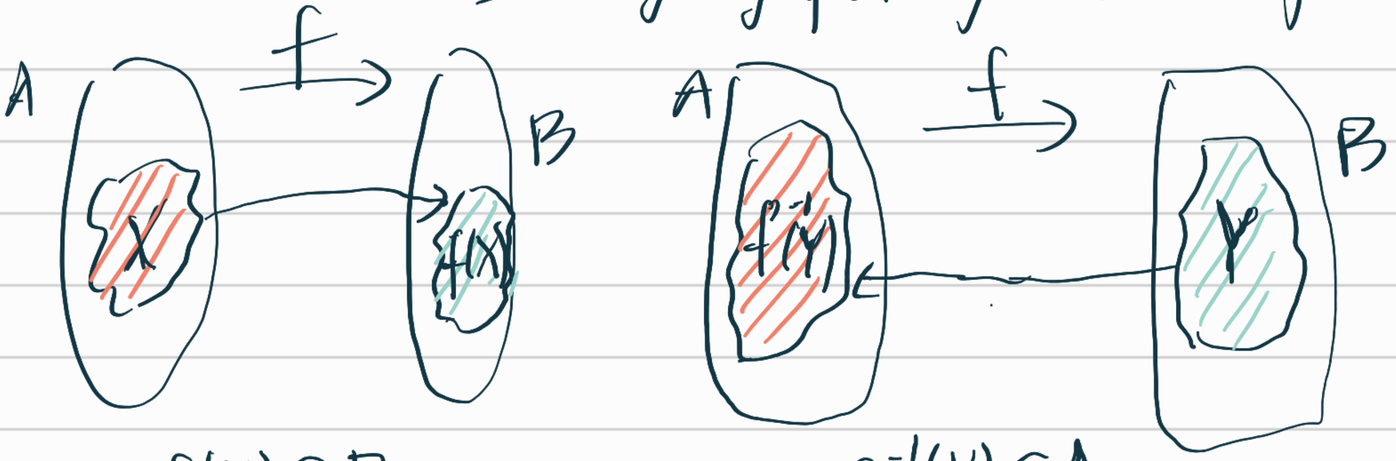
Clase 3 _ Funciones (parte 2)

Clase anterior: → Dominio, codominio, Recorrido (imagen)
→ Inyectividad, sobreyectividad, biyectividad.
→ Imagen y preimagen de un conjunto.



$f(X) \subseteq B$ $\qquad$ $f^{-1}(Y) \subseteq A$

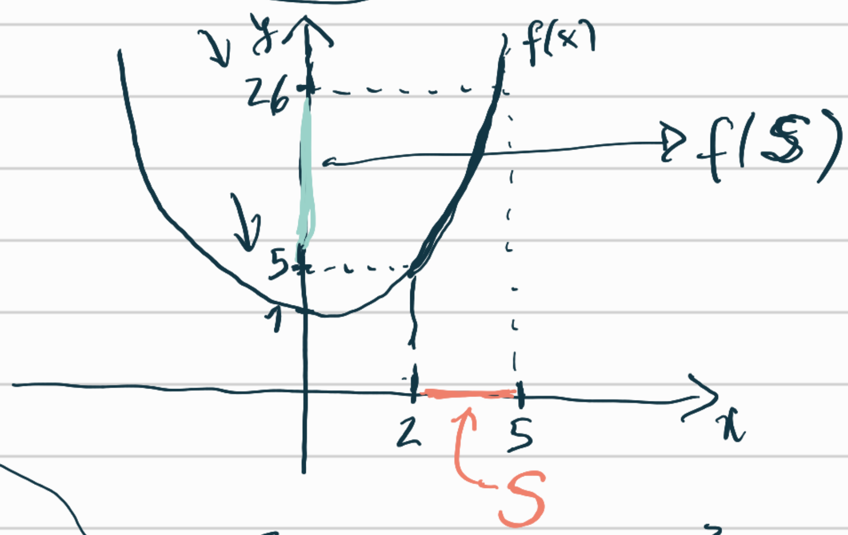
Ejercicio: $f: [-10,10] \to \mathbb{R} \;/\; f(x) = x^2+1$

2.0.14 $\quad S = [2,5] \subseteq [-10,10]$

$R = [82, 101] \subseteq \mathbb{R}$

Determinar $f(S)$ y $f^{-1}(R)$.

