Calculo de limites

$$\lim_{x \to 0} \frac{x^5 - x^3 + 5x^2}{x^7 - x^7 + x^2} = \frac{0}{0} \quad \text{indeterminacion}$$

$$\lim_{x \to 0} \frac{\cancel{x^2}(x^3 - x + 5)}{\cancel{x^2}(x^5 - x^2 + 1)} = \lim_{x \to 0} \frac{x^3 - x + 5}{x^5 - x^2 + 1} = \frac{5}{1}$$

$$\lim_{x\to 3} \frac{x^2-5x+6}{x^2-4x+3} = \frac{0}{0} \quad \text{ind.}$$

	1	-5	6
3		3	-6
	1	-2	0

$$\lim_{x\to 3} \frac{(x-3)(x-2)}{(x-3)(x-1)} = \lim_{x\to 3} \frac{x-2}{x-1} = \frac{1}{2}$$

	1	-4	3
3		3	-3
	1	-1	0

$$\lim_{x \to -2} \frac{x^3 + 3x^2 - 4}{(x+2)^2}$$

$$\lim_{x \to -2} \frac{(x+2)^2(x-1)}{(x+2)^2} = -3$$

	1	3	0	-4
-2		-2	-2	4
	1	1	-2	0
1		1	2	
	1	2	0	

$$\lim_{x \to 1} \frac{x^3 - 3x^2 + 3x - 1}{x^3 - 2x^2 + x} = \lim_{x \to 1} \frac{(x-1)^3}{x(x-1)^2} = \lim_{x \to 1} \frac{x-1}{x} = 0$$

	1	-3	3	-1
1		1	-2	1
	1	-2	1	0
1		1	-1	
	1	-1	0	

	1	-2	1
1		1	-1
	1	-1	0

$\boxed{4.3.1.i}$

$$\lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1} = \frac{0}{0} \quad \text{indeterminado}$$

$$\lim_{x \to 1} \frac{(\sqrt{x} - 1)}{(x - 1)} \cdot \frac{(\sqrt{x} + 1)}{(\sqrt{x} + 1)} = \lim_{x \to 1} \frac{\cancel{(x - 1)} \cancel{- \sqrt{x} + \sqrt{x}}^{\,0}}{\cancel{(x - 1)} (\sqrt{x} + 1)} = \lim_{x \to 1} \frac{1}{\sqrt{x} + 1}$$

$$(\sqrt{x} - 1)(\sqrt{x} + 1) = (x - 1)$$

multiplico por conjugado

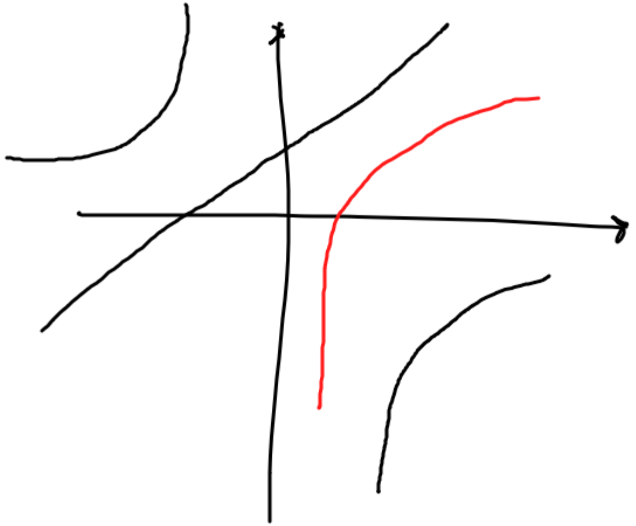
$$\lim_{x \to 1} \frac{1}{\sqrt{x} + 1} = \frac{1}{2}$$

$\boxed{4.3.2.h}$

$$\lim_{x \to 0^+} \frac{1}{\left\lfloor \frac{1}{x} \right\rfloor} = 0$$

6

$$\lim_{x \to 1} \frac{\log(x)}{x-1}$$



$$\frac{x-1}{x} \le \log \le x-1$$

$$\frac{x-1}{x(x-1)} \le \frac{\log(x)}{x-1} \Rightarrow \frac{1}{x} \le \frac{\log(x)}{x-1} \Rightarrow \lim_{x \to 1} \frac{1}{x} = 1 \le \lim_{x \to 1} \frac{\log(x)}{x-1}$$

$$\frac{x-1}{x-1} \ge \frac{\log(x)}{x-1} \Rightarrow 1 \ge \frac{\log(x)}{x-1} \Rightarrow \lim_{x \to 1} \frac{\log(x)}{x-1} \le 1$$

$$\lim_{x \to 1} \frac{\log(x)}{x-1} = 1 \quad \text{por teo. del}$$

$$\lim_{x\to 0^+} \underbrace{x^a}_{\to 0} \underbrace{\log x}_{\to -\infty} \quad \forall a > 1 \implies \text{indeterminado } (0 \cdot \infty)$$

