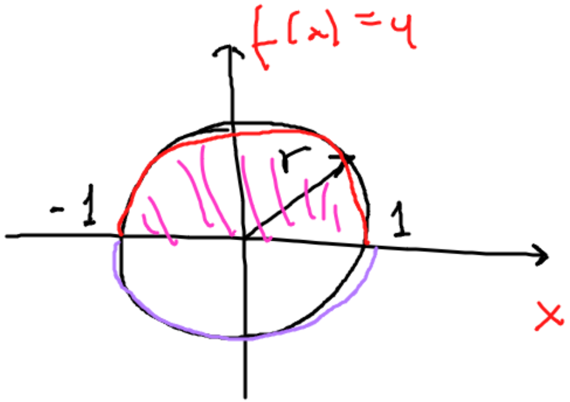
Cambio de variable

1. $y^2 + x^2 = 1 \Rightarrow y^2 = 1 - x^2 \Rightarrow y = \pm\sqrt{1-x^2}$



$$\int_{-1}^{1} \sqrt{1-x^2}\,dx = \frac{\pi}{2}$$

$$\int_{-1}^{1} \sqrt{r - x^2}\,dx = \frac{\pi r^2}{2} \Rightarrow \text{area} \quad 2\int_{-1}^{1} \sqrt{r - x^2}\,dx = \pi r^2$$

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$$\int \cos(x)\,dx = \text{sen}(x) + C \quad ; \quad \int \text{sen}(x)\,dx = -\cos(x) + C$$

a)

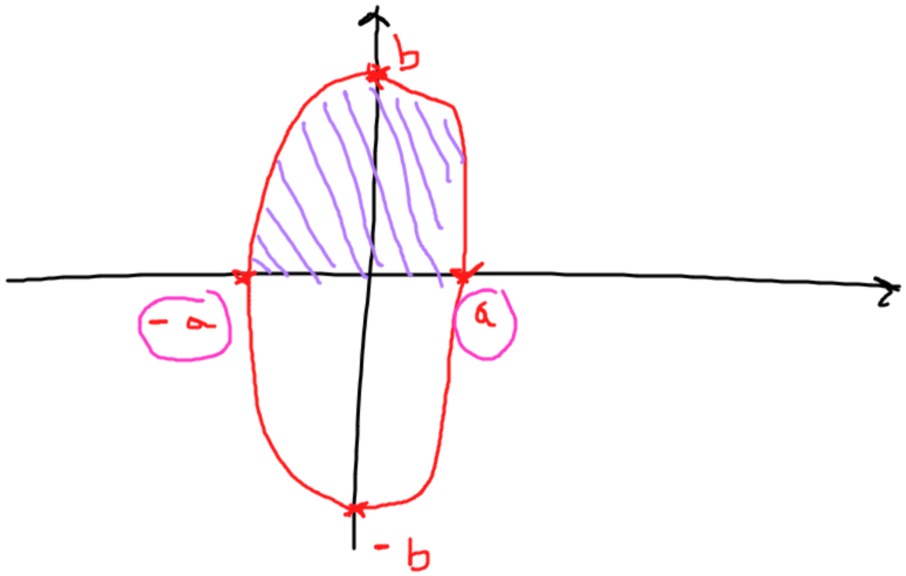
$$\int_0^{2\pi} \text{sen}(kt) + 5 \; dt = \underbrace{\int_0^{2\pi} \text{sen}(kt)\,dt}_{\boxed{\frac{-\cos(kt)}{k}}} + \underbrace{\int_0^{2\pi} 5\,dt}_{5.2\pi} = 10\pi$$

$2\pi \simeq 0$

0

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \left(\frac{y}{b}\right)^2 = 1 - \left(\frac{x}{a}\right)^2 \Rightarrow \frac{y}{b} = \pm\sqrt{1 - \left(\frac{x}{a}\right)^2}$$

$$y = \pm\sqrt{1 - \left(\frac{x}{a}\right)^2} \cdot b$$



$$\int_{-a}^{a} \sqrt{1 - \left(\frac{x}{a}\right)^2}\, b$$

$$\int_a^b f(x)\,dx = \int_{a+p}^{b+p} f(x-p)\,dx$$

$x = a$

extremos de integracion

$ax = a \Rightarrow x = \frac{a}{a} = 1$

$f(xa)$

$1 = a/a$

$$\int_{-a}^{a} \sqrt{1-\left(\frac{x}{a}\right)^2}\, b = b\int_{-a}^{a} \sqrt{1-\left(\frac{x}{a}\right)^2} = b\int_{-1}^{1} \sqrt{1-\left(\frac{xa}{a}\right)^2} = \frac{b\pi}{2}$$

