⑥ b) d. Calcular $(f^{-1})'(0)$

$$f(x) = \int_1^{\operatorname{arctg}(x)^{=4}} \sqrt{t^2+e^t}\,dt = x \longrightarrow \text{cuesta sacar } f^{-1}$$

$f(x)$; $g(x) = f^{-1}(x)$

$g(f(x)) = x$ ✓

derivemos

$$g'(f(x))\,f'(x) = 1$$

$$\Rightarrow \boxed{g'(f(x)) = \frac{1}{f'(x)}}$$

$$f(x) = \int_1^{\operatorname{arctg}(x)} \underbrace{\sqrt{t^2+e^t}}_{h(x)}\,dt = H(\operatorname{arctg}) - H(1) \qquad (f^{-1})'(f(x)) = 1/f'(x)$$

$$(f^{-1})'(0)$$

$$f(x) = 0 = \int_1^{\operatorname{actg} x} \sqrt{t^2+e^t}\,dt \quad 1 = \operatorname{arctg}(x) \Rightarrow \boxed{x = \operatorname{tg}(1)}$$

$$f'(x) = H'(\operatorname{arctg})(\operatorname{arctg})' = \frac{h(\operatorname{arctg})}{1+x^2} = \frac{\sqrt{\operatorname{arctg}^2(x) + e^{\operatorname{arctg}(x)}}}{1+x^2}$$

$$\left(f^{-1}\right)'(x) = \frac{1}{f'(x)} = \frac{1+x^2}{\sqrt{\operatorname{arctg}^2(x) + e^{\operatorname{arctg} x}}}$$

$$\left(f^{-1}\right)'(x=0) = \frac{1+\operatorname{tg}^2(1)}{\sqrt{\operatorname{arctg}^2(\operatorname{tg}(1)) + e^{\operatorname{arctg}(\operatorname{tg}(1))}}} = \frac{1+\operatorname{tg}(1)}{\sqrt{1+e}}$$

$$\Rightarrow \left(f^{-1}\right)'(0) = \frac{1+\operatorname{tg}(1)}{\sqrt{1+e}}$$

12 $\displaystyle\int_a^b \frac{1}{1+x^2}\,dx$

$tg\,u = \dfrac{sen\,u}{cos\,u} \Longrightarrow$ derivada

$tg'u = \dfrac{cos^2 u + sen^2 u}{cos^2 u} = \dfrac{1}{cos^2 u} = sec^2 u$

$arc\,tg\,(x) = u$

$tg(u) = x$

$\dfrac{1}{sen\,x} = CSC\ x\ ;\ \dfrac{1}{cos\,x} = sec\ x\ ;\ \dfrac{1}{tg\,x} = ctg\,x$

derivamo

$tg'(u)\,u' = 1$

$sec^2 x - tg^2 x = 1 \Longrightarrow sec^2 x = 1 + tg^2 x$

$tg'u = 1 + tg^2 u \Longrightarrow tg'u = 1 + tg^2(arctg(x)) = 1 + x^2$

$$\operatorname{tg}'(u)\,u' = 1 \quad ; \quad \operatorname{tg}'(u) = 1 + x^2$$

$$u' = \frac{1}{\operatorname{tg}'(u)} \Rightarrow \operatorname{arctg}'(x) = \frac{1}{1+x^2}$$

$$\int_a^b \frac{1}{1+x^2}\,dx = \operatorname{arctg}(b) - \operatorname{arctg}(a)$$

7.2 Partes

$$\int u\,dv = uv - \int v\,du$$

I → inversas

L → logartitmicas

A → algebraicas ← u

T → trigonometricas ← v

E → exponenciales

1 $\int x \operatorname{sen} x\,dx = -x\cos x + \int \cos x = \operatorname{sen} x - x\cos x$

