \int \frac{du}{u+1} = \ln(u) - \ln(u+1)$$

$$\Longrightarrow \ln(e^x) - \ln(e^x + 1)$$

$$P_n(f, a) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \dots$$

$$P_n(f, a) = \sum_{i=0}^{n} f^{(i)}(a) \frac{(x-a)^i}{i!}$$

8.1 (1) $f(0), f'(0), \dots, f^{(4)}(0)$

$$P_4(f,0) = 3 - 5x + 4x^2 - x^3 - 2x^4$$

$$P_4(f,0) = f(0) + f'(0)x + f''(0)\frac{x^2}{2} + f'''(0)\frac{x^3}{6} + f^{(4)}(0)\frac{x^4}{24}$$

$$3 - 5x + 4x^2 - x^3 - 2x^4 = f(0) + f'(0)x + f''(0)\frac{x^2}{2} + f'''(0)\frac{x^3}{6} + f^{(4)}(0)\frac{x^4}{24}$$

$$\Rightarrow \begin{cases} 3 = f(0) \\ -5 = f'(0) \\ 4 = \dfrac{f''(0)}{2} \Rightarrow f''(0) = 8 \end{cases} \quad \left| \; -1 = \frac{f'''(0)}{6} \Rightarrow f'''(0) = -6 \right. \quad \left\{ \; -2 = \frac{f^{(4)}(0)}{24} \Rightarrow f^{(4)}(0) = -48 \right.$$

8.1 2 a)

$f(x) = x^4 - x^3 + 2$

$a = 0 \qquad a = 1$

$n = 2$

$f'(x) = 4x^3 - 3x^2$

$f''(x) = 12x^2 - 6x$

$$P_2(f,0) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!}$$

$$P_2(f,0) = 2 + 0 + 0$$

$$P_2(f,1) = 2 + 1x + \frac{6x^2}{2}$$

(c)

$$f(x) = \operatorname{sen}(x)$$

$$a = \pi$$

$$n = 6$$

$$P_6(f,\pi) = \overset{0}{\cancel{f(\pi)}} + f'(\pi)(x-\pi) + \overset{0}{\cancel{f''(\pi)}}\frac{(x-\pi)^2}{2} + \cdots$$

$$\cdots + f'''(\pi)\frac{(x-\pi)^3}{6} + \overset{0}{\cancel{f^{(4)}(\pi)}}\frac{(x-\pi)^4}{24} + f^{(5)}(\pi)\frac{(x-\pi)^5}{120}$$

$$+ \overset{0}{\cancel{f^{(6)}(\pi)}}\frac{(x-\pi)^6}{720}$$

$$f'(x) = \cos(x)$$

$$f''(x) = -\operatorname{sen}(x)$$

$$f'''(x) = -\cos(x)$$

$$f^{(4)}(x) = \operatorname{sen}(x)$$

$$f^{(5)}(x) = \cos(x)$$

$$f^{(6)}(x) = -\operatorname{sen}(x)$$

$$\boxed{P_6(f,\pi) = -1(x-\pi) + \frac{(x-\pi)^3}{6} - \frac{(x-\pi)^5}{120}}$$

i) $f(x) = \int_1^{x^2} e^{-t^2}\, dt$; $a=1$; $n=3$ $\quad \frac{\partial x^2}{\partial x}$

$$\frac{\partial e^{-x^4}}{\partial x} = \frac{\partial f(g(x))}{\partial x}$$

$f(x) = e^x \quad f'(x) = e^x$

$g(x) = -x^4 \quad g'(x) = -4x^3$

$$f'(x) = 2x\, e^{-x^4}$$

$$f''(x) = 2e^{-x^4} + \left(-4x^3 e^{-x^4}\right) 2x = 2e^{-x^4}(1-4x^4)$$

$$\frac{\partial f(g(x))}{\partial x} = g'(x) f'(g(x))$$

$$f'''(x) = -4x^3 e^{-x^4}\, 2(1-4x^4) + (-4 \cdot 4x^3) 2e^{-x^4}$$

$f'''(1) = -8e^{-1}$

$$\frac{\partial e^{-x^4}}{\partial x} = -4x^3 e^{-x^4}$$

$f(1) = \int_1^1 e^{-t^2}\, dt = 0 \quad f'(1) = 2e^{-1} \quad f''(1) = -6e^{-1}$

$$P_3(f,1) = 0 + \frac{2}{e}(x-1) + \left(\frac{-6}{e}\right)\frac{(x-1)^2}{2} + \left(\frac{-8}{e}\right)\frac{(x-1)^3}{3!}$$

$3! = 3 \cdot 2 \cdot 1 = 6$

4 a $\lim_{x\to 0} \dfrac{x - \log(x+1)}{x^2} = \lim_{x\to 0} \dfrac{x - P_2(f,0)}{x^2}$

