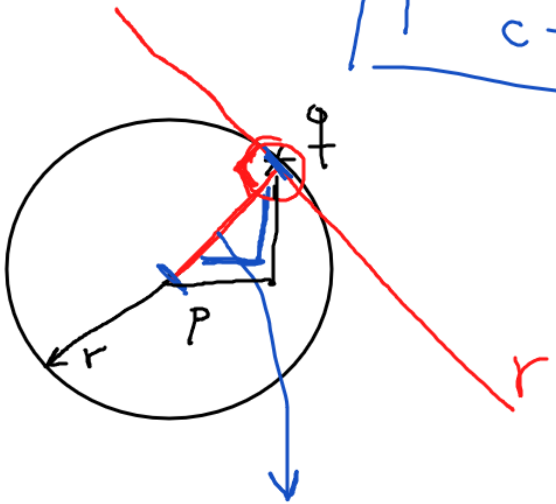
Recta tg

③ $C(p,r)$

$q \in C(p,r)$

$p = (a,b)$

$q = (c,d)$



$$\boxed{y = \frac{d-b}{c-a}(x-a)+b}$$

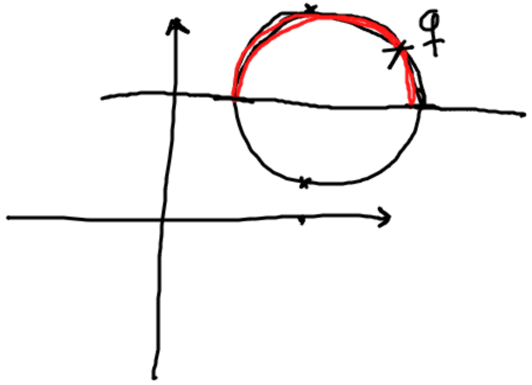
$(\cdot)\ y - b = m(x-a)$

$\longrightarrow m = \frac{(d-b)}{(c-a)}$

$$p = (a, b)\ ,\ r$$

$$m_p = \frac{-1}{m}$$

$$(x-a)^2 + (y-b)^2 = r^2$$



$$f(x) = y = b \pm \sqrt{r^2 - (x-a)^2}$$

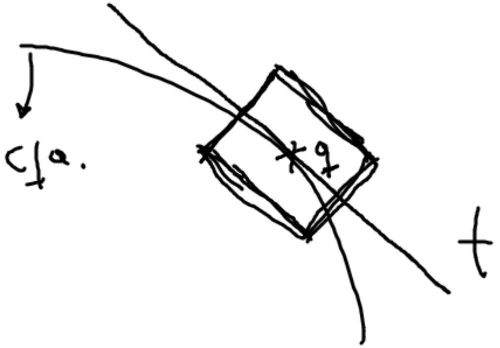
$$q = (c, d) \in C(p, r)$$

$$(c-a)^2 + (d-b)^2 = r^2$$

$$m_p = f'(q)$$

$$f(x) = b \pm \sqrt{r^2-(x-a)^2}$$

$$m = \frac{(d-b)}{(c-a)}$$



$$f'(x) = \frac{-2(x-a)}{2\sqrt{r^2-(x-a)^2}}$$

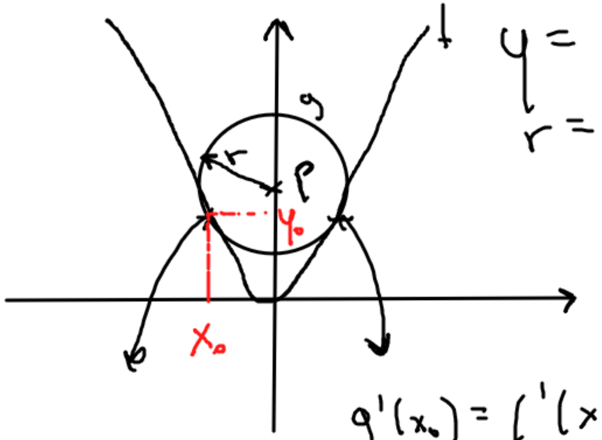
$$f'(q) = \frac{-(c-a)}{\sqrt{r^2-(c-a)^2}} = \frac{-(c-a)}{\sqrt{(c-a)^2+(d-b)^2-(c-a)^2}} = \frac{-(c-a)}{(d-b)}$$

$$\boxed{m_p = \frac{-1}{m}} = \frac{-(c-a)}{(d-b)} \Rightarrow$$

se cumple la relacion $m_p = \frac{-1}{m}$

son perpendiculares

1.