* $\log(x) < 0 \quad \forall x \in (0,1) \implies x^a \log(x) < 0$

* $x^a = \dfrac{1}{t^{2a}} \implies \displaystyle\lim_{t\to+\infty} \frac{1}{t^2}\log\left(\frac{1}{t^2}\right) = \lim_{t\to\infty} \frac{1}{t^2}\Big(\overbrace{\log(1)}^{0} - \overbrace{\log(t^2)}^{2\log(t)}\Big) = \lim_{t\to\infty} \frac{-2\log(t)}{t^2}$

$$\left.\begin{array}{l}\log(x) \le x-1\\ x-1 < x\end{array}\right\} \quad \log(x) < x \implies -\log(x) > -x \implies \frac{-2\log(x)}{x^2} > \frac{-2x}{x^2}$$

-1

$$\frac{-2\log(t)}{t^2} > \frac{-2t}{t^2} = \frac{-2}{t} \implies \lim_{x \to 0^+} x \log(x) = 0$$

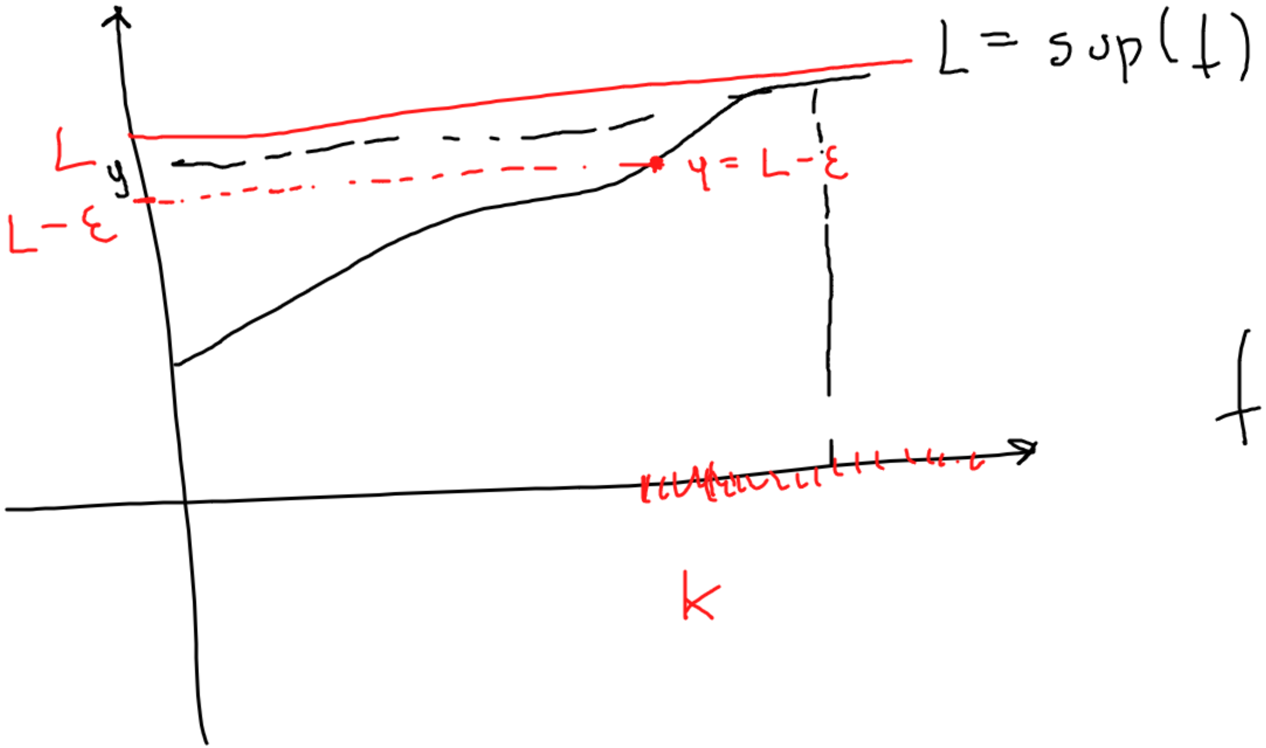
$$0 > x\log(x) = \frac{-2\log(t)}{t^{2a}} > \frac{-2}{t^{a}}$$

