7.2.3

$$\int \frac{x \quad dx}{(x+1)(x+2)(x+3)} = \int \frac{A}{(x+1)} dx + \int \frac{B}{(x+2)} dx + \int \frac{C}{(x+3)} dx$$

$$\frac{x}{(x+1)(x+2)(x+3)} = \frac{A(x+2)(x+3) + B(x+1)(x+3) + C(x+1)(x+2)}{(x+1)(x+2)(x+3)}$$

$$1x = A(x^2+5x+6) + B(x^2+4x+3) + C(x^2+3x+2)$$

$$\begin{cases} A+B+C=0 \\ 5A+4B+3C=1 \\ 6A+3B+2C=0 \end{cases} \qquad \begin{array}{l} A=-B-C \\ 6(-B-C)+3B+2C=0 \\ -3B=4C \Rightarrow (\ldots) \end{array} \qquad \begin{cases} A=-1/2 \\ B=+2 \\ C=-3/2 \end{cases}$$

$$-\frac{1}{2}\int \frac{1}{x+1}\,dx + 2\int \frac{1}{x+2}\,dx - \frac{3}{2}\int \frac{1}{x+3}\,dx$$

$x+1 = u$ $\quad$ $x+2 = v$ $\quad$ $x+3 = z$

$dx = du$ $\quad$ $dx = dv$ $\quad$ $dx = dz$

$$-\frac{1}{2}\int \frac{1}{u}\,du + 2\int \frac{1}{v}\,dx - \frac{3}{2}\int \frac{1}{z}\,dz$$

$$-\frac{1}{2}\operatorname{Ln}(u) + 2\operatorname{Ln}(v) - \frac{3}{2}\operatorname{Ln}(z) = -\frac{1}{2}\operatorname{Ln}(x+1) + 2\operatorname{Ln}|x+2) - \frac{3}{2}\operatorname{Ln}|x+3)$$

$$\int \frac{x\,dx}{(x+1)(x+2)(x+3)} = \int \frac{A}{(x+1)}dx + \int \frac{B}{(x+2)}dx + \int \frac{C}{(x+3)}dx$$

A "tapamos" x+1 (el denominador en la ec. original

eval en x = raíz

$$A = \frac{x}{(x+2)(x+3)}\bigg|_{x=-1} = \frac{-1}{(-1+2)(-1+3)} = -\frac{1}{2}$$

(...)

$$B = \frac{x}{(x+1)(x+3)}\bigg|_{x=-2} = \frac{-2}{(-2+1)(-2+3)} = 2\ ; \quad C = \frac{x}{(x+1)(x+2)}\bigg|_{x=-3} = -\frac{3}{2}$$

$$\int \frac{1}{(x^2+1)(x^2+2)} = \int \frac{Ax+B}{(x^2+1)}\,dx + \int \frac{Cx+D}{(x^2+2)}\,dx$$

$$\frac{1}{(x^2+1)(x^2+2)} = \frac{(Ax+B)(x^2+2) + (Cx+D)(x^2+1)}{(x^2+1)(x^2+2)}$$

$$1 = Ax^3 + 2Ax + Bx^2 + 2B + Cx^3 + Cx + Dx^2 + D$$

$$1 = (A+C)x^3 + (B+D)x^2 + (2A+C)x + (2B+D)$$

$$\begin{cases} A+C=0 \rightarrow A=-C \\ B+D=0 \rightarrow B=-D \\ 2A+C=0 \\ 2B+D=1 \Rightarrow 2B-B=1 \Rightarrow B=1 \Rightarrow D=-1 \end{cases}$$

$$2A-A=0 \Rightarrow A=0 \Rightarrow C=0$$

$$\underbrace{\int \frac{1}{x^2+1}\,dx}_{\operatorname{arctg}(x)} - \int \frac{1\,dx}{x^2+2} = \operatorname{arctg}(x) - \frac{\operatorname{arctg}(x/\sqrt{2})}{\sqrt{2}}$$

pag. sig.

$$\int \frac{1}{x^2+2}\,dx = \int \frac{2}{2}\,\frac{dx}{x^2+2} = \int \frac{1}{2}\,\frac{dx}{\frac{x^2+2}{2}}$$

$$\int \frac{1}{2}\,\frac{dx}{\frac{x^2}{2}+\frac{2}{2}} = \int \frac{1}{\sqrt{2}}\,\underbrace{\frac{1}{\sqrt{2}}\,dx}_{=du}\,\frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2+1}$$

$$\frac{1}{2} = \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} \qquad \frac{x^2}{2} = \left(\frac{x}{\sqrt{2}}\right)^2$$