Solución:

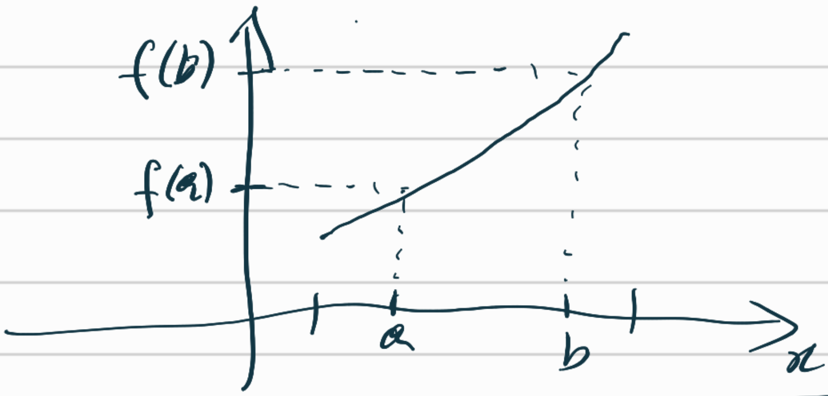


$$2 \leq x \leq 5 \Leftrightarrow 2^2+1 \leq x^2+1 \leq 5^2+1 \Leftrightarrow 5 \leq f(x) \leq 26$$

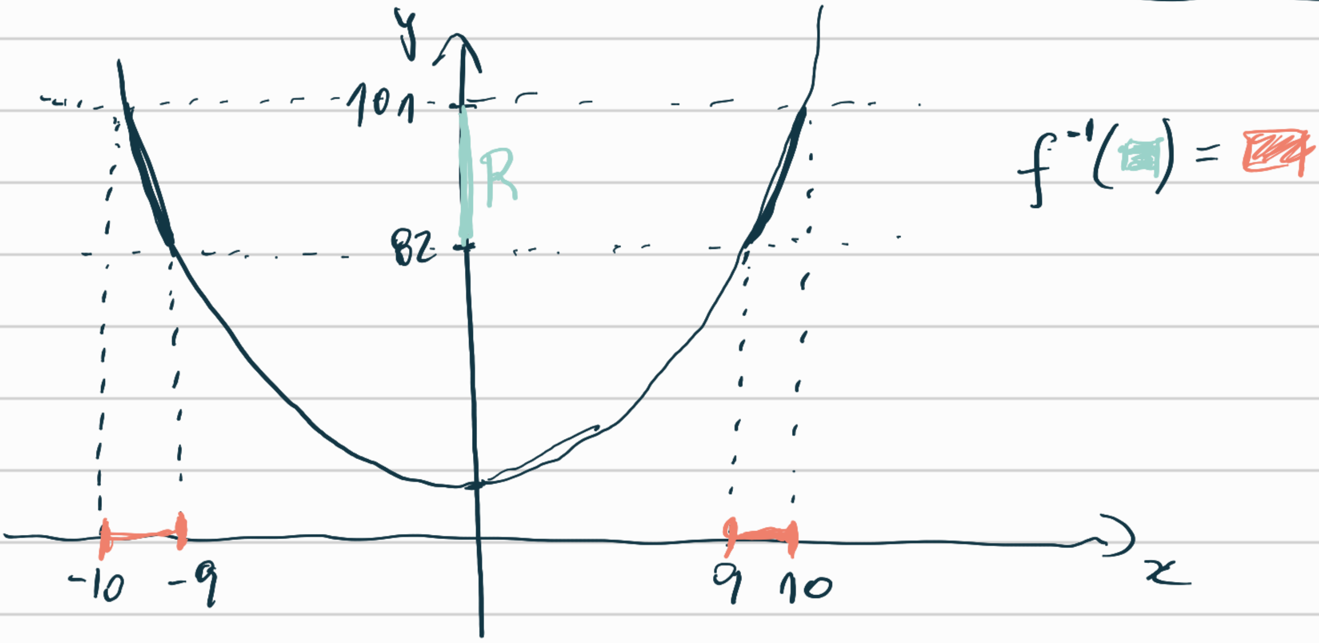
f(x) es creciente en $[0,+\infty)$

$\therefore f([2,5]) = [5,26] \quad (f(S) = [5,26])$

$f: I \to \mathbb{R}$ <u>creciente</u> significa que si $a, b \in I$, $\underbrace{a \leq b} \Rightarrow \underbrace{f(a) \leq f(b)}$



Si $f: I \to \mathbb{R}$ es derivable y $f'(x) > 0 \quad \forall x \in I$
$\Rightarrow f$ es creciente en I