$$\lim_{x \to 0^+} 0 > \lim_{x \to 0^+} x\log(x) = \lim_{t \to +\infty} \frac{-2\log(t)}{t^2} > \lim_{t \to +\infty} \frac{-2}{t} = 0$$

4.4.10

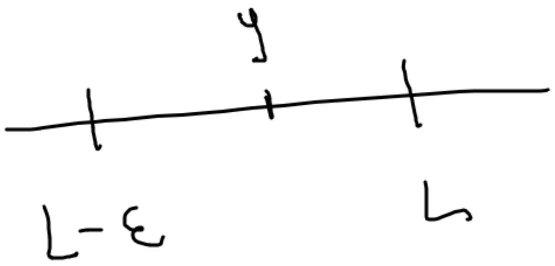
$$\lim_{x\to\infty} f(x) = L \in \mathbb{R} \implies \forall \varepsilon > 0 \ \exists K \in \mathbb{R} \ \forall x \in \mathbb{R} \quad x > K \implies |f(x) - L| < \varepsilon$$

Sup. creciente para fijar ideas $(x<y \Rightarrow f(x)<f(y))$



funcion cont.

Sea $L = \sup\{ f(x) / x \in \mathbb{R} \}$ pq f es acotada



$\exists y \in \{ f(x) / x \in \mathbb{R} \} / L - \varepsilon < y \leq L$

tomemos $k \in \mathbb{R} \mid f(k) = y \Longrightarrow$ f creciente $\forall x \geq k \; f(x) \geq \overbrace{f(k)}^{y}$

$\therefore \forall x \geq k \quad L - \varepsilon < f(x) \leq L$

$y > L - \varepsilon \Longrightarrow f(x) > L - \varepsilon$

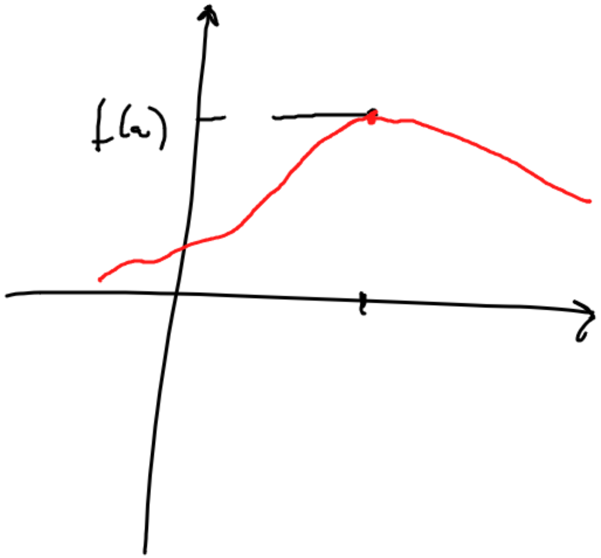
$|f(x) - L| < \varepsilon$

* $L = \sup\{f\} \; \forall x \in \mathbb{R} \quad f(x) \leq L$

Continuidad

3- det a y b para f cont.

$$\lim_{x \to a^-} f(x) = f(a) = \lim_{x \to a^+} f(x)$$



b) $f(x) \begin{cases} x^2 + 3x + 2 & x \leq 1 \\ ax^2 + bx + 1 & x > 1 \end{cases}$

$$\lim_{x \to 1^-} x^2 + 3x + 2 = \lim_{x \to 1^+} ax^2 + bx + 1 = f(1) = 6$$

$$\Downarrow$$

$$a + b + 1 = 6 \implies \boxed{a + b = 5}$$

$$f(x) = \begin{cases} \log(x+1) & x > 0 \\ (x+a)^2 & x \leq 0 \end{cases}$$

$$f(0) = a^2 = \lim_{x \to 0^+} \log(x+1) = 0$$

$$\Rightarrow \boxed{a = 0}$$