7.2.3 c

$$\int \frac{x^4\,dx}{\underbrace{(x+1)(x+2)(x+3)}}$$

$N \,\lfloor\, D$

$R \quad C$

$\underline{N = D.C + R}$

$$(x^2+3x+2)(x+3) = x^3+3x^2+2x+3x^2+9x+6 = x^3+6x^2+11x+6$$

$$\begin{array}{l|l} x^4 & \boxed{}x^3+6x^2+11x+6 \\ \hline -\;x^4+6x^3+11x^2+6x & x \quad -6 \\ \hline -6x^3-11x^2-6x & \\ -\;-6x^3-36x^2-66x-36 & \\ \hline 25x^2+60x+36 & \end{array}$$

$$x^4 = (x^3+6x^2+11x+6)(x-6) + (25x^2+60x+36)$$

$$\int \frac{\overbrace{(x^3+6x^2+11x+6)}(x-6)\,d}{\underbrace{(x+1)(x+2)(x+3)}_{x^3+6x^2+11x+6}} + \int \frac{(25x^2+60x+36)\,dx}{(x+1)(x+2)(x+3)}$$

$$\int (x-6)\,dx + \int \frac{(25x^2+60x+36)\,dx}{(x+1)(x+2)(x+3)}$$

$$\int x-6 \, dx = \frac{x^2}{2} - 6x$$

$$\int \frac{25x^2+60x+36}{(x+1)(x+2)(x+3)} \, dx = \int \frac{A}{(x+1)} + \int \frac{B}{(x+2)} + \int \frac{C}{(x+3)}$$

$$A = \frac{25(-1)^2+60(-1)+36}{(-1+2)(-1+3)} = \frac{1}{2} \qquad B = \frac{25(-2)^2+60(-2)+36}{(-2+1)(-2+3)} = -16$$

$$C = \frac{25(-3)^2+60(-3)+36}{(-3+1)(-3+2)} = \frac{81}{2}$$

$$\frac{1}{2}\int \frac{1}{x+1}\,dx - 16\int \frac{1}{x+2}\,dx + \frac{81}{2}\int \frac{1}{x+3}\,dx$$

$$\frac{81}{2}\int \frac{1}{x+3}\,dx = \frac{81\ln(x+3)}{2}$$

$x+1=u$

$dx=du$

$$\frac{1}{2}\int \frac{1\,du}{u} = \frac{\ln(u)}{2} = \frac{\ln(x+1)}{2}$$

$$-16\int \frac{1}{x+2}\,dx = -16\ln(x+2)$$

$$\Rightarrow \int \frac{x^4}{(x+1)(x+2)(x+3)}\,dx = \frac{x^2}{2} - 6x + \frac{\ln(x+1)}{2} - 16\ln(x+2) + \frac{81}{2}\ln(x+3)$$

f)

$$\int \frac{4x-3}{3x^2+3x+2}\,dx$$

$$\frac{-3 \pm \sqrt{9-4.3.2}}{6} \rightarrow 9-24=-15$$

$3x^2+3x+2=u$

$6x+3\,dx=du$

$$(3x^2+3x+2)' = 6x+3$$

$$\frac{4}{6}\underbrace{(6x+3)}_{\text{quiero}}-5 = \underbrace{4x-3}$$

$= \frac{2}{3}$

$$\Longrightarrow \frac{2}{3}\int \frac{\overbrace{6x+3\,dx}^{du}}{\underbrace{3x^2+3x+2}_{=u}} - 5\int \frac{1}{3x^2+3x+2}\,dx$$

tengo

$$\frac{2}{3}\int \frac{1}{u} - \underbrace{5\int \frac{1\,dx}{3x^2+3x+2}}_{\frac{5}{3}\int \frac{1\,dx}{x^2+x+\frac{2}{3}}}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$a = x$$

$$(x+b)^2 = x^2 + \underbrace{2xb}_{b=\frac{1}{2}} + b^2$$

$$\frac{2}{3} = \frac{1}{4} + t$$

$$t = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$$

$$x^2 + x + \frac{2}{3} = \underbrace{\left(x + \frac{1}{2}\right)^2}_{x^2+x+\frac{1}{4}} + \frac{5}{12}$$

$$\frac{5}{3}\int \frac{1\,dx}{\left(x+\frac{1}{2}\right)^2+\frac{5}{12}} = \frac{5}{3}\int \frac{1\,dx}{\frac{5}{12}\left[1+\underbrace{\left[\sqrt{\frac{12}{5}}\left(x+\frac{1}{2}\right)\right]^2}_{\frac{12}{5}\left(x+\frac{1}{2}\right)^2}\right]} =$$

$$4 = \frac{12\cdot 5}{3\cdot 5}\int \frac{1\,dx}{1+\left(\sqrt{\frac{12}{5}}\left[x+\frac{1}{2}\right]\right)^2} =$$

CdeV $\quad u = \sqrt{\frac{12}{5}}\left(x+\frac{1}{2}\right) = \sqrt{\frac{12}{5}}\,x + \sqrt{\frac{12}{5}}\cdot\frac{1}{2}$