$y = x^2$

$r = 1$

$g'(x_0) = f'(x_0)$

$P = (0, K)$

$x^2 + (y - K)^2 = 1$

$y = K \pm \sqrt{1 - x^2}$

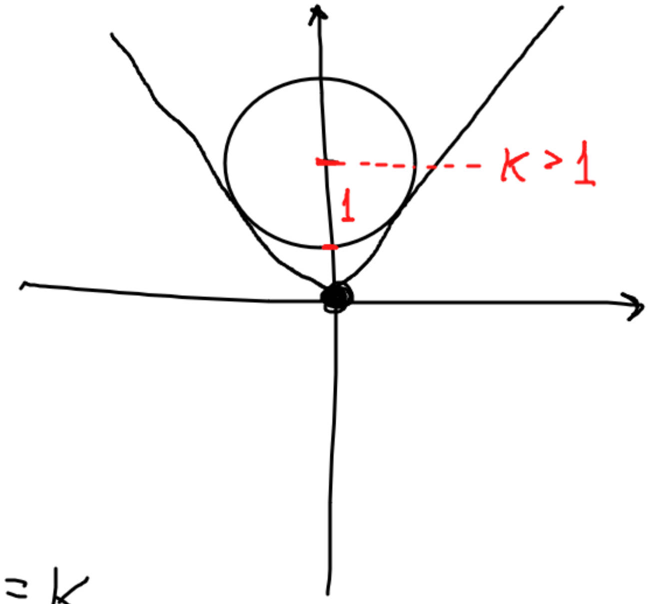
$$\left.\begin{aligned} g'(x) &= \pm \frac{2x}{2\sqrt{1-x^2}} \\ f'(x) &= 2x \end{aligned}\right\} \quad \frac{x}{\sqrt{1-x^2}} = 2x \Rightarrow x_0 = \pm\sqrt{\frac{3}{4}}$$

$$\sqrt{1-x^2} = \frac{1}{2}$$

$$1 - x^2 = \frac{1}{4} \Rightarrow x^2 = \frac{3}{4}$$

$$y = K \pm \sqrt{1-x^2} \longrightarrow y_0 = K \pm \sqrt{1-x_0^2}$$

$$x_0 = \pm\sqrt{\frac{3}{4}} \longrightarrow y_0 = x_0^2 = \frac{3}{4}$$



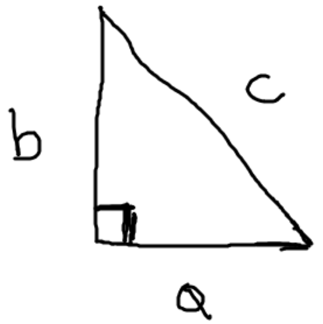
$$\frac{3}{4} = K \pm \sqrt{1-\frac{3}{4}}$$

$$\frac{3}{4} = K \pm \frac{1}{2} \Rightarrow K = \frac{3}{4} \pm \frac{2}{4} \begin{cases} 1/4 = K \\ 5/4 = K \end{cases}$$

