

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$A + C = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

$$A * B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \end{pmatrix}$$

$$A * C = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$v = (v_1, v_2, \dots, v_n)$$

$$\|v\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$$

$$v \text{ dot } v = 0$$

$$v \text{ dot } v = v_1^2$$

$$v \text{ dot } v = v_1^2 + v_2^2$$

⋮

$$v \text{ dot } v = v_1^2 + v_2^2 + \dots + v_n^2$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} & \dots & a_{1M} \\ a_{21} & a_{22} & & a_{2n} & \dots & a_{2M} \\ \vdots & & & a_{in} & & \\ a_{k1} & a_{k2} & \dots & a_{kn} & \dots & a_{kM} \end{pmatrix}$$

$$V = [v_1, v_2, v_i, \dots, v_k]$$

$$\text{retval} = \begin{pmatrix} a_{11} & a_{12} & \dots & v_1 & \dots & a_{1M} \\ a_{21} & a_{22} & & v_2 & & a_{2M} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{k1} & a_{k2} & \dots & v_k & \dots & a_{kM} \end{pmatrix}$$

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$$A = \begin{pmatrix} 2 & 3 & 0 \\ 2 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathcal{B} = \{ \overset{v_1}{(2, 2, 0)}, \overset{v_2}{(3, -1, 0)}, \overset{v_3}{(0, -2, 1)} \}$$

$$\langle (x_1, y_1, z_1), (x_2, y_2, z_2) \rangle = x_1 x_2 + y_1 y_2 + z_1 z_2$$

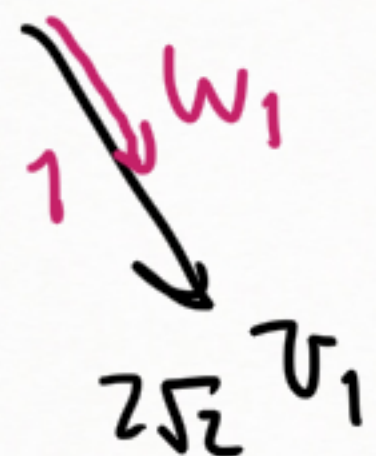
$$\mathcal{B} \rightarrow \mathcal{B}' \text{ bon } \quad \mathcal{B}' = \{ w_1, w_2, w_3 \} \quad \|w_i\| = 1 \quad \langle w_i, w_j \rangle = 0 \quad i \neq j$$



w<sub>1</sub>:

$$w_1 = \frac{v_1}{\|v_1\|} = \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right)$$

$$\|v_1\| = \sqrt{4+4} = 2\sqrt{2}$$



w<sub>2</sub>:

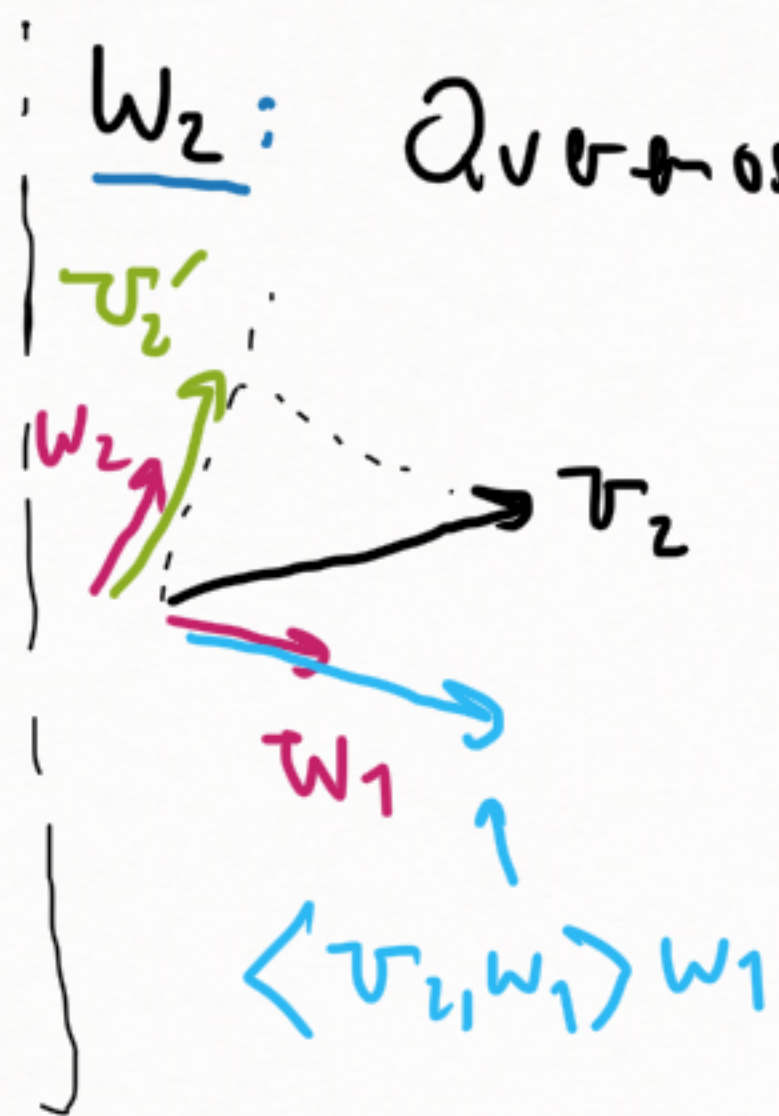
Quereforderung: 1)  $\langle w_1, w_2 \rangle = 0$       2)  $\|w_2\| = 1$

$$1: v_2' = v_2 - \langle v_2, w_1 \rangle w_1 = (3, -1, 0) - \sqrt{2} \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right)$$

$$\langle v_2, w_1 \rangle = \langle (3, -1, 0), \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right) \rangle = \frac{3}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \sqrt{2} \quad (= \frac{2}{\sqrt{2}})$$

$$v_2' = (2, -2, 0) \leftarrow \text{cumpel } \langle v_2', w_1 \rangle = 0$$

$$w_2 = v_2' / \|v_2'\| = \left( \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right)$$



