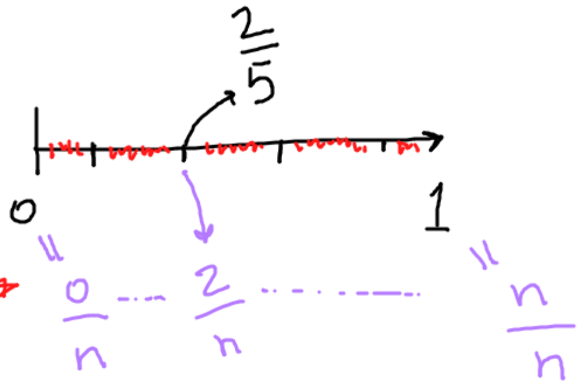
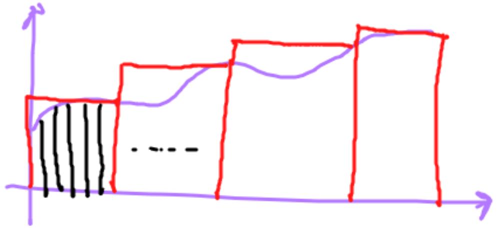
Ejercicio 6 $\quad F(x) = \int_0^x f(t)\,dt \qquad n \in \mathbb{N}$

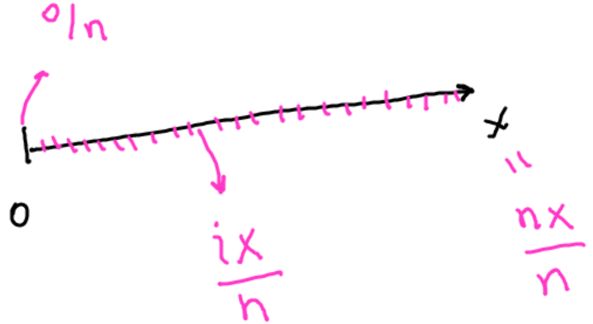
$$f(t) = \max\{t, 2-t\} \qquad P_n = \frac{1}{n}$$

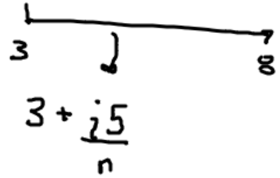
$$f(t) = \begin{cases} t & t > 1 \\ 2-t & t < 1 \end{cases}$$



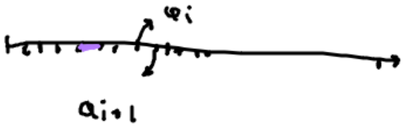


$$\int_3^8 \cdots$$





$f(t) = t$ $\qquad P_n = \frac{1}{n}$

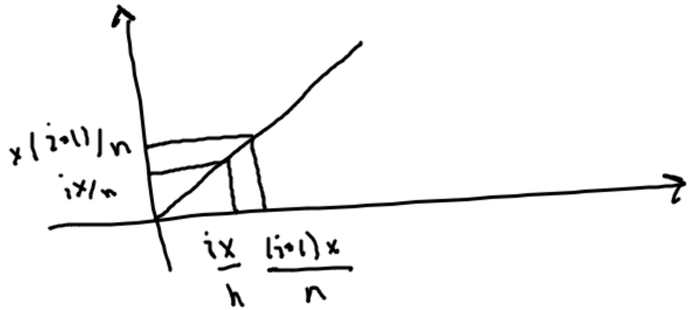


$\frac{(i+1)}{n} - \frac{i}{n} = \frac{1}{n}$

$$S_*(f, P_n) = \sum_{i=0}^{n-1} (a_{i+1} - a_i) \inf(f, [a_i, a_{i+1}]) = \sum_{i=0}^{n-1} \frac{x}{n} \inf(f, [a_i, a_{i+1}]) =$$

$$= \frac{x}{n} \cdot \inf\left(f, \left[\frac{0}{n}, \frac{x}{n}\right]\right) + \dots + \frac{x}{n} \inf\left(f, \left[\frac{x(n-1)}{n}, \frac{xn}{n}\right]\right) =$$

$$\frac{x}{n} f(0) + \dots + \frac{x}{n} f\left(\frac{xn-1}{n}\right) = \frac{x}{n} f\left(\frac{x}{n}\right)$$



$$\sup(S_*) \le \int f(t)\,dt \le \overbrace{\inf(S^*)}^{\text{analogamente}}$$

$$\frac{x^2}{2} - \frac{x^2}{2n}$$

$$I_* \le \int f(t)\,dt \le I^*$$

$$\frac{x^2}{2} - \frac{x^2}{2},\ \frac{x^2}{2} - \frac{x^2}{7},\ \dots,\ \frac{x^2}{2} - \frac{x^2}{20000},\ \dots$$

$$\frac{x^2}{2} - \xi \le \int f(t)\,dt \le \frac{x^2}{2} + \xi \implies \int_0^t f(t)\,dt = \frac{x^2}{2}$$

