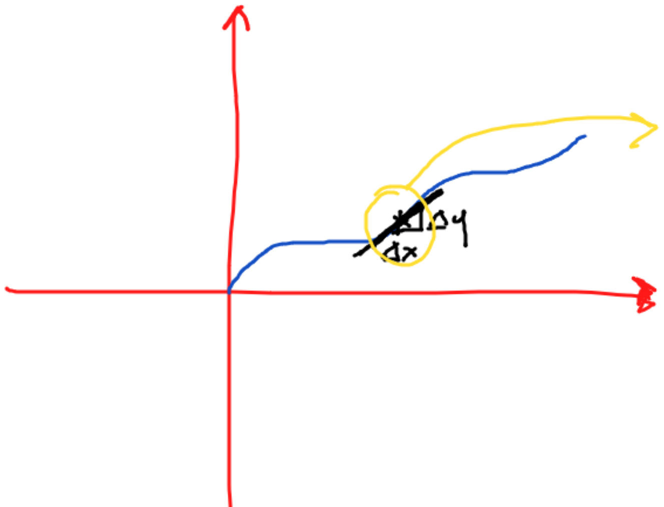
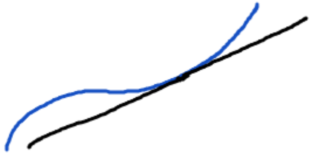
Derivación

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$





$$f'(x) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$\lim_{x \to x_0^+} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \to x_0^-} \frac{f(x) - f(x_0)}{x - x_0}$$

1 $f(x) = x^2 \Rightarrow f'(x)$ $\quad (f'(x) = 2x)$

$$\lim_{h \to 0} \frac{\overbrace{f(x+h)}^{=(x+h)^2} - \overbrace{f(x)}^{=x^2}}{h} = \lim_{h \to 0} \frac{x^2 + h^2 + 2xh - x^2}{h} =$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} 2x + h = 2x$$

$$f(x) = x^3 \Rightarrow f'(x) = 3x^2$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{x^3} + \overset{\to 0}{3xh^{\cancel{2}}} + 3x^2\cancel{h} + \overset{\to 0}{h^{\cancel{3}\,2}} - \cancel{x^3}}{\cancel{h}} = 3x^2$$

$f(x)=\sqrt{x}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=$$

$$\lim_{h\to 0}\frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}=$$

$$=\lim_{h\to 0}\frac{h}{h(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to 0}\frac{1}{\underbrace{\sqrt{x+h}}_{\sqrt{x}}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$

$$+\sqrt{x}=2\sqrt{x}$$

$$f(x) = \frac{1}{x}$$

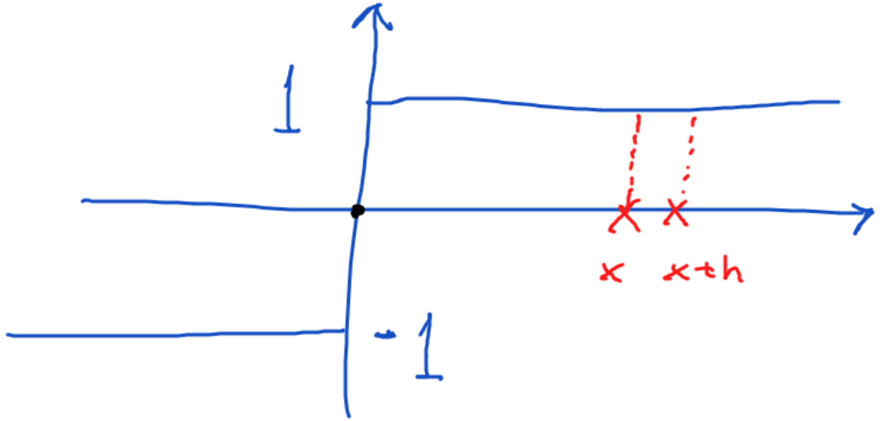
$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \left(\frac{1}{x+h} - \frac{1}{x} \right) \cdot \frac{1}{h} =$$

$$= \lim_{h \to 0} \frac{\cancel{x} - \cancel{x} - h}{x(x+h)h} = \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}x(x+h)} = \lim_{h \to 0} \frac{-1}{x(x+h)} = \frac{-1}{x^2}$$

②

$$\boxed{\begin{array}{c}\text{derivable} \Longrightarrow \text{continua} \\ \text{no cont.} \Longrightarrow \text{no deriv.}\end{array}}$$

$f(x) = \mathrm{sig}(x)$

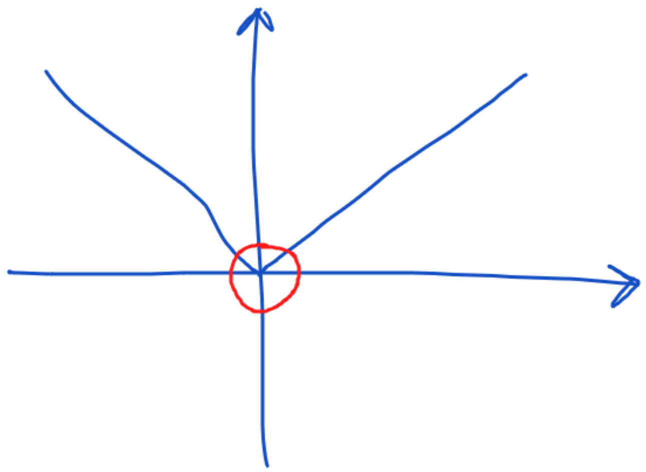


en 0 $f(x)$ no es cont. $\Longrightarrow$ no es deriv.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\overbrace{1 - 1}^{=0}}{\underbrace{h}_{\to 0}} = 0$$

tomemos el caso positivo y el negativo es analogo

$$f(x) = |x| = \begin{cases} x & \text{si } x > 0 \\ -x & \text{si } x < 0 \end{cases}$$

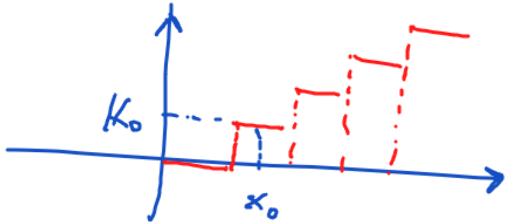


$$\lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \frac{x}{x} = 1$$

$$\lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-} \frac{-x}{x} = -1$$

$$\lim_{x \to 0^+} \ldots \neq \lim_{x \to 0^-} \ldots \Longrightarrow \nexists f'(0)$$

$$f(x) = \lfloor x \rfloor$$



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{K_0 - K_0}{h} = 0$$

$K_0 = 0 + K$ con $K \in \mathbb{N}$

$$\Rightarrow f'(x) = 0$$