Recta - tangente

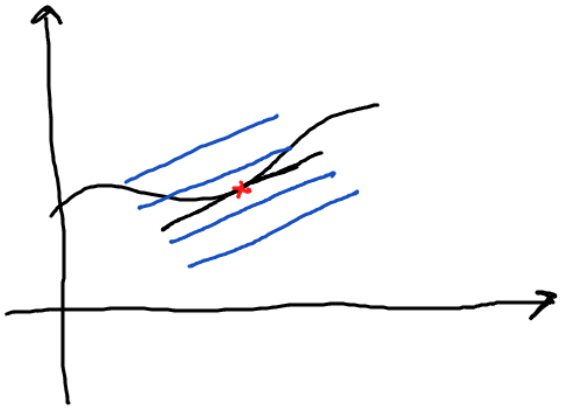
1. a

$$f(x)=x^2\ ,\ p=\left(\overset{x}{\overset{\shortparallel}{3}},\ \underset{\underset{f(x)}{\shortparallel}}{\overset{y}{\overset{\shortparallel}{9}}}\right)$$

$$f'(x)=2x$$

$$m = f'(x=3) = 6$$

$$f'(x)$$



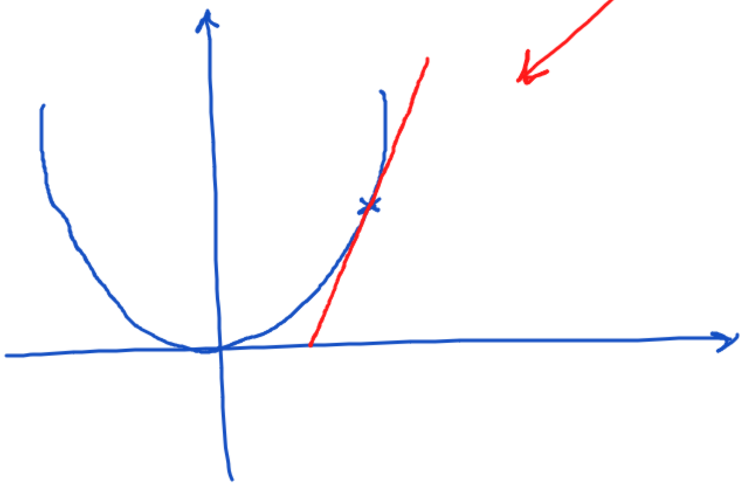
$$y = m\,x + n$$

$$m \downarrow f'(x_0)$$

$m = f'(x=3) = 6$

$f'(x) = 2x$

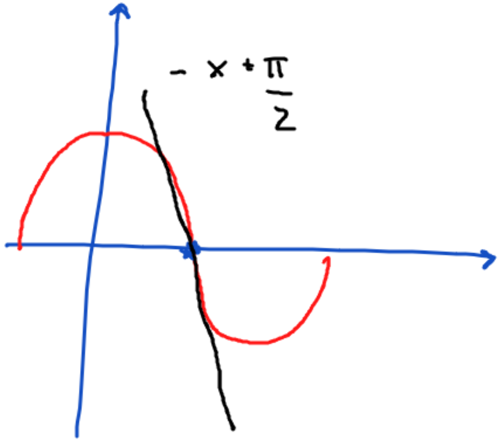
$y - 9 = 6(x-3) \longrightarrow y = 6x - 9 \longrightarrow n = 9$



1.b $f(x) = \cos(x) \,, \quad p\left(\frac{\pi}{2}, 0\right)$

$f'(\;\;) = -\operatorname{sen}(x) \Rightarrow f'\left(x = \frac{\pi}{2}\right) = -1$

$y - 0 = -1\left(x - \frac{\pi}{2}\right) \Rightarrow y = -x + \frac{\pi}{2}$



1.c $f(x) = \dfrac{x}{x^2+1}$; $P = (0,0)$

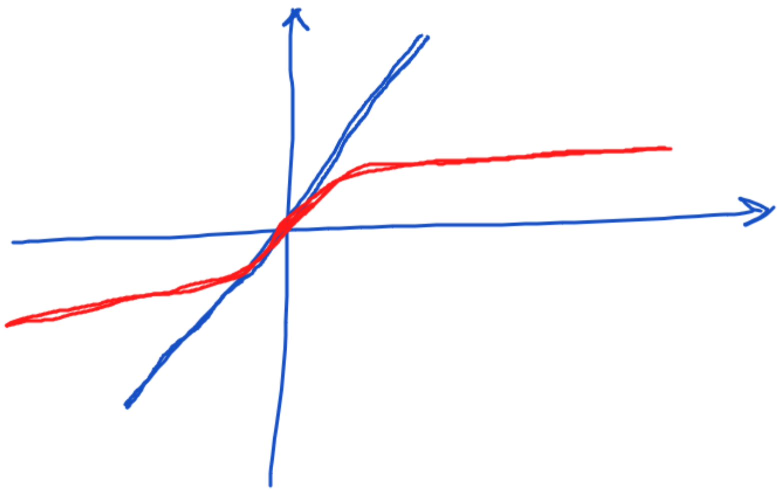
$h(x) = \dfrac{f(x)}{g(x)} \Rightarrow h'(x) = \dfrac{f'g - g'f}{g^2}$

$$f'(x) = \frac{x^2+1-2x\cdot x}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)}$$

$f'(x=0) = 1$

$$y - 0 = 1(x-0)$$

$$y = x$$



2.