$n \to \infty$

$\frac{x^2}{2}$ - chico

$$\frac{x}{n} f(0) + \frac{x}{n} f\left(\frac{x}{n}\right) + \frac{x}{n} f(2x/n) + \cdots + \frac{x}{n} f\left(\frac{(n-1)x}{n}\right) =$$

$$= 0 + \underbrace{\frac{x}{n}\cdot\frac{x}{n}}_{\frac{x^2}{n^2}} + \underbrace{\frac{x}{n}\cdot\frac{2x}{n}}_{2x^2/n^2} + \cdots + \underbrace{\frac{x}{n}\,\frac{(n-1)x}{n}}_{\frac{x^2(n-1)}{n}} =$$

$$\sum_{i=0}^{(n-1)} \frac{i\,x^2}{n^2} = \frac{x^2(n-1)n}{2n^2} = \frac{x^2n^2 - x^2n}{2n^2} = \frac{x^2}{2} - \frac{x^2}{2n} \quad n \in \mathbb{N}$$

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2} \qquad n \to n-1$$

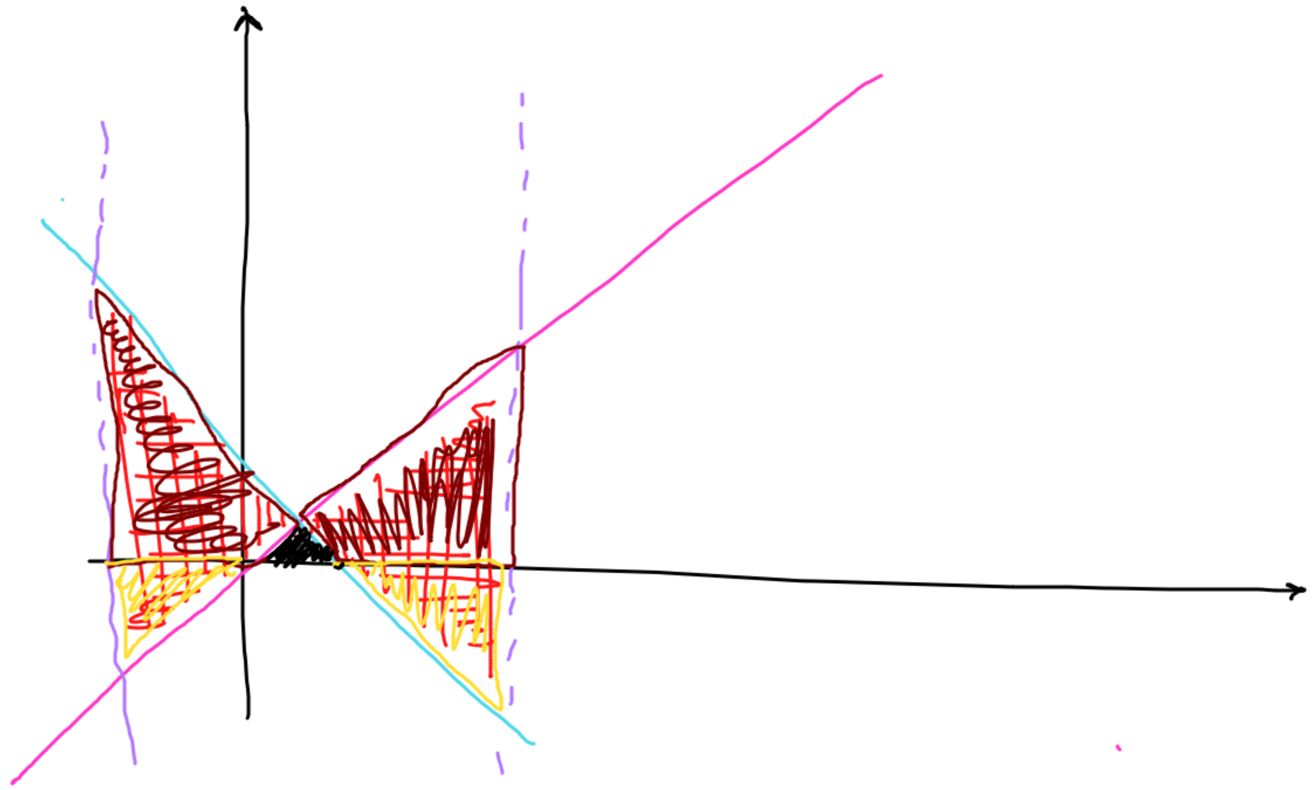
$$\sup\{S_*\}_{n\in\mathbb{N}} = \sup\left\{\frac{x^2}{2} - \frac{x^2}{2n}\right\}_{n\in\mathbb{N}} = \frac{x^2}{2} - \underbrace{\text{algo}(n)}_{\to 0} = \frac{x^2}{2}$$