$x \in [-10, 10] \; / \; 82 \leq f(x) \leq 101$

$82 \leq x^2 + 1 \leq 101$

$81 \leq x^2 \leq 100$

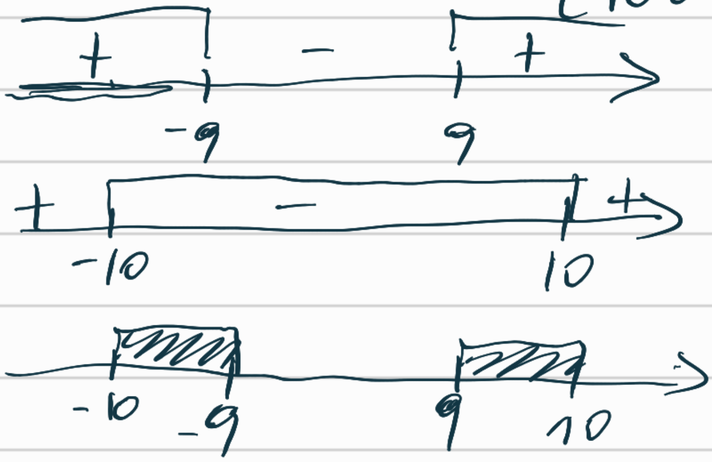
$9^2 \leq |x|^2 \leq 10^2$

$9 \leq |x| \leq 10$

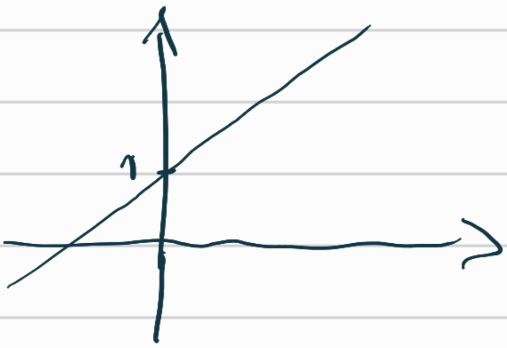
$x \in [9, 10] \cup [-10, -9] \subseteq [-10, 10]$

$\Rightarrow f^{-1}(R) = [-10, -9] \cup [9, 10]$

Otro: $81 \le x^2 \le 100 \iff \begin{cases} 81 \le x^2 \\ 100 \ge x^2 \end{cases} \iff \begin{cases} x^2 - 81 \ge 0 \\ x^2 - 100 \le 0 \end{cases}$



Observación:



$f(x) = x+1$
es creciente en $\mathbb{R}$

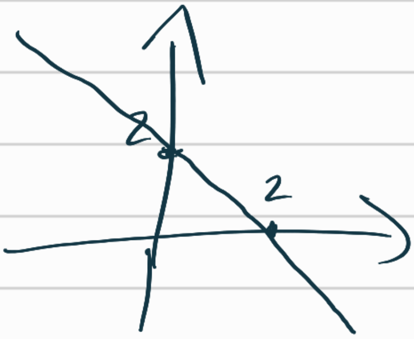
$$a \le b \longrightarrow f(a) \le f(b) \rightarrow a+1 \le b+1$$

Otra: $a \le b \rightarrow b - a \ge 0 \rightarrow$

$$(b+1) - (a+1) = b + 1 - a - 1 = b - a \ge 0$$

$$\rightarrow b + 1 \ge a + 1$$

Obs: $f(x) = 2 - x$



decreciente

$$a \le b \rightarrow 2 - a \ge 2 - b$$

Gráfico de una función

En este curso consideramos funciones

$f: X \subseteq \mathbb{R} \to \mathbb{R}$

Ejemplo: $f: \mathbb{R} \to \mathbb{R} \ / \ f(x) = x^2+1$

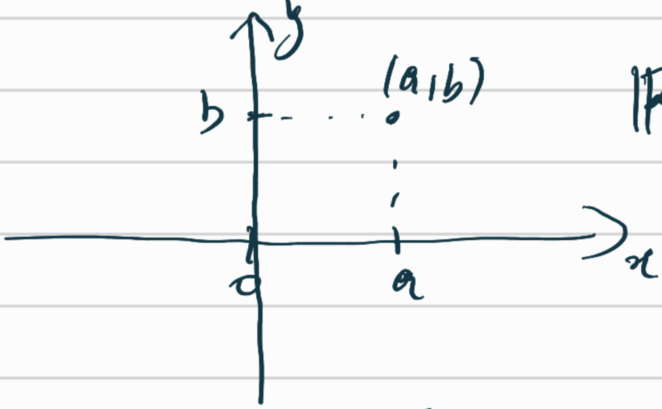
$f: \mathbb{R}\setminus\{0\} \to \mathbb{R} \ / \ f(x) = 1/x$

$f: \mathbb{R}^+ \to \mathbb{R} \ / \ f(x) = \log(x)$.