sol. $K = 5/4$

6.6

3



función a det. $A = \dfrac{ab}{2}$

$$(*)\quad a^2 + b^2 = c^2 \longrightarrow (h_0 - c)^2 + b^2 = c^2$$

$$(*)\quad c + a = h_0 \longrightarrow a = h_0 - c$$

$$h_0^2 + c^2 - 2h_0 c + b^2 = c^2$$

$$b^2 = 2h_0 c - h_0^2$$

$$b = \sqrt{2h_0 c - h_0^2}$$

$$A(c) = \frac{(h_0 - c)\left(\sqrt{2h_0 c - h_0^2}\right)}{2}$$

$$A'(c) = \left[-1\left(\sqrt{2h_0 c - h_0^2}\right) + \frac{2h_0 (h_0 - c)}{2\sqrt{2h_0 c - h_0^2}}\right] \cdot \frac{1}{2} = 0$$

$$\frac{1}{2}\left(-1\left(\sqrt{2h_0c-h_0^2}\right)+\frac{h_0(h_0-c)}{\sqrt{2h_0c-h_0^2}}\right)=0$$

$$\sqrt{2h_0c-h_0^2}=\frac{h_0(h_0-c)}{\sqrt{2h_0c-h_0^2}} \longrightarrow \Rightarrow \quad 2h_0c-h_0^2=h_0^2-h_0c$$

$$3h_0c=2h_0^2$$

$$\Longrightarrow \quad c=\frac{2h_0}{3}\,;\; b=\frac{h_0}{\sqrt{3}}$$

$$\sqrt{a}\sqrt{a}=\sqrt{a}^2=a$$

$$b^2+a^2=c^2 \longrightarrow \boxed{\sqrt{\frac{4}{9}h_0^2-\frac{h_0^2}{9}}=\sqrt{\frac{h_0^2}{3}}}$$