$$\left(\frac{x}{\sqrt{2}}\right) \to u$$

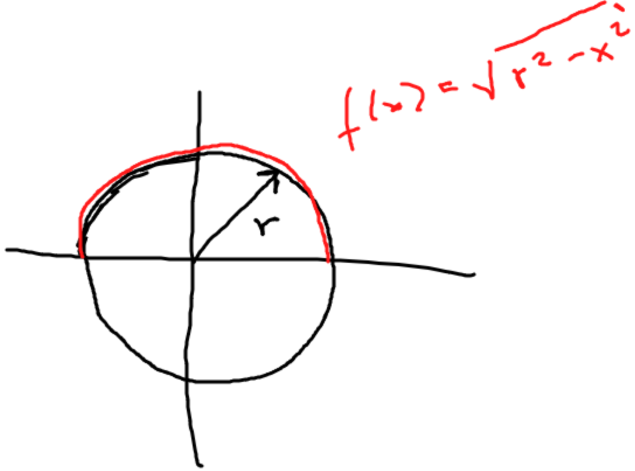
$$\frac{x}{\sqrt{2}} = u$$

$$\frac{dx}{\sqrt{2}} = du$$

$$\int \frac{1}{\sqrt{2}}\,\frac{du}{u^2+1} = \frac{1}{\sqrt{2}}\,\text{arctg}(u) = \frac{\text{arctg}(x/\sqrt{2})}{\sqrt{2}}$$

7.3.3

$$A_{e(0,r)} = r^2 A_{e(0,1)} = r^2 \int_{-1}^{1} \sqrt{1-x^2}\,dx$$



$$x^2 + y^2 = r^2 \Rightarrow y = \pm\sqrt{r^2 - x^2}$$

$$2\int_{-r}^{r} \sqrt{r^2 - x^2}\,dx = 2\frac{r}{r}\int_{-r}^{r} \sqrt{r^2 - x^2}\,dx = 2r\int_{-r}^{r} \sqrt{\frac{r^2 - x^2}{r^2}}\,dx$$

$$2r\int_{-r}^{r}\sqrt{\frac{r^2}{r^2}-\frac{x^2}{r^2}}\,dx = 2r\int_{-r}^{r}\sqrt{1-\left(\frac{x}{r}\right)^2}\,dx$$

ojo

$$= \frac{2r^2}{r}\int_{-r}^{r}\sqrt{1-\left(\frac{x}{r}\right)^2}\,dx \quad = du$$

$$\frac{x}{r} = u$$

$$\frac{dx}{r} = du$$

$$= 2r^2\int_{\frac{-r}{r}}^{\frac{r}{r}}\sqrt{1-u^2}\,du = A_{e(0,1)}\,r^2$$

$$1 = \text{sen}^2(x) + \cos^2(x)$$

$$A_{e(0,r)} = r^2 \underbrace{A_{e(0,1)}}$$

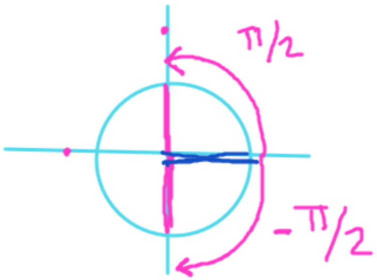
$$\frac{A_{e(0,1)}}{2} = \int_{-1}^{1} \sqrt{1-u^2}\,du = \int_{-\pi/2}^{\pi/2} \sqrt{1-\text{sen}^2(x)}\,\cos(x)\,dx$$

$$= \int_{-\pi/2}^{\pi/2} \sqrt{\cos^2(x)}\,\cos(x)\,dx$$

$$u = \text{sen}(x)$$

$$du = \cos(x)\,dx$$

$$= \int_{-\pi/2}^{\pi/2} \cos(x)\cos(x)\,dx = \int_{-\pi/2}^{\pi/2} \cos^2(x)\,dx$$



$$\int_{-\pi/2}^{\pi/2} \cos^2(x)\,dx = I$$

$$\int u\,dx = uv - \int v\,du$$

$$u = \cos x \longrightarrow du = -\operatorname{sen} x$$

$$dv = \cos x \longrightarrow v = \operatorname{sen} x$$

$$\cos x \operatorname{sen} x \Big|_{-\pi/2}^{\pi/2} + \int_{-\pi/2}^{\pi/2} \overbrace{\operatorname{sen}^2(x)}^{1-\cos^2 x}\,dx$$

$$\cos x \operatorname{sen} x \Big|_{-\pi/2}^{\pi/2} + \int_{-\pi/2}^{\pi/2} 1\,dx - \underbrace{\int_{-\pi/2}^{\pi/2} \cos^2(x)\,dx}_{I} = I$$

$$2I = \overbrace{\cos(x)\operatorname{sen}(x)\Big|_{-\pi/2}^{\pi/2}}^{0} + \int_{-\pi/2}^{\pi/2} 1\,dx$$

$$2I = \int_{-\pi/2}^{\pi/2} 1\,dx = x\Big|_{-\pi/2}^{\pi/2} = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

$$\Longrightarrow I = \int_{-1}^{1} \sqrt{1-u^2}\,du = \frac{\pi}{2} \Longrightarrow A_{e(0,1)} = 2I = \pi$$

$$A_{e(0,r)} = r^2 A_{e(0,1)} = r^2 \pi$$