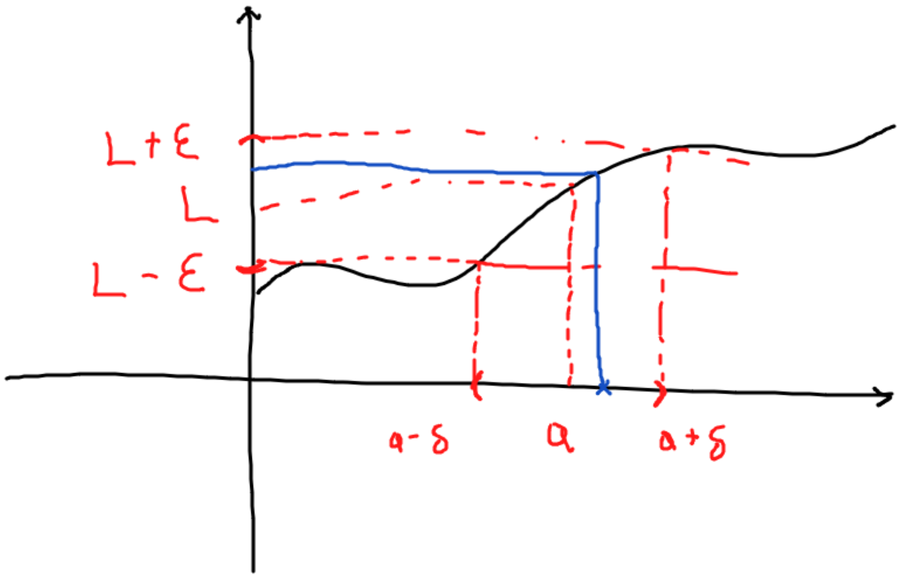
Definicion y prop. basicas de los limites

Def. $\lim\limits_{x \to a} f(x) = L \iff \forall \varepsilon > 0 \; \exists \delta > 0 \; \forall x \; |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon$

