Numero real

Axioma de cuerpo ordenado

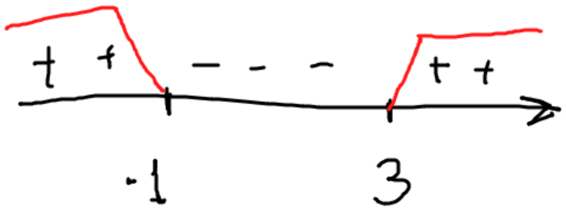
Ejercicio 2

	1	-2	-3
-1		-1	3
	1	-3	0

a) $x^2 - 2x > 3$

$x^2 - 2x - 3 > 0 \Rightarrow x^2 - 2x - 3 = 0 = (x+1)(x-3) = f(x)$

sig $f(x)$



punto testigo $f(x=0) = -3$

$x \in (-\infty, -1) \cup (3, +\infty)$

c) $\underbrace{x(x-1)(x-2)(x-3)}_{f(x)} < 0$

		0		1		2		3	
sig$(f(x))$	+ +		- -		+ +		- -		+ +

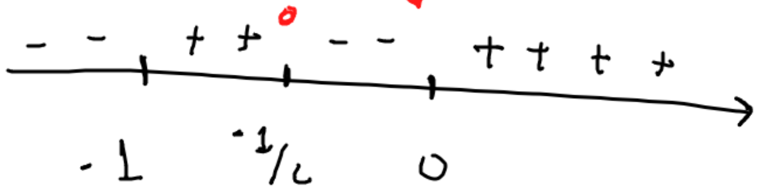
$x \in (0,1) \cup (2,3)$

e) $$\frac{1}{x} + \frac{1}{x+1} \geq 0$$

$$\frac{x+1+x}{x(x+1)} \geq 0$$

$2x+1=0 \Rightarrow x = -1/2$

sig



$x \in (-1, -1/2) \cup (0, +\infty)$

$x \in (-1, -1/2] \cup (0, +\infty)$

g) $\sqrt{x+4} < x-1 \Rightarrow x+4 < (x-1)^2$

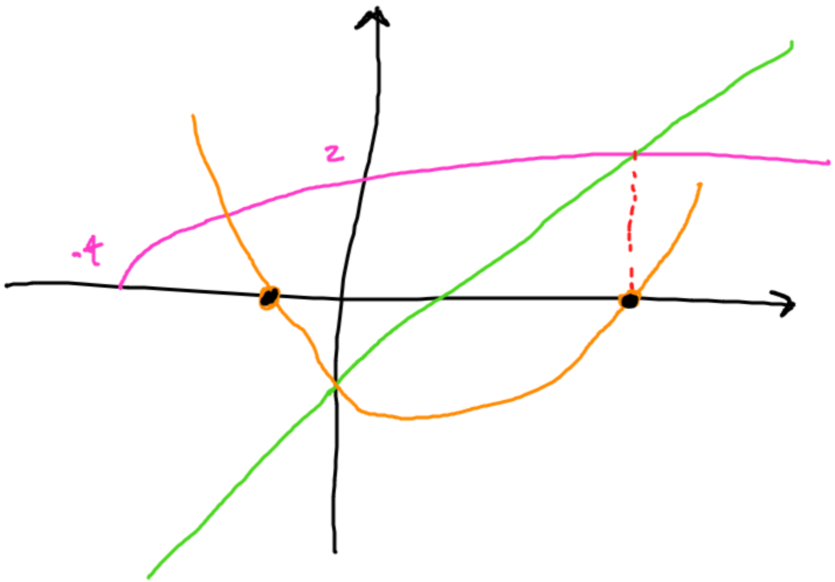
$(\sqrt{x})^2 = |x|$

$x+4>0 \Rightarrow \boxed{x>-4}$

$x^2-2x+1 > x+4$

$x^2-3x-3>0$

$$\frac{3\pm\sqrt{9+12}}{2} = \boxed{\begin{array}{l}\frac{3+\sqrt{21}}{2} = 3,8 \\ \frac{3-\sqrt{21}}{2} = -0,8\end{array}}$$