$u = x \rightarrow du = 1$

$dv = \operatorname{sen} x \rightarrow \int dv = v = -\cos x$

$$\int x \log x \, dx$$

I
L $\longrightarrow \log x = u \longrightarrow du = 1/x$
A $\longrightarrow x = dv \longrightarrow \int dv = v = \frac{x^2}{2} + C$
T
E

$$\int u\,dv = uv - \int v\,du$$

$$\boxed{\int x^n = \frac{x^{n+1}}{n+1}}$$

$$\int x \log x = \log x \left(\frac{x^2}{2} + C \right) - \overbrace{\int \left(\frac{x^2}{2} + C \right) \cdot \frac{1}{x}}^{\int \frac{x}{2} + \int \frac{C}{x}}$$

$$\int x \log x = \log x \cdot \frac{x^2}{2} + \cancel{\log x\, C} - \frac{x^2}{4} + K - \cancel{C \log(x)} = \log x \cdot \frac{x^2}{2} - \frac{x^2}{4} + K$$

$$\int \cos(x)\, e^{2x}\, dx$$

$$\int \underbrace{u\,dv}_{2} = \underbrace{uv - \int v\,du}_{2}$$

I

L

A

T $\rightarrow \cos x = u \rightarrow du = -\operatorname{sen} x$

E $\rightarrow e^{2x} = dv \rightarrow \int dv = v = \dfrac{e^{2x}}{2}$

$$dv = e^{2x} \rightarrow v = \frac{e^{2x}}{2}$$

$$\operatorname{sen} x = u \rightarrow du = \cos x$$

$$\int \cos x\, e^{2x}\, dx = \cos x \frac{e^{2x}}{2} + \int \frac{e^{2x}}{2} \operatorname{sen} x$$

$$1 + \frac{1}{4} = \frac{5}{4}\, x_0$$

$$\underbrace{\int \cos x\, e^{2x}\, dx}_{= x_0} = \cos x \frac{e^{2x}}{2} + \frac{\operatorname{sen} x\, e^{2x}}{2 \cdot 2} - \underbrace{\int \frac{e^{2x}}{2} \frac{\cos x}{2}}_{= \frac{x_0}{2}} \rightarrow \frac{5}{4} \int \cos x\, e^{2x}\, dx = \cos x \frac{e^{2x}}{2} + \operatorname{sen} x \frac{e^{2x}}{4}$$

$$\int \cos x \, e^{2x} \, dx = \frac{4}{5} \frac{e^{2x}}{2} \left(\cos x + \frac{\operatorname{sen} x}{2} \right) = \frac{2e^{2x}}{5} \left(\cos x + \frac{\operatorname{sen} x}{2} \right)$$

② Sustitución

$$\frac{-2}{-2}\int x\sqrt{1-x^2}\,dx = \int \frac{\sqrt{u}\,du}{-2} = \frac{1}{-2}\int u^{1/2}\,du = \frac{1}{-2}\frac{u^{1/2+1}}{\left(\frac{1}{2}+1\right)}$$

$$u = 1-x^2$$

$$du = -2x\,dx$$

$$= \frac{1}{-2}\frac{u^{3/2}}{3}2 = -\frac{u^{3/2}}{3} = \frac{-\left(\sqrt{u}\right)^3}{3} =$$

$$-\frac{\left(\sqrt{1-x^2}\right)^3}{3} = \frac{-\sqrt{(1-x^2)^3}}{3}$$

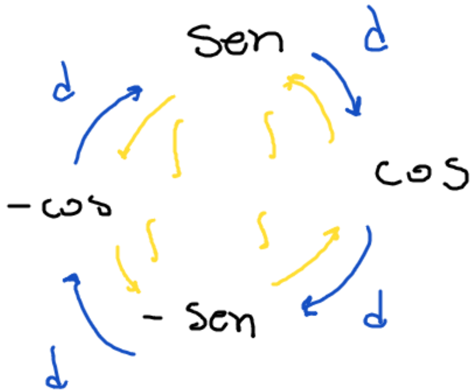
$$\int e^x \operatorname{sen}(e^x)\, dx = \int \operatorname{sen} u \, du = -\cos u = -\cos e^x$$

$$u = e^x$$

$$du = e^x dx$$

$$\int 1\, dx = \int dx = x + C$$

$$\int x\, du = x \int du = x(u + C)$$



$$\int \frac{e^x\,dx}{1+e^{2x}} = \int \frac{du}{1+u^2} = \operatorname{arctg}(u) = \operatorname{arctg}(e^x)$$

$$e^{2x} = (e^x)^2$$

$$u = e^x$$

$$du = e^x\,dx$$