Ejercicio 7

$f(x) = x$

$g(x) = 1 - 2x$

$[-1, 2]$



$\frac{3}{2} \cdot \frac{3}{2} = \frac{9}{4}$ tri. sup. izq.

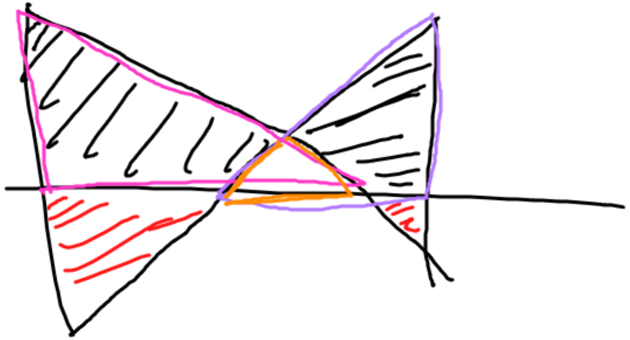
$\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{12}$ area tri chiquito

$\frac{2 \cdot 2}{2} = 2$ tri. sup. der.

$\Rightarrow \frac{9}{4} + 2 - \frac{9}{4} - \frac{1}{2} - 2 \cdot \frac{1}{12} = \frac{3}{2} - \frac{1}{6} = \frac{8}{6}$

$\frac{3}{2} \cdot \frac{3}{2} = \frac{9}{4}$ tir. inf. der

$\frac{1 \cdot 1}{2} = \frac{1}{2}$ tri. inf. izq.

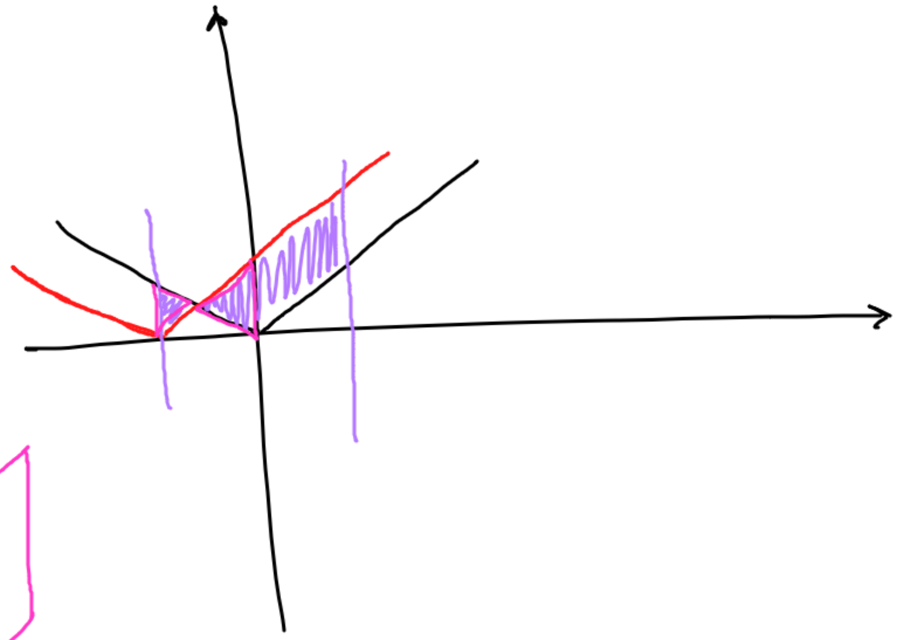


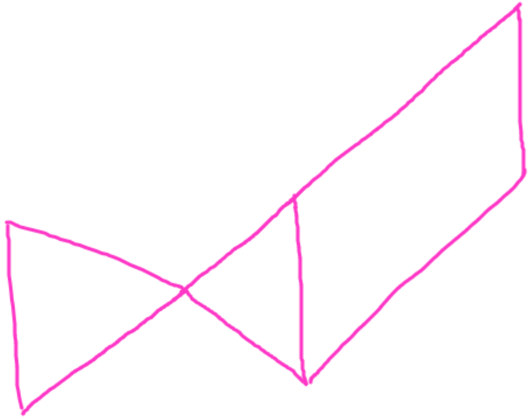
c)