$\boxed{x>3,8}$

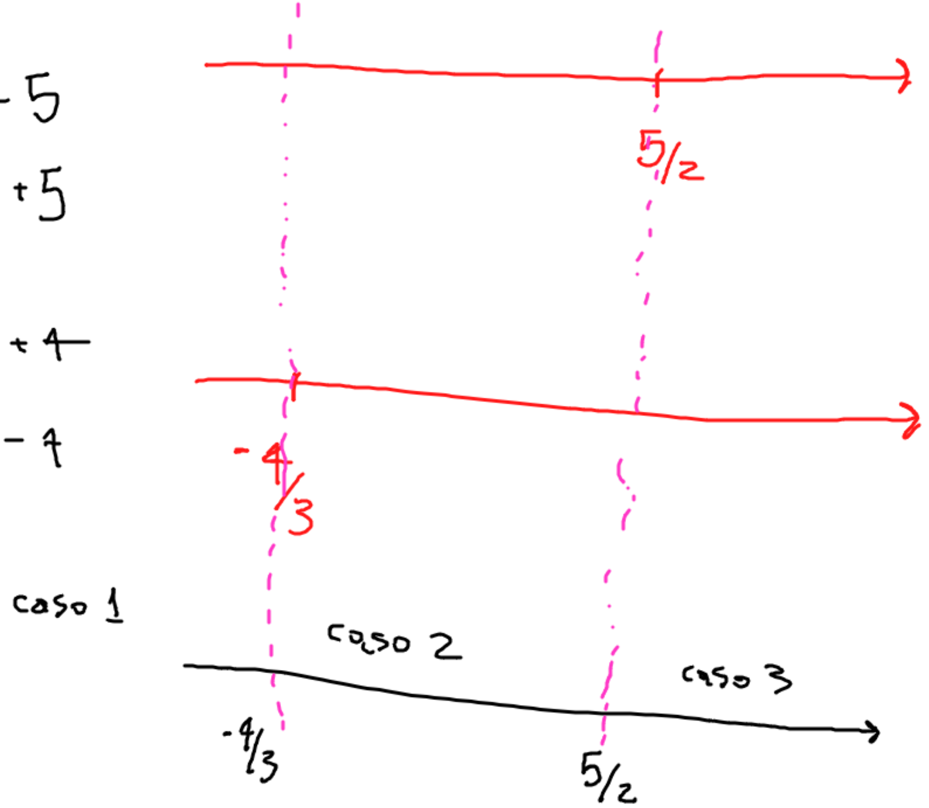
$x \in (-4,+\infty) \cap (3,8,+\infty)$

$x \in (3,8,+\infty)$

i) $\underbrace{|2x-5|}_{f(x)} < \underbrace{|3x+4|}_{g(x)}$

$$2x-5 > 0 \Rightarrow x \geq 5/2 \Rightarrow f(x) = 2x-5$$
$$x < 5/2 \Rightarrow f(x) = -2x+5$$

$$3x+4 > 0 \Rightarrow x \geq -4/3 \Rightarrow g(x) = 3x+4$$
$$x < -4/3 \Rightarrow g(x) = -3x-4$$



caso 1 $\quad -2x+5<-3x-4 \Rightarrow x<-9$

$x \in (-\infty,-9) \cap (-\infty,-4/3) \Rightarrow x \in (-\infty,-9)$

caso 2 $\quad -2x+5<3x+4 \Rightarrow 5x>1 \Rightarrow x>1/5$

$x \in (1/5,+\infty) \cap [-4/3, 5/2) \Rightarrow x \in (1/5, 5/2)$

caso 3 $\quad 2x-5<3x+4 \Rightarrow x>-9$

$x \in (-9,+\infty) \cap [5/2,+\infty) \Rightarrow x \in [5/2,+\infty)$

$x \in (-\infty,-9) \cup (1/5,+\infty)$

Ejercicio 3 $\quad 0 < a < b \Rightarrow a < H < G < A < b$

$a < H$ $\quad a < \dfrac{2}{\frac{1}{a} + \frac{1}{b}} = \dfrac{2}{\frac{b+a}{ab}} = \dfrac{2ab}{b+a} \iff$

