Teo: $A \in M_n(\mathbb{R})$ $\exists$ 1) P matriz de permutación

2) L matriz triang. inferior con 1's en la diagonal

3) U matriz triangular superior

tq $PA = LU$ P, L, U vienen del proceso de escalerización

$$L = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ l_{21} & 1 & 0 & & \vdots \\ l_{31} & l_{32} & 1 & & \\ \vdots & & & \ddots & 0 \\ l_{n1} & \cdots & & l_{n(n-1)} & 1 \end{pmatrix} \quad U = \begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ 0 & u_{22} & u_{23} & \cdots & u_{2n} \\ 0 & 0 & u_{33} & \cdots & u_{3n} \\ 0 & & \cdots & 0 & u_{nn} \end{pmatrix}$$

PA hace una permutación de las filas de A.

se forma con los coeficientes que se usan al escalerizar

es la matriz escalerizada

P hace las permutaciones de filas que se realizaron al escalerizar

$$PA = LU$$

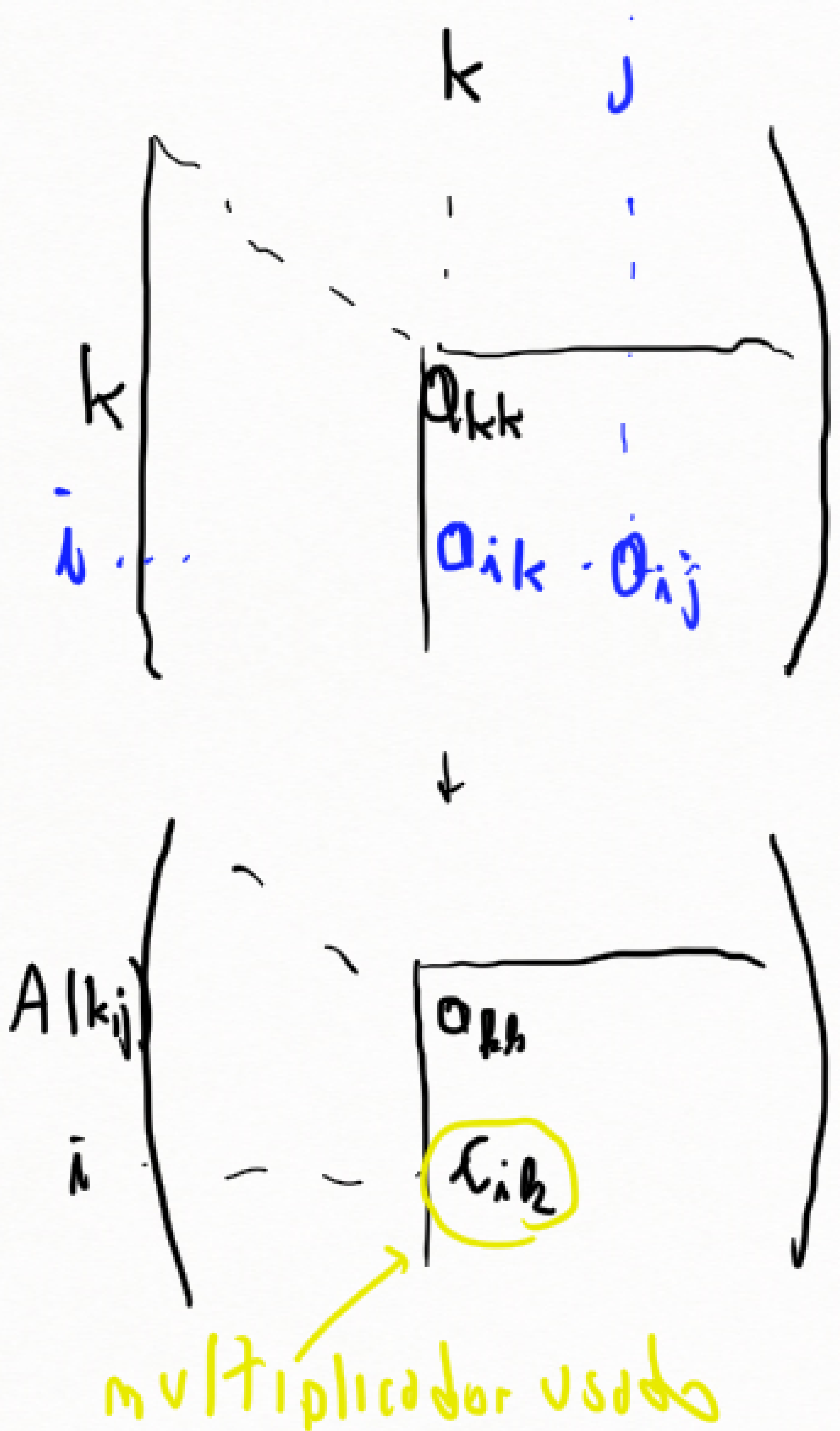
$P \leftarrow$ se representa con un vector p.