$\dfrac{x^2}{2} \cdot \dfrac{1}{x^2}$

$//$

$\cancel{x} - \cancel{x} + \frac{x^2}{2} / x^2$

pol. de Taylor

$\log(x+1) = f$

$\lim_{x\to 0} \dfrac{x - \left(x - \frac{x^2}{2}\right)}{x^2} = \lim_{x\to 0} \dfrac{x^2}{2x^2} = \dfrac{1}{2}$

$$\boxed{P_2(f,0) = 0 + 1x - \frac{x^2}{2} = x - \frac{x^2}{2}}$$

$\lim_{x\to 0} \dfrac{x - \left(x - \frac{x^2}{2}\right)}{x^2} = \lim_{x\to 0} \dfrac{\overbrace{x - x}^{0} + \frac{x^2}{2}}{x^2}$ ⓐ

$f'(x) = \dfrac{1}{x+1} = \dfrac{1}{u} = u^{-1}$

$P = f(0) + f'(0)x + f''(0)\dfrac{x^2}{2!} + f'''(0)\dfrac{x^3}{3!}$

$f''(x) = \dfrac{-1}{(x+1)^2}$

ⓐ $\lim_{x\to 0} \dfrac{\frac{x^2}{2}}{x^2} = \lim_{x\to 0} \dfrac{\cancel{x^2}}{2\cancel{x^2}} = \dfrac{1}{2}$

5

$$\lim_{x\to 0} \frac{x-\operatorname{arctg}(x)}{\underbrace{\operatorname{sen}^3(x)}_{g(f(x))}} \simeq \lim_{x\to 0} \frac{x-\left(x-\frac{x^3}{3}\right)}{g(x)} = \lim_{x\to 0} \frac{x-x+x^3/3}{x^3}$$

$g = x^3$

$f = \operatorname{sen}$

$$= \lim_{x\to 0} \frac{x^3/3}{x^3} = \frac{1}{3}$$

$\operatorname{sen}(x) = f(x)$

$$P_1(f,0) = \overset{\operatorname{sen}(0)}{\overset{\shortparallel}{f(0)}} + \overset{\cos(0)}{\overset{\shortparallel}{f'(0)}}\,x = x \simeq \operatorname{sen}(x) = f(x)$$

$\operatorname{arctg}(x) = h(x)$

$$P_3(h,0) = \overbrace{h(0)}^{0} + \overbrace{h'(0)}^{1}\,x + \overbrace{h''(0)}^{0}\,\frac{x^2}{2} + \overbrace{h'''(0)}\,\frac{x^3}{6} = \overbrace{0 + x + 0 - \frac{2}{6}x^3}^{x-\frac{x^3}{3}}$$

$$h'(x) = \frac{1}{1+x^2} \;;\quad h''(x) = \frac{-2x}{(1+x^2)^2} \;;\quad h'''(x) = \frac{-2(1+x^2)^2 + 2(1+x^2)2x}{(1+x^2)^4}$$

6 d.

$$\log(x+1) - \frac{ax+bx^2}{cx+1} = \underbrace{x - \frac{x^2}{2} - \frac{ax+bx^2}{cx+1}}_{} = f(x)$$

$$\lim_{x\to 0} \frac{f(x)}{x^{n}} \boxed{= 0}$$

orden de infinitesimo

$$\boxed{\begin{array}{l} c=0 \\ a=1 \\ b=-\frac{1}{2} \end{array}}$$

$$\lim_{x\to 0} \frac{x + \frac{x^2}{2} + R_3 - \frac{ax+bx^2}{cx+1}}{x^3}$$

$$\lim_{x\to 0} \frac{x - \frac{x^2}{2} + R_2 - \frac{ax+bx^2}{cx+1}}{x^{3}} = 0 \qquad x^{3} = n$$

$R_3 = o(4) \simeq x^4 \ldots$ → orden de pol. Taylor

$$\lim_{x\to 0} \frac{R_3}{x^3} + \frac{\left(x - \frac{x^2}{2}\right)(cx+1) - ax + bx^2}{x^3(cx+1)}$$

$\frac{R_3}{x^3} \to 0$

$$\lim_{x\to 0} \frac{1}{(cx+1)} \cdot \frac{\overbrace{x - \frac{x^2}{2} + cx^2 - \frac{cx^3}{2} - ax + bx^2}^{-\frac{c}{2}x^3 + \left(c+b-\frac{1}{2}\right)x^2 + (1-a)x}}{x^3} = 0$$

$\frac{1}{(cx+1)} \xrightarrow{x\to 0} 1$

$$\lim_{x\to 0} \underbrace{\frac{-\frac{c}{2}x^3}{x^3}} + \underbrace{\frac{(b+c-\frac{1}{2})x^2}{x^3}} + \underbrace{\frac{(1-a)x}{x^3}} = 0$$

$$-\frac{c}{2} = 0 \qquad \frac{b+c-\frac{1}{2}}{x} \qquad \frac{1-a}{x^2} = 0 \Rightarrow a = 1$$

$$\boxed{a=1,\ b=\frac{1}{2},\ c=0}$$

$$c = 0$$

$$b+c-\frac{1}{2} = 0 \Rightarrow b+c = \frac{1}{2} \Rightarrow b = \frac{1}{2}$$