(1, 1, 0)

$$v_3 = (0, -2, 1)$$

$$w_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \quad w_2 = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$$

$w_3$ : 
$$v_3' = v_3 - \langle v_3, w_1 \rangle w_1 - \langle v_3, w_2 \rangle w_2 = (0, -2, 1) + \sqrt{2} \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) - \sqrt{2} \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$$

$$\langle v_3, w_1 \rangle = -\frac{2}{\sqrt{2}} = -\sqrt{2} \qquad \qquad \qquad = (0, -2, 1) + (1, 1, 0) + (-1, 1, 0) = (0, 0, 1)$$

$$\langle v_3, w_2 \rangle = \frac{2}{\sqrt{2}} = \sqrt{2} \qquad \qquad \qquad \langle v_3', w_1 \rangle = 0 \quad \langle v_3', w_2 \rangle = 0$$

$$w_3 = \frac{v_3'}{\lambda} \qquad w_3 = (0, 0, 1)$$

$\lambda = \|v_3'\|$

$$\mathcal{B}' = \left\{ \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right), (0, 0, 1) \right\}$$

$$w_1 = \frac{1}{\|v_1\|} v_1 \rightarrow v_1 = \|v_1\| w_1$$

$$w_2 = \frac{1}{\|v_2'\|} v_2' = \frac{1}{\|v_2'\|} (v_2 - \langle v_2, w_1 \rangle w_1) = \frac{1}{\|v_2'\|} v_2 - \frac{\langle v_2, w_1 \rangle}{\|v_2'\|} w_1$$

despejo  $v_2$

$$\rightarrow v_2 = \|v_2'\| w_2 + \langle v_2, w_1 \rangle w_1$$

$$w_3 = \frac{1}{\|v_3'\|} (v_3 - \langle v_3, w_1 \rangle w_1 - \langle v_3, w_2 \rangle w_2) \rightarrow v_3 = \|v_3'\| w_3 + \langle v_3, w_1 \rangle w_1 + \langle v_3, w_2 \rangle w_2$$

$$v_1 = \|v_1\| w_1$$

ya los calculamos son números

$$v_2 = \langle v_2, w_1 \rangle w_1 + \|v_2'\| w_2$$

$$v_3 = \langle v_3, w_1 \rangle w_1 + \langle v_3, w_2 \rangle w_2 + \|v_3'\| w_3$$

$$A = (v_1 | v_2 | v_3)$$

$$R = \begin{pmatrix} \|v_1\| & \langle v_2, w_1 \rangle & \langle v_3, w_1 \rangle \\ 0 & \|v_2'\| & \langle v_3, w_2 \rangle \\ 0 & 0 & \|v_3'\| \end{pmatrix}$$

$$Q = (w_1 | w_2 | w_3)$$

$$A = QR$$

ortogonal xq las columnas son bon



$$QR = \left( \overbrace{w_1}^{\text{purple}} \mid \overbrace{w_2}^{\text{green}} \mid \overbrace{w_3}^{\text{red}} \right) \begin{pmatrix} \underbrace{\|v_1\|}_{\text{purple}} & \underbrace{\langle v_2, w_1 \rangle}_{\text{blue}} & \underbrace{\langle v_3, w_1 \rangle}_{\text{pink}} \\ \underbrace{0}_{\text{orange}} & \underbrace{\|v_2\|}_{\text{blue}} & \underbrace{\langle v_3, w_2 \rangle}_{\text{green}} \\ \underbrace{0}_{\text{yellow}} & \underbrace{0}_{\text{cyan}} & \underbrace{\|v_3\|}_{\text{red}} \end{pmatrix}$$

$$= \begin{pmatrix} \underbrace{\|v_1\| w_1}_{\tau_1} \mid \underbrace{\langle v_2, w_1 \rangle w_1 + \|v_2\| w_2}_{\tau_2} \mid \underbrace{\langle v_3, w_1 \rangle w_1 + \langle v_3, w_2 \rangle w_2 + \|v_3\| w_3}_{\tau_3} \end{pmatrix}$$

$$\begin{pmatrix} \sigma & \sigma & \sigma & \rightarrow \\ 0 & \sigma & \sigma & \rightarrow \\ 0 & 0 & 0 & \\ 1 & 0 & 0 & \end{pmatrix} \begin{pmatrix} + & + & + \\ 0 & - & - \\ 0 & 0 & + \end{pmatrix} \begin{pmatrix} 2 & 3 & 0 \\ 2 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= (\tau_1 \mid \tau_2 \mid \tau_3) = A$$