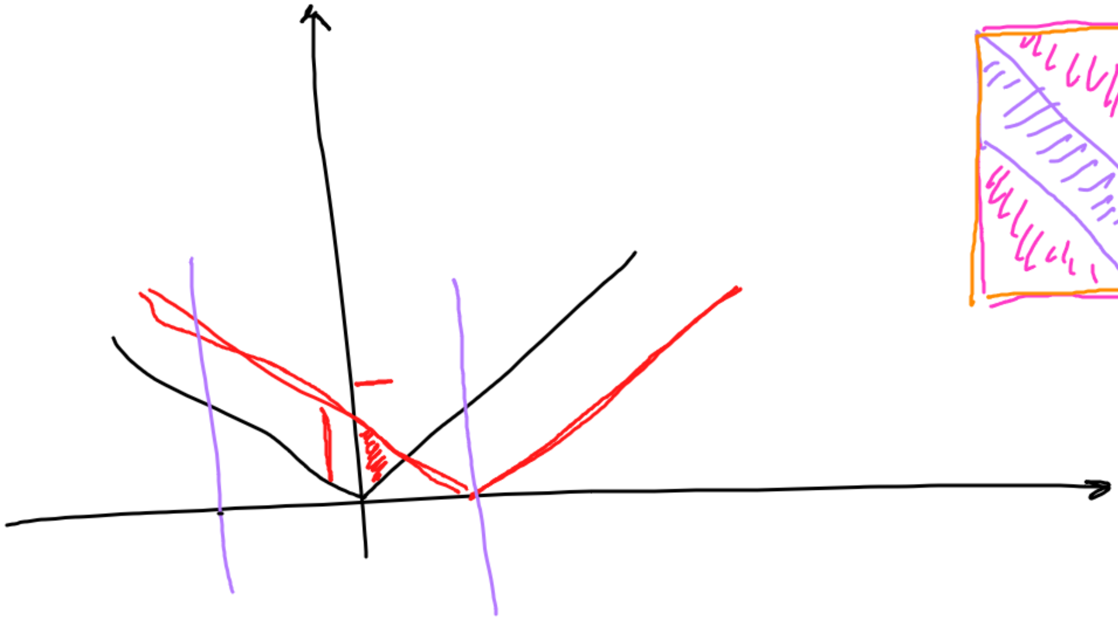
$f(x) = |x|$

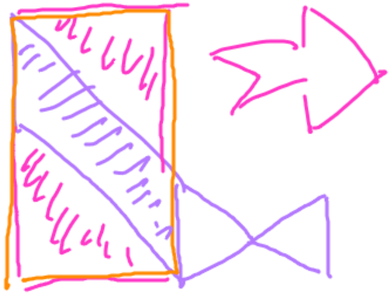
$g(x) = |x-1|$

$[-1, 1]$









rectangulo 2

area rose 1

$2 - 1 = \boxed{1}$

$$\frac{1 \cdot 1}{2} = \frac{1}{2}$$

$$1 \cdot 2 = 2$$

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

area 2 tri. $\boxed{\frac{1}{2}}$

$$\frac{1}{2} + 1 = \frac{3}{2}$$

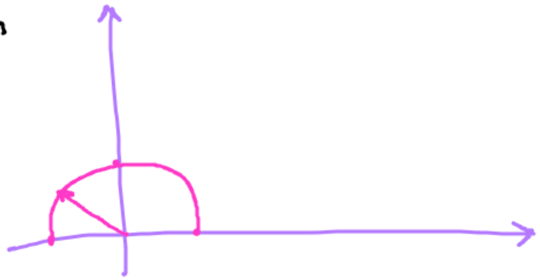
3.2 Ejercicio 1

$$\frac{\sqrt{3}}{4} \le \int_0^{1/2} \sqrt{1-x^2}\,dx \le \frac{1}{2} \to \frac{1}{2} \cdot 1$$

$$= \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$

$$f(1/2) = \sqrt{1-\frac{1}{4}} = \sqrt{3/4}$$

$$f(1/2) = \frac{\sqrt{3}}{2}$$



$r=1$