$-1 = a/a$

$\frac{\pi}{2}$

area de elipse $\frac{2b\pi}{2} = b\pi$

Calculo de integrales

prop. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(2) $\int_2^4 f(x)\,dx$

$$\Rightarrow \int_2^8 f(x)\,dx = \int_2^4 f(x)\,dx + \int_4^8 f(x)\,dx$$

$$-\left(-\int_8^4 f(x)\,dx\right)$$

$$\int_2^4 f(x)\,dx = \int_2^8 f(x)\,dx - \int_4^8 f(x)\,dx = \underbrace{\int_2^8 f(x)\,dx}_{20} + \underbrace{\int_8^4 f(x)\,dx}_{12} = 32$$

③

a) $\int_2^2 g(x)\,dx = 0$ prop. $\int_a^a f(x)\,dx = 0$

b) $\int_5^1 g(x)\,dx = -\int_1^5 g(x)\,dx = -8$ prop. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

c) $\int_1^2 3f(x)\,dx = 3\int_1^2 f(x)\,dx = -12$ prop. $\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$

d) $\int_2^5 f(x)\,dx = \int_1^5 f(x)\,dx - \int_1^2 f(x)\,dx = 10$ prop. $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

③

e) $\int_1^5 \left(f(x) - g(x)\right) dx = \int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx = -2$

f) $\int_1^5 \left(4f(x) - g(x)\right) dx = 4\int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx = 16$

prop. $\int_a^b \left[\alpha f(x) + \beta g(x)\right] dx = \alpha \int_a^b f(x)\,dx + \beta \int_a^b g(x)\,dx$

⑤ a) $\int_1^4 3x-2\,dx = 3\int_1^4 x\,dx - 2\int_1^4 1\,dx = \frac{3}{2}(4^2-1^2) - 2(4-1) = \frac{33}{2}$

d) $\int_0^2 2x^2+x-3\,dx = 2\int_0^2 x^2\,dx + \int_0^2 x\,dx - 3\int_0^2 1\,dx = \frac{2}{3}(2^3-0) + \frac{2^2}{2} - 3\cdot 2 = \frac{4}{3}$

g) $\int_0^2 x(x+1)\,dx = \int_0^2 x^2+x\,dx = \underbrace{\int_0^2 x^2\,dx}_{\frac{2^3-0^3}{3}} + \int_0^2 x\,dx = \frac{2^3}{3} + \frac{2^2}{2} = \frac{14}{3}$

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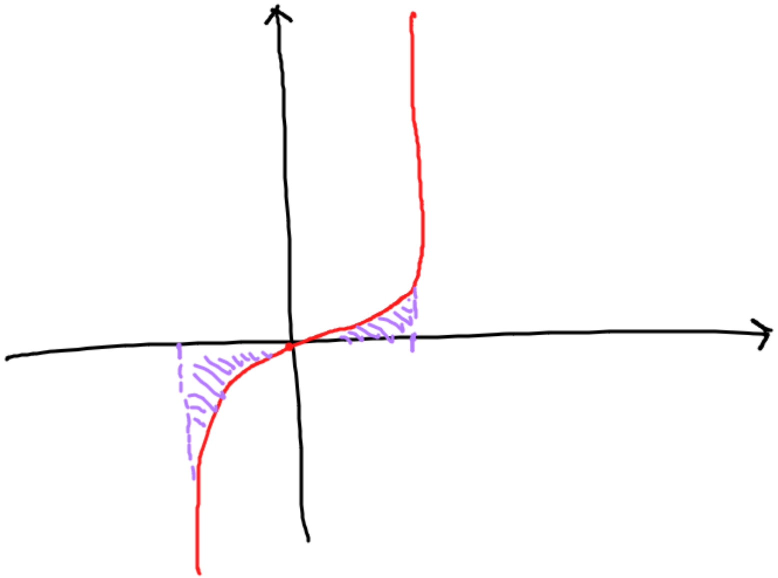
$$\int_{-4}^{4} \frac{x^7}{4}\,dx = \frac{1}{4}\int_{-4}^{4} x^7\,dx = 0$$



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$$\int_a^b \frac{1}{x}\,dx = \ln(b) - \ln(a)$$

$$\int_2^5 \frac{1}{2x}\,dx = \frac{1}{2}\int_2^5 \frac{1}{x}\,dx = \frac{\ln(5) - \ln(2)}{2}$$

o) $$\int_1^5 \frac{1}{x+1}\,dx = \int_{1+1}^{5+1} \frac{1}{u}\,du = \int_2^6 \frac{1}{u}\,du = \ln(6) - \ln(2)$$

$f(x-1)$

$u = x+1 \rightarrow x = u-1$

d) $$\int_0^1 \frac{x-1}{x+1}\,dx = \int_0^1 \frac{x}{x+1}\,dx - \int_0^1 \frac{1}{x+1}\,dx$$