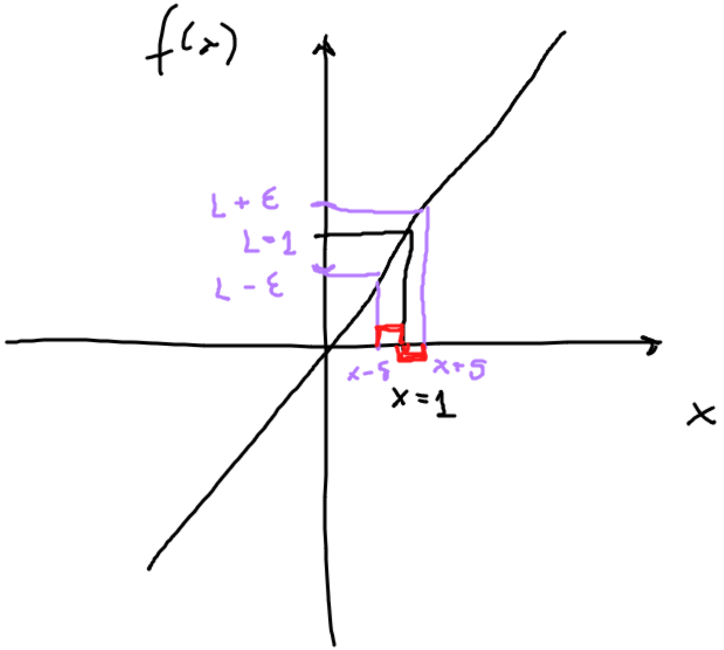


$f(x) = x$

$f(1) = 1$

$\boxed{a = 1}$

$\varepsilon = 10^{-5}$



$|f(x) - L| < \varepsilon$

$|x - a| < \delta$

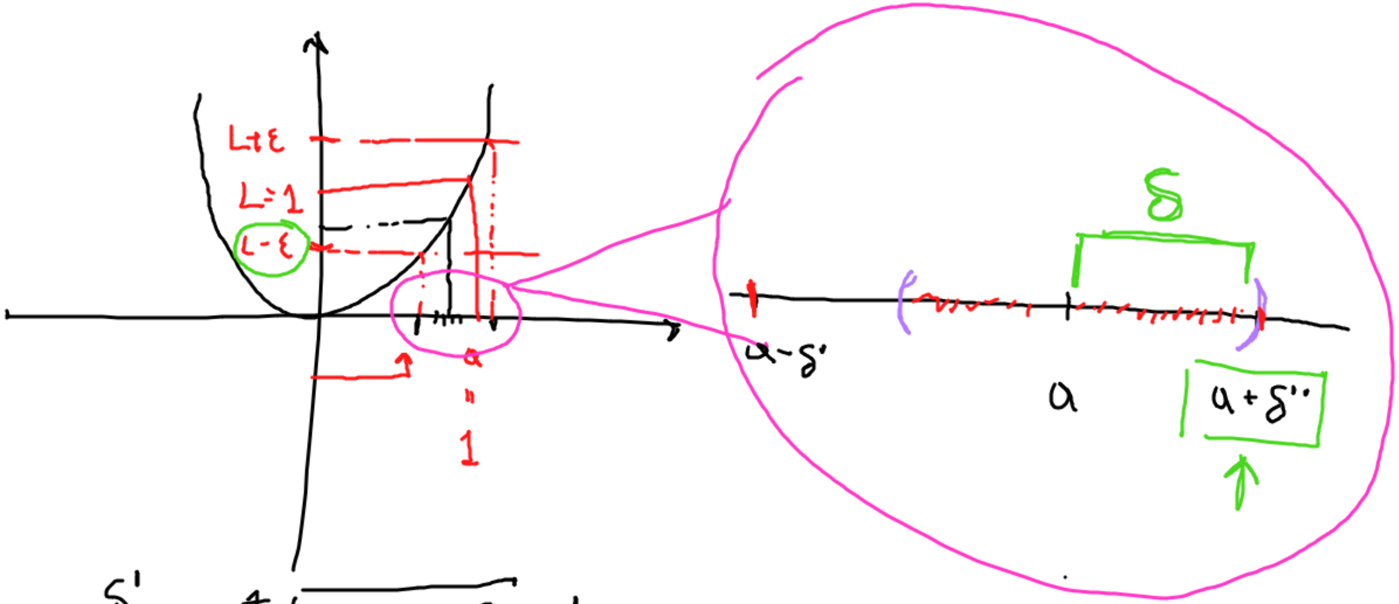
$L = 1$

$\delta = \varepsilon$

$f(x) = x^4$

$a = 1$

$\varepsilon = 10^{-5}$



$\sqrt[4]{1 + 10^{-5}}$ posibles δ

$\sqrt[4]{1 - 10^{-5}}$

$\delta' = \sqrt[4]{1 + 10^{-5}} - 1$

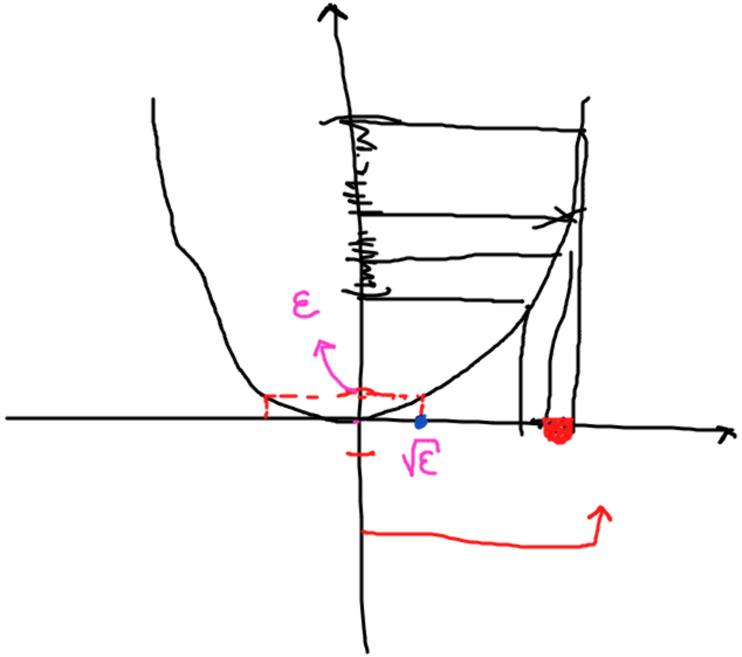
$\delta'' = 1 - \sqrt[4]{1 - 10^{-5}}$

$\delta = \min\{\delta', \delta''\}$

① a) $f(x) = x^2$, $a = 0$, $\varepsilon = 10^{-2}, 10^{-5}, 10^{-10}$

$$\lim_{x \to 0} x^2 = L = f(0) = 0 \Rightarrow L = 0$$

$$|x - \overset{a}{\overset{\|}{0}}| < \delta \Rightarrow |\overset{f(x)}{\overset{\|}{x^2}} - \overset{L}{\overset{\|}{0}}| < \varepsilon$$