$$a=\frac{h_0}{3}\,;\; b=\sqrt{c^2-a^2}$$

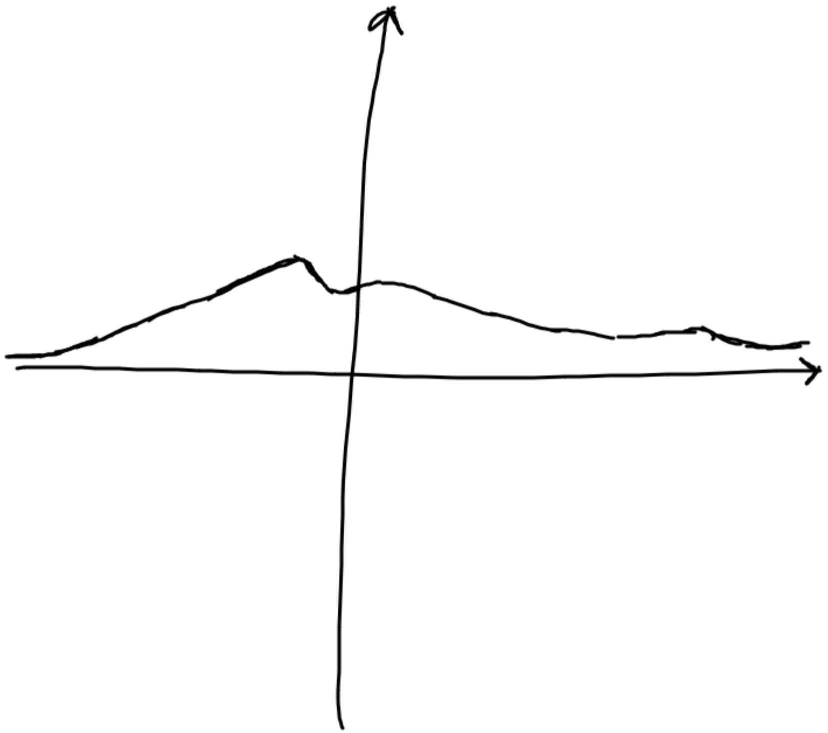
6.5

2) e) $f(x) = \frac{1}{1+|x|} + \frac{1}{1+|x+a|}$

$a > 0$

$$\lim_{x \to +\infty} \frac{1}{1+|x|} + \frac{1}{1+|x+a|} = 0^+$$

$$\lim_{x \to -\infty} \frac{1}{1+|x|} + \frac{1}{1+|x+a|} = 0^+$$

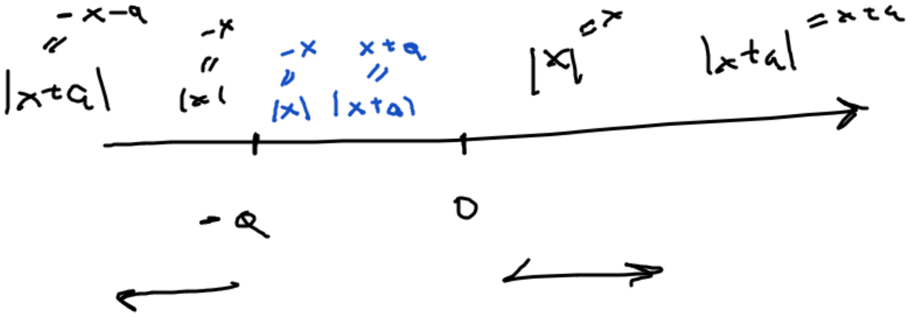


$|x| \quad x>0 \quad x$

$\boxed{|x+a| \quad x>-a \quad x+a}$

$\boxed{|x| \quad x<0 \quad -x}$

$|x+a| \quad x<-a \quad -x-a$



$$f(x) = \begin{cases} \text{si } x<-a \ ; & \dfrac{1}{1-x} + \dfrac{1}{1-x-a} \\ \text{si } -a<x<0 \ ; & \dfrac{1}{1-x} + \dfrac{1}{1+x+a} \\ \text{si } x>0 \ ; & \dfrac{1}{1+x} + \dfrac{1}{1+x+a} \end{cases}$$

$$
f'(x) = \begin{cases} \dfrac{1}{(1-x)^2} + \dfrac{1}{(1-x-a)^2} & \text{si } x < -a \\[2ex] \dfrac{1}{(1-x)^2} - \dfrac{1}{(1+x+a)^2} & \text{si } -a < x < 0 \\[2ex] -\dfrac{1}{(1+x)^2} - \dfrac{1}{(1+x+a)^2} & \text{si } x > 0 \end{cases}
$$

$$
\lim_{x \to -a^+} f'(x) = \lim_{x \to -a^+} \frac{1}{(1-x)^2} - \frac{1}{(1+x+a)^2} = \frac{1}{(1+a)^2} - 1
$$

$$
\lim_{x \to -a^-} f'(x) = \lim_{x \to -a^-} \frac{1}{(1-x)^2} + \frac{1}{(1-x-a)^2} = \frac{1}{(1+a)^2} + 1
$$

$$\lim_{x\to 0^+} \frac{1}{(1+x)^2} - \frac{1}{(1+a+x)^2} = \underline{1} - \frac{1}{(1+a)^2}$$

$$\lim_{x\to 0^-} \frac{-1}{(1-x)^2} - \frac{1}{(1+a+x)^2} = -\underline{1} - \frac{1}{(1+a)^2}$$

candidatos a maximo $0, -a, f'(x)=0$

$$\frac{1}{(1-x)^2} = \frac{1}{(1+x+a)^2} \Rightarrow 1+x+a = 1-x \Rightarrow 2x = 1-a \Rightarrow x = \frac{1-a}{2}$$

$$f(-a) = \frac{1}{1+a} + 1$$

$$f(0) = 1 + \frac{1}{1+a}$$

$$\Bigg\} \quad f(0) = f(-a)$$

$$f\left(\frac{1-a}{2}\right) = \frac{8+4a}{a^2+4a+4}$$

sup. $1 + \dfrac{1}{1+a} > \dfrac{8+4a}{a^2+4a+4}$

$$\frac{1+a+1}{1+a} > \frac{8+4a}{a^2+4a+4}$$

$$2a^2+8a+8+a^3+4a^2+4a$$

$$> 8+4a+8a+4a^2$$

$$a^3+2a^2 > 0 \quad \checkmark \text{ pq. } a>0$$

$\Rightarrow$ max $= 1 + \dfrac{1}{1+a}$ se va a dar en $x=-a$ y $x=0$