$U = x+1$

$$\parallel$$

$$\int_0^1 \frac{x-1+1-1}{x+1}\,dx = \int_0^1 \frac{x+1-2}{x+1}\,dx = \int_0^1 \frac{x+1}{x+1}\,dx - \int_0^1 \frac{2}{x+1}\,dx = \int_0^1 1\,dx - 2\int_0^1 \frac{1}{x+1}\,dx =$$

$$1 - 2\int_1^2 \frac{1}{u}\,du = 1 - 2\left[\ln(2) - \underbrace{\ln(1)}_{0}\right] = 1 - 2\ln(2) = \ln(e) - \ln(2^2)$$

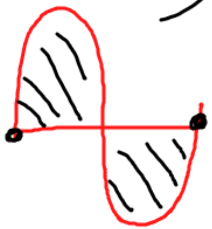
$$\Rightarrow \ln(e/4)$$

4

b) $\displaystyle\int_0^{2\pi} \cos(Kt) + 5 \, dt = \int_0^{2\pi} \cos(Kt)\, dt + \int_0^{2\pi} 5 \, dt = 10\pi$

$= 0$

$0, 2\pi, 4\pi \ldots$



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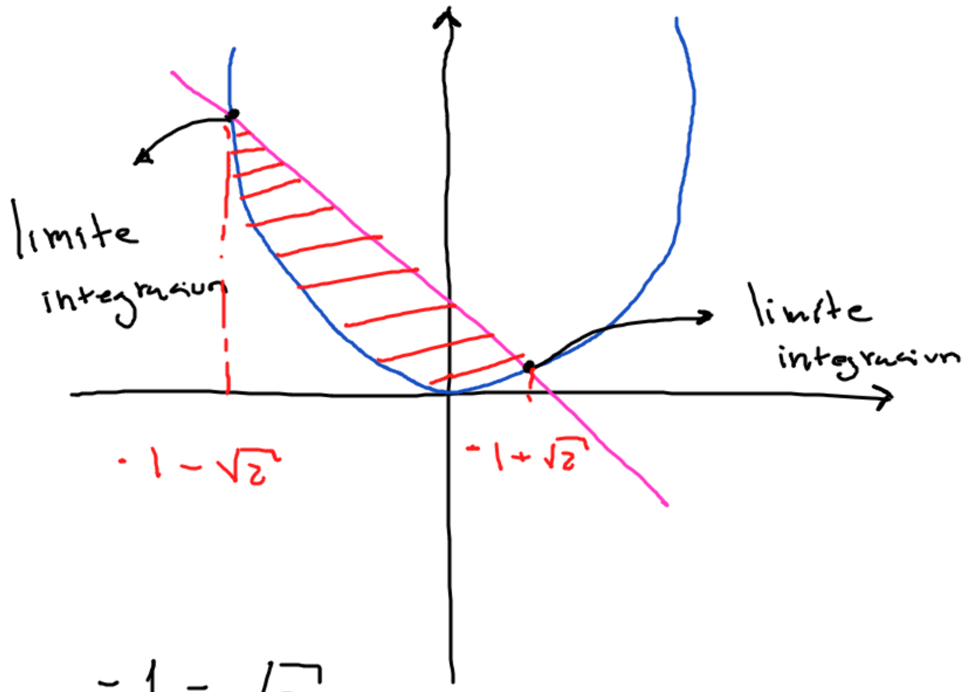
$f(x) = x^2$

a) $g(x) = -2x + 1$



$$x^2 = -2x + 1$$

$$x^2 + 2 - 1 = 0$$

$$\frac{-2 \pm \sqrt{4+4}}{2} = -1 + \sqrt{2} \text{ y } -1 - \sqrt{2}$$

$$\int_{-1-\sqrt{2}}^{-1+\sqrt{2}} -2x+1\,dx - \int_{-1-\sqrt{2}}^{-1+\sqrt{2}} x^2\,dx = \int_{-1-\sqrt{2}}^{-1+\sqrt{2}} 1\,dx - 2\int_{-1-\sqrt{2}}^{-1+\sqrt{2}} x\,dx - \int_{-1-\sqrt{2}}^{-1+\sqrt{2}} x^2\,dx$$

$$\underbrace{-1+\sqrt{2}-(-1-\sqrt{2})}_{\underbrace{-1+\sqrt{2}+1+\sqrt{2}}_{2\sqrt{2}}} - 2\,\underbrace{\frac{\left[(-1+\sqrt{2})^2-(-1-\sqrt{2})^2\right]}{2}}_{(-1+\sqrt{2})^2-(-1-\sqrt{2})^2} - \frac{(-1+\sqrt{2})^3-(-1-\sqrt{2})^3}{3}$$

$$= \frac{8\sqrt{2}}{3}$$