$$x^2 = \varepsilon \Rightarrow x = \sqrt{\varepsilon}$$

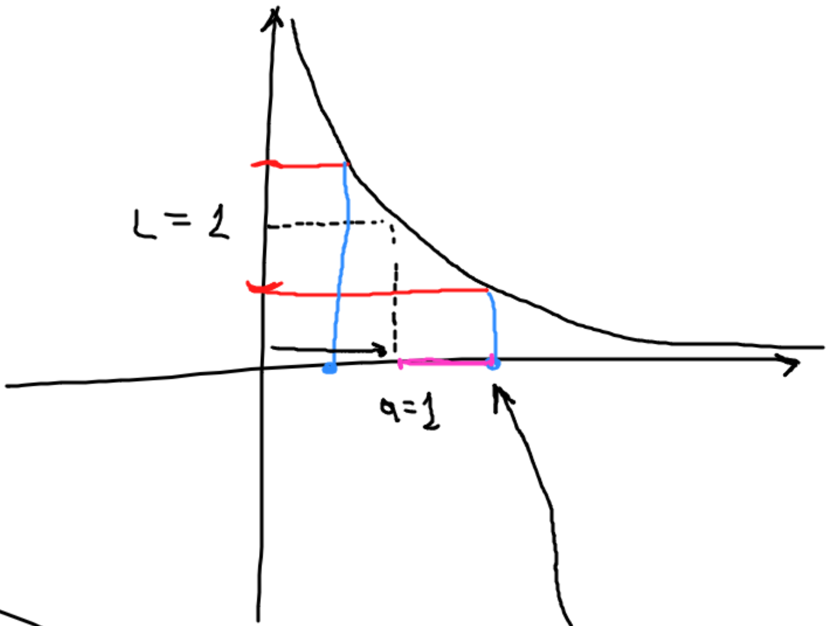
$$\boxed{|x = \delta|} \Rightarrow \delta < \sqrt{\varepsilon} < \sqrt{10^{-2}}$$

$\delta = \sqrt{10^{-5}}$ con $\varepsilon = 10^{-5}$

$\delta = \sqrt{10^{-10}}$ con $\varepsilon = 10^{-5}$

1 b) $f(x) = \frac{1}{x}$, $a = 1$, $\varepsilon = 10^{-2}$

$\lim_{x \to 1} 1/x = L = f(1) = 1 \Rightarrow L = 1$



$|x - \overset{a}{1}| < \delta \Rightarrow |f(x) - L| < \varepsilon$

(+) $\frac{1}{x} - 1 < \varepsilon$

(−)

$\frac{1}{x} = \varepsilon + 1$

$1 = (\varepsilon + 1)x$

$x = \frac{1}{\varepsilon + 1}$

$\frac{1}{x} - 1 = \varepsilon \Rightarrow \boxed{x = \frac{1}{1+\varepsilon}}$; $-\frac{1}{x} + 1 = \varepsilon \Rightarrow \boxed{x = \frac{1}{1-\varepsilon}}$

$$\delta = \min\left\{ \frac{1}{1-\varepsilon} - 1 \,;\, 1 - \frac{1}{1+\varepsilon} \right\} = \frac{1}{1-10^{-2}} - 1$$

no esta chequeado

7) $\lim_{x\to p} f(x) = a$; $\lim_{x\to p} g(x) = b$

a) $\lambda \in \mathbb{R}$, $h(x) = f(x) + \lambda \implies \lim_{x\to p} h(x) = \lambda + a$

Dem.

$$\lim_{x\to p} f(x) + \lambda = \overbrace{\lim_{x\to p} f(x)}^{a} + \overbrace{\lim_{x\to p} \lambda}^{\lambda} = a + \lambda$$

b) $h(x) = f(x) + g(x) \Rightarrow \lim_{x\to p} h(x) = a + b$

$$\lim_{x\to p} f(x) + g(x) = \lim_{x\to p} f(x) + \lim_{x\to p} g(x) = a + b$$

c) $\exists \lim_{x\to p} f(x) + h(x) \iff \exists \lim_{x\to p} h(x)$

$$\exists\ \overbrace{\lim_{x\to p} f(x)}^{\text{hip } \exists \lim} + \lim_{x\to p} h(x) \iff \exists \lim_{x\to p} h(x)$$