Def: $\mathbb{R}^2 = \{(x,y) : x, y \in \mathbb{R}\}$

$\mathbb{R}$ (recta real)

$f: X \to \mathbb{R}$
$X \subseteq \mathbb{R}$



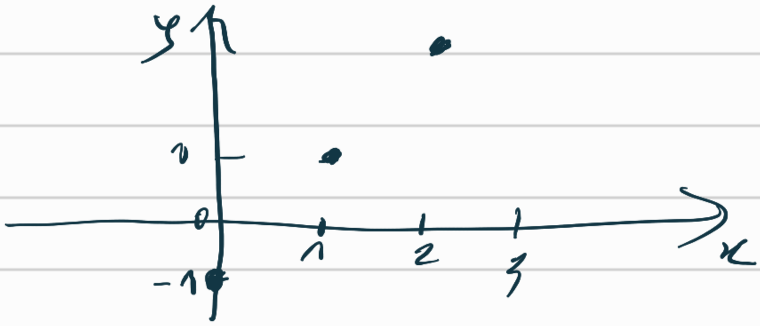
$\mathbb{R}^2$ = plano cartesiano

Def: Si $\boxed{f: X \subseteq \mathbb{R} \to \mathbb{R}}$ entonces su gráfico

es $G(f) = \{(x, f(x)) : x \in X\}$
$= \{(x,y) \in \mathbb{R}^2 : y = f(x)\}$

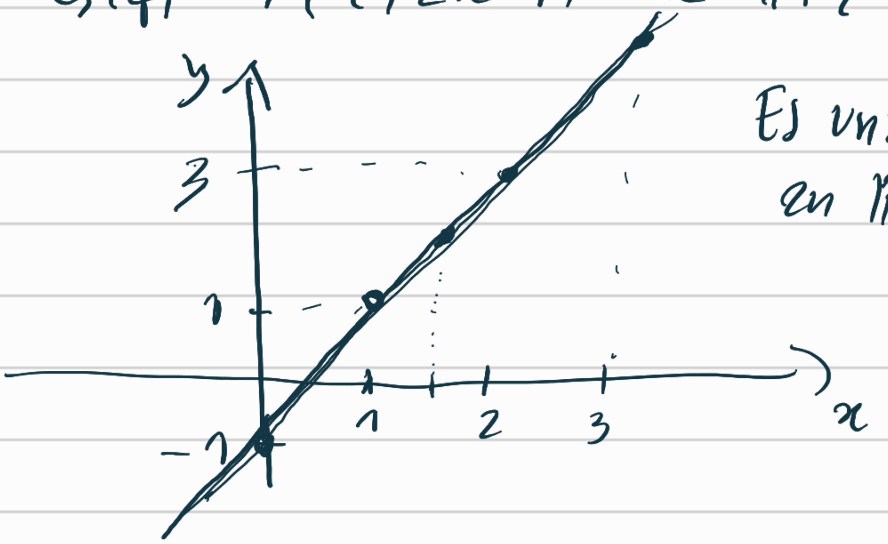
Ejemplo: $f: \{0,1,2,3\} \to \mathbb{R} \ / \ f(x) = 2x-1$