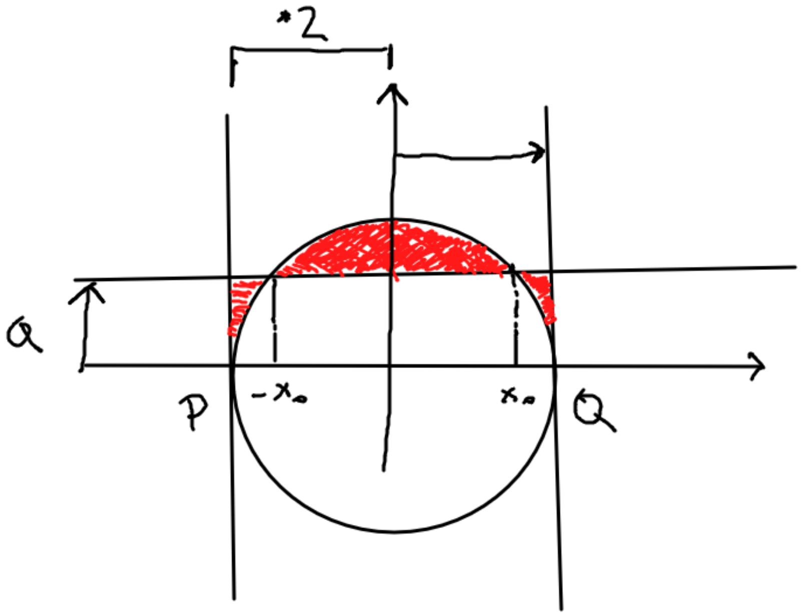
$$du = \sqrt{\frac{12}{5}}\,dx$$

$$4\sqrt{\frac{5}{12}}\int \frac{\overbrace{\sqrt{\frac{12}{5}}\,dx}^{=du}}{1+\Big(\underbrace{\sqrt{\tfrac{12}{5}}\left(x+\tfrac{1}{2}\right)}_{=u}\Big)^2} = \frac{4\cdot\sqrt{5}}{\sqrt{12}}\int \frac{du}{1+u^2}$$

$$\frac{2\sqrt{5}}{\sqrt{3}}\operatorname{arctg}(u) = \frac{2\sqrt{5}}{\sqrt{3}}\operatorname{arctg}\left(\frac{\sqrt{12}}{\sqrt{5}}\left(x+\frac{1}{2}\right)\right)$$

$$\int \frac{4x-3}{3x^2+3x+2}\,dx = \frac{2\sqrt{5}}{\sqrt{3}}\operatorname{arctg}\left(\frac{\sqrt{12}}{\sqrt{5}}\left[x+\frac{1}{2}\right]\right)+\frac{2}{3}\ln\left(3x^2+3x+2\right)$$

7.6.7



$$f(x) = \sqrt{1-x^2}$$

$$g(x) = a$$

$$f(x_0) = g(x_0)$$

$$\sqrt{1-x_0^2} = a \Rightarrow x_0 = \sqrt{1-a^2}$$

$$h(a) = \int_{P=-1}^{-x_0} g(x) - f(x)\,dx + \int_{-x_0}^{x_0} f(x) - g(x)\,dx + \int_{x_0}^{Q=1} g(x) - f(x)\,dx$$

TFC× TFC×

$$h(a) = \left[\int_0^{\sqrt{1-a^2}} \sqrt{1-x^2} - a \, dx + \int_{\sqrt{1-a^2}}^{1} a - \sqrt{1-x^2} \, dx \right] 2$$

$$h(a) = \left[\int_0^{\sqrt{1-a^2}} \sqrt{1-x^2} \, dx - \int_0^{\sqrt{1-a^2}} a \, dx + \int_{\sqrt{1-a^2}}^{1} a \, dx - \int_{\sqrt{1-a^2}}^{1} \sqrt{1-x^2} \, dx \right] 2$$

TFC $\qquad = a\sqrt{1-a^2} \qquad = a\left(1-\sqrt{1-a^2}\right) \qquad$ TF $\quad \dfrac{-a}{\sqrt{1-a^2}} \cdot \underbrace{\sqrt{1-\sqrt{1-a^2}^{\,2}}}_{\underbrace{\sqrt{1-(1-a^2)}}_{\sqrt{a^2}=a}}$

$a =$

$$h'(a) = \left[\sqrt{1-\sqrt{1-a^2}^{\,2}} \cdot \frac{-a}{\sqrt{1-a^2}} - \left(\sqrt{1-a^2} - \frac{a^2}{\sqrt{1-a^2}} \right) + 1 - \left(\sqrt{1-a^2} - \frac{a^2}{\sqrt{1-a^2}} \right) + \right] 2$$

$$\left(\sqrt{1-a^2}\right)' = \frac{-2a}{2\sqrt{1-a^2}} = \frac{-a}{\sqrt{1-a^2}}$$

$$h'(a) = \left[-\frac{a^2}{\sqrt{1-a^2}} - \sqrt{1-a^2} + \frac{a^2}{\sqrt{1-a^2}} + 1 - \sqrt{1-a^2} + \frac{a^2}{\sqrt{1-a^2}} - \frac{a^2}{\sqrt{1-a^2}} \right] 2$$

$$h'(a) = \left(1 - 2\sqrt{1-a^2}\right) 2 \Rightarrow h'(a) = 0$$

$$1 = 2\sqrt{1-a^2} \Rightarrow \frac{1}{2} = \sqrt{1-a^2} \Rightarrow \frac{1}{4} = 1 - a^2$$

$$a^2 = 1 - \frac{1}{4} \Rightarrow a^2 = \frac{3}{4} \Rightarrow \boxed{a = \frac{\sqrt{3}}{2}}$$