d) $\tilde{f}(x) = \lambda f(x) \Rightarrow \lim_{x \to p} \tilde{f}(x) = \lambda a$

$$\lim_{x \to p} \tilde{f}(x) = \lim_{x \to p} \lambda f(x) = \underbrace{\lim_{x \to p} \lambda}_{\lambda} \cdot \overbrace{\lim_{x \to p} f(x)}^{a} = \lambda a$$

f) $\lim_{x \to p} \frac{f(x)}{g(x)} = \frac{a}{b} \Rightarrow \frac{\lim_{x \to p} f(x)}{\lim_{x \to p} g(x)} = \frac{a}{b}$

$\boxed{b \neq 0}$

g) $\lim\limits_{x \to p} \dfrac{f(x)}{g(x)} = 1$, $h(x) \Rightarrow \lim\limits_{x \to p} \dfrac{h(x)}{g(x)} = \lim\limits_{x \to p} \dfrac{h(x)}{f(x)}$

<u>Dem.</u> $\dfrac{\lim\limits_{x \to p} f(x)}{\lim\limits_{x \to p} g(x)} = 1 \Rightarrow \lim\limits_{x \to p} f(x) = \lim\limits_{x \to p} g(x)$

$\dfrac{\lim\limits_{x \to p} h(x)}{\lim\limits_{x \to p} g(x)} = \dfrac{\lim\limits_{x \to p} h(x)}{\lim\limits_{x \to p} f(x)} \Rightarrow \lim\limits_{x \to p} h(x) = \lim\limits_{x \to p} h(x)$

h) $h(x)$ acotada $a = 0 = \lim_{x \to p} f(x)$; $r(x) = f(x)h(x)$

$$\lim_{x \to p} r(x) = 0$$

Dem. $\lim_{x \to p} f(x)h(x) = \underbrace{\lim_{x \to p} f(x)}_{=0} \cdot \underbrace{\lim_{x \to p} h(x)}_{\leq M} \leq 0 . M = 0$

$$\left| \lim_{x \to p} r(x) \right| \leq 0 \Longrightarrow \lim_{x \to p} r(x) = 0$$

5 $\lim_{x\to a} f(x)$

a) $\lim_{x\to 3} \frac{2f(x)}{x} = 1 \Rightarrow \frac{\lim_{x\to 3} 2f(x)}{\lim_{x\to 3} x} = 1 \Rightarrow \frac{2\lim_{x\to 3} f(x)}{\lim_{x\to 3} x} = 1$

$2\lim_{x\to 3} f(x) = \lim_{x\to 3} x \Rightarrow \lim_{x\to 3} f(x) = \frac{\lim_{x\to 3} x}{2} = \frac{3}{2} \Rightarrow \lim_{x\to 3} f(x) = \frac{3}{2}$

5 b)

$$\lim_{x \to -2} \frac{f(x)}{x^2} = 1 \Rightarrow \frac{\lim_{x \to -2} f(x)}{\lim_{x \to -2} x^2} = 1$$

$$\lim_{x \to -2} f(x) = \lim_{x \to -2} x^2 = 4 \Rightarrow \boxed{\lim_{x \to -2} f(x) = 4}$$

d)
$$\lim_{x \to 0} \frac{f(x)}{\operatorname{sen}(x)} = 5 \Rightarrow \frac{\lim_{x \to 0} f(x)}{\lim_{x \to 0} \operatorname{sen}(x)} = 5$$

$$\lim_{x \to 0} f(x) = 5 \lim_{x \to 0} \operatorname{sen}(x) = 0 \Rightarrow \boxed{\lim_{x \to 0} f(x) = 0}$$

g) $$\lim_{x\to 2} \frac{f(x^3)-1}{x} = 1 \; ; \quad a = 8 \qquad \boxed{\lim_{x\to 8} f(x)} = \lim_{x\to 2} f(x^3)$$

$$\frac{\lim_{x\to 2} f(x^3) - 1}{\lim_{x\to 2} x} = 1 \Rightarrow \lim_{x\to 2} f(x^3) - 1 = \lim_{x\to 2} x \quad \lim_{x\to 2} f(x^3) = \lim_{x\to 2} 3$$

$$\lim_{x\to 2} f(x^3) = \underbrace{\lim_{x\to 2} x}_{2} + 1 = 3$$

$$\boxed{\lim_{x\to 2} f(x^3) = 3}$$