$G(f) = \{(0,-1), (1,1), (2,3), (3,5)\} \subseteq \mathbb{R}^2$



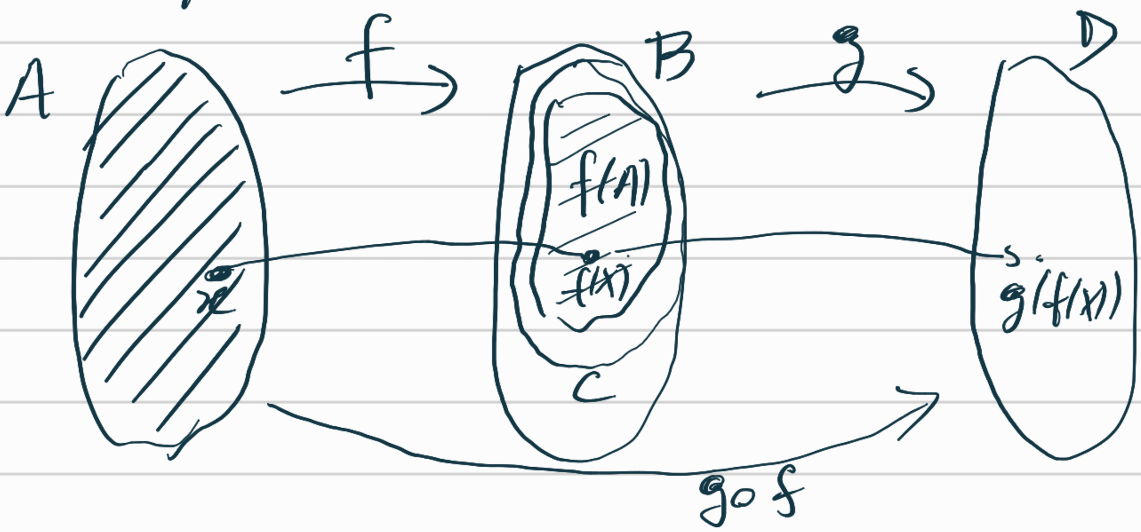
Ejemplo: $f : \mathbb{R} \to \mathbb{R} \,/\, f(x) = 2x - 1$

$$G(f) = \{(x, 2x-1) : x \in \mathbb{R}\} \subseteq \mathbb{R}^2$$



Es una recta en $\mathbb{R}^2$

Composición de funciones

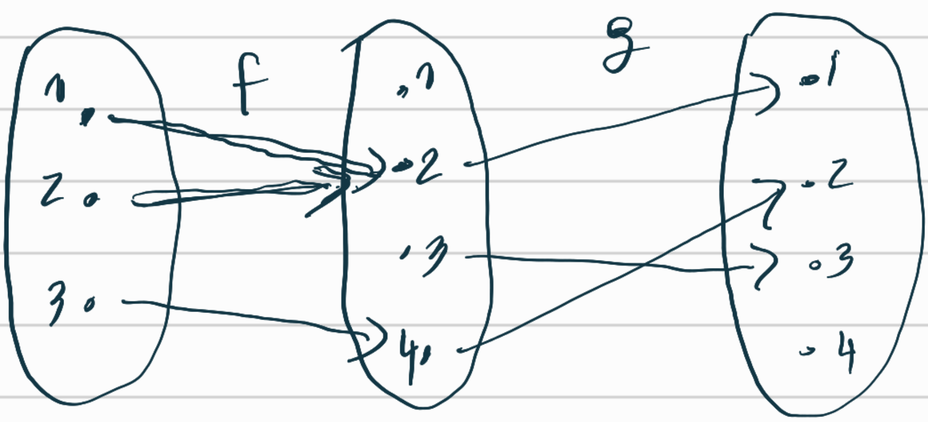


Def: Sean $f : A \to B$, $g : C \to D \,/\, f(A) \subseteq C$

Entonces se define $g \circ f : A \to D \,/$

$(g \circ f)(x) = g(f(x))$.

Ejemplo:



$g \circ f : \{1,2,3\} \to \{1,2,3,4\}$

$(g \circ f)(1) = g(f(1)) = g(2) = 1$

$(g \circ f)(2) = 1$

$(g \circ f)(3) = 2$

Ejemplo: $f : \mathbb{R} \to \mathbb{R} \;/\; f(x) = x^2 + 1 \geq 1$

$g : \mathbb{R}^+ \to \mathbb{R} \;/\; g(x) = \log(x) + 1$

$(f \circ g)(x) \overset{\text{def.}}{=} f(g(x)) = f(\log(x)+1) = (\log(x)+1)^2 + 1$

$= \log^2(x) + 2\log(x) + 2$

$(g \circ f)(x) = g(f(x)) = g(x^2+1) = \log(x^2+1) + 1$

$f(\mathbb{R}) = [1, +\infty) \subseteq \mathbb{R}^+ = \mathrm{dom}(g)$ ✓

Obs: Si $f : \mathbb{R} \to \mathbb{R} \;/\; f(x) = x + 1$

$g : \mathbb{R}^+ \to \mathbb{R} \;/\; g(x) = \log(x) + 1$

$\Rightarrow$ $f \circ g$ está definido pero $g \circ f$ no está definido.

$\boxed{g(\mathbb{R}^+) \subseteq \mathbb{R} = \mathrm{dom}(f)}$

$\boxed{f(\mathbb{R}) = \mathbb{R} \not\subseteq \mathrm{dom}(g)}$