$\iff \dfrac{a}{a} < \dfrac{2ab}{(b+a)a} \iff 1 < \dfrac{2b}{b+a} \iff b + a < 2b \iff$

$\iff a < b$

$\sqrt{ab}\sqrt{ab} = ab$ $\qquad$ $x > 0 \Rightarrow (\sqrt{x})^2 = x$

$H < G$

$$\frac{2ab}{b+a} < \sqrt{ab} \iff \frac{2\sqrt{ab}\sqrt{ab}}{b+a} < \sqrt{ab} \iff \boxed{2\sqrt{ab} < b+a}$$

$$(2\sqrt{ab})^2 = 2^2(\sqrt{ab})^2$$

$$2^2(\sqrt{ab})^2 < (b+a)^2 \iff 4ab < b^2 + 2ab + a^2 \iff \underbrace{b^2 - 2ab + a^2}_{(b-a)^2} > 0$$

$G < A$ $\quad \boxed{\sqrt{ab} < \frac{a+b}{2}}$

$A < b$ $\quad \frac{a+b}{2} < b \iff \frac{a+b}{2} < \frac{b+b}{2} \iff a < b$

$a < c < b$ $\quad a < \sqrt{\frac{a^2+b^2}{2}} \iff a^2 < \frac{a^2+b^2}{2} \iff 2a^2 < a^2+b^2$

$\iff a^2 < b^2$ $\quad a > 0 \quad b > 0 \quad \Rightarrow \quad a < b \iff a^2 < b^2$

$\sqrt{\frac{a^2+b^2}{2}} < b \iff \frac{a^2+b^2}{2} < b^2 \iff a^2 < b^2$

$A < C$

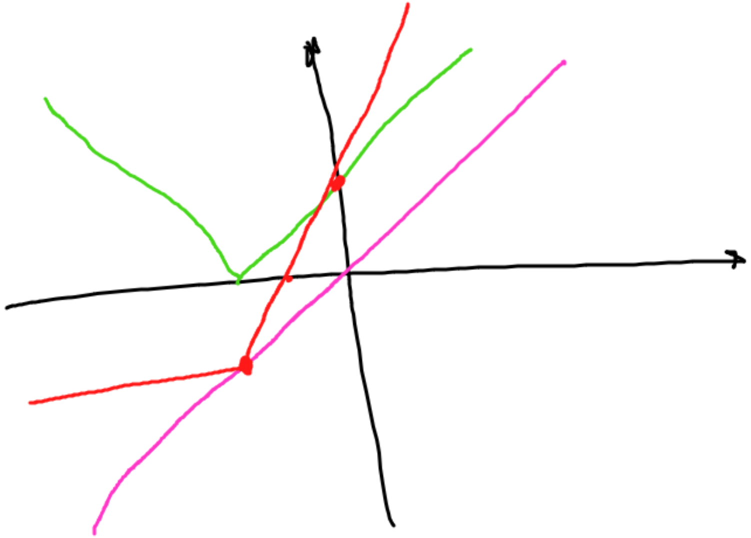
$$\frac{a+b}{2} < \sqrt{\frac{a^2+b^2}{2}} \iff \frac{(a+b)^2}{4} < \frac{a^2+b^2}{2} \iff$$

$$\iff \frac{(a+b)^2}{2} < a^2+b^2 \iff a^2+2ab+b^2 < 2a^2+2b^2 \iff a^2-2ab+b^2 > 0$$

$$(a-b)^2$$

Funciones reales

Ejercicio 1 $f(x) = x + |x+1|$



$f(x) = |x| + |x+1|$