$f(x) = x^2 + ax + b \longrightarrow f'(x) = 2x + a$

$g(x) = cx - x^2 \longrightarrow g'(x) = c - 2x$

$p = (3,3)$

$\rightarrow g'(3) = f'(3) = -2 = m = 2.3 + (-8) = -2$

$6 + a = c - 6 \Rightarrow a = c - 12 = -8$

$f(3) = 3 = 9 + 3a + b \longrightarrow b = 3 - 9 - 3a = 18$

$g(3) = 3 = 3c - 9 \longrightarrow \boxed{c = 4}$

$f(x) = x^2 - 8x + 18$

$g(x) = 4x - x^2$

$f'(3) = g'(3)$

$y - 3 = m(x - 3)$

recta tangente $y - 3 = -2(x-3) \Rightarrow y = -2x + 9$

Calculo de derivadas

1. c

$$f(x) = \frac{\overbrace{x^2+3x+2}^{g(x)}}{\underbrace{x^4+x^2+1}_{h(x)}}$$

$$f'(x) = \frac{g'(x)\,h(x) - g(x)\,h'(x)}{h^2(x)}$$

$$\left.\begin{aligned} g'(x) &= 2x+3 \\ h'(x) &= 4x^3+2x \end{aligned}\right\} \Rightarrow f'(x) = \frac{(2x+3)(x^4+x^2+1) - (x^2+3x+2)(4x^3+2x)}{(x^4+x^2+1)^2}$$

1. f $\quad f(x) = \left(\sqrt[5]{x+1}\right)^2 = \left((x+1)^{1/5}\right)^2 = (x+1)^{2/5}$ $\quad \boxed{(x^K)' = Kx^{K-1}}$

$$f'(x) = \frac{2}{5}(x+1)^{\left(\frac{2}{5}-1\right)} = \frac{2}{5}(x+1)^{-3/5} = \frac{2}{5\left(\sqrt[5]{x+1}\right)^3}$$

ok

regla de la cadena

$$h(x) = g(f(x)) \Longrightarrow h'(x) = f'(x)\,g'(f(x))$$

1.g) $h(x) = \operatorname{sen}^3(x) = \underbrace{[\operatorname{sen}(x)]}_{= f(x)}^{3} = g$

$\textcircled{x}^3$

$f(x) = \operatorname{sen}(x)$

$h'(x) = \underbrace{3\operatorname{sen}^2(x)}_{g'(f(x))} . \underbrace{\cos(x)}_{f'(x)}$

1.h) $f(x) = x^3 \operatorname{sen}(x)$

$f(x) = g(x)h(x) \Rightarrow f'(x) = g'(x)h(x) + h'(x)g(x)$

$f'(x) = 3x^2 \operatorname{sen}(x) + \cos(x)\, x^3$

1.i) $f(x) = \operatorname{sen}(x)\cos(x)$

$f'(x) = \cos^2(x) - \operatorname{sen}^2(x)$

1.k) $f(x) = \operatorname{sen}\left(e^{\overset{z}{\boxed{\frac{x+1}{x-2}}}}\right)$

$f(x) = \underset{\text{sen}}{h}\Big(g\big(\underbrace{z(x)}_{\frac{x+1}{x-2}}\big)\Big)$ $g(z(x)) = e^{\frac{x+1}{x-2}}$

$$f'(x) = \underbrace{\frac{x-2-x-1}{(x-2)^2}}_{z'}\;\underbrace{e^{\frac{x+1}{x-2}}}_{g'(z(x))}\;\underbrace{\cos\left(e^{\frac{x+1}{x-2}}\right)}_{h'(g(z(x)))} = -\frac{3\,e^{\frac{x+1}{x-2}}}{(x-2)^2}\cdot\cos\left(e^{\frac{x+1}{x-2}}\right)$$

1.n) $f(x) = (\underbrace{e^x+2}_{\text{variable}})^{100}$ $f'(x) = 100\,(e^x+2)^{99}\cdot e^x$

$$\left(\operatorname{sen}^3(3x)\right)' = 3.3\,\operatorname{sen}^2(3x)\cos(x)$$

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5. $\underline{\alpha \text{ es raiz multiple de } P} \iff \alpha$ es raiz de P' y P

<u>Dem.</u> $(\Longrightarrow)$ $\quad p(x) = \sum_{i=0}^{n} a_i x^i = (x-\alpha)^k q(x)$ con $k > 1$

↳ polinomio que no tiene raiz α

$$p(\alpha) = (\alpha-\alpha)^k q(\alpha) = 0$$

$$p'(x) = k(x-\alpha)^{k-1} q(x) + (x-\alpha)^k q'(x) = (x-\alpha)^{k-1}\underbrace{\left[k q(x) + (x-\alpha) q'(x)\right]}_{r(x)}$$

$$p'(\alpha) = (\alpha-\alpha)^{k-1} r(\alpha) = 0$$

$(\Leftarrow)$

$$p(x) = (x-\alpha)^k g(x) \quad \text{con} \quad k \geq 1$$

$$p'(x) = (x-\alpha)^z r(x) \quad \text{con} \quad z \geq 1 \leftarrow$$

$$p'(x) = k(x-\alpha)^{k-1} g(x) + (x-\alpha)^k g'(x) = (x-\alpha)^{k-1}\left[k g(x) + (x-\alpha) g'(x)\right]$$

$$(x-\alpha)^z r(x) = (x-\alpha)^{k-1}\left[\overbrace{k g(x) + (x-\alpha) g'(x)}^{r(x)}\right]$$

$z = k-1 \geq 1 \rightarrow k \geq 2$ ó $k > 1$ $p(x)$ tiene a α como raiz multiple ∎