$p = [3,1,2]$

```
For k=1:n-1
  For i = k+1:n
    A(i,k) = A(i,k)/A(k,k)
    For j = k+1:n
      A(i,j) = A(i,j) - A(i,k)*A(k,j)
```

coef

$$\left(F_i - \frac{a_{ik}}{a_{kk}} F_k\right)$$



Resultado

escalerizada

$$A = \begin{pmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ l_{21} & u_{22} & \cdots & u_{2n} \\ \vdots & & \ddots & \\ l_{n1} & l_{n2} & \cdots & u_{nn} \end{pmatrix}$$

coeficientes

eye(n,n)

$$L = \begin{pmatrix} 1 & & & \\ l_{21} & 1 & & \\ \vdots & & \ddots & \\ l_{n1} & \cdots & l_{n(n-1)} & 1 \end{pmatrix} \qquad U = \begin{pmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ 0 & u_{22} & \cdots & u_{2n} \\ \vdots & & \ddots & \\ 0 & \cdots & & u_{nn} \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \to \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22}' & a_{23}' \\ 0 & a_{32}' & a_{33}' \end{pmatrix} \to \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22}' & a_{23}' \\ 0 & 0 & a_{33}'' \end{pmatrix}$$

- $l_{21} = \dfrac{a_{21}}{a_{11}}$ $\quad a_{22}' = a_{22} - l_{21}a_{12}$ $\quad a_{23}' = a_{23} - l_{21}a_{13}$
- $l_{31} = \dfrac{a_{31}}{a_{11}}$ $\quad a_{32}' = a_{32} - l_{31}a_{12}$ $\quad a_{33}' = a_{33} - l_{31}a_{13}$
- $l_{32} = \dfrac{a_{32}'}{a_{22}'}$ $\quad a_{33}'' = a_{33}' - l_{32}a_{23}'$

① $l_{21}a_{11} = \dfrac{a_{21}}{a_{11}} a_{11} = a_{21}$ ✓

④ $l_{31}a_{11} = \dfrac{a_{31}}{a_{11}} \cdot a_{11} = a_{31}$ ✓

⑥ Piénsalo

② $l_{21}a_{12} + a_{22}' = \cancel{l_{21}a_{12}} + a_{22} - \cancel{l_{21}a_{12}} = a_{22}$ ✓

③ $l_{21}a_{13} + a_{23}' = l_{21}a_{13} + a_{23} - l_{21}a_{13} = a_{23}$ ✓

⑤ $l_{31}a_{12} + l_{32}a_{22}' = l_{31}a_{12} + a_{32}' = a_{32}$

$$U = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22}' & a_{23}' \\ 0 & 0 & a_{33}'' \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}$$

$$LU = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22}' & a_{23}' \\ 0 & 0 & a_{33}'' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ {}^{1}\,l_{21}a_{11} & {}^{2}\,l_{21}a_{12} + a_{22}' & {}^{3}\,l_{21}a_{13} + a_{23}' \\ {}^{4}\,l_{31}a_{11} & {}^{5}\,l_{31}a_{12} + l_{32}a_{22}' & {}^{6}\,l_{31}a_{13} + l_{32}a_{23}' + a_{33}'' \end{pmatrix}$$

$$\parallel$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$PA = LU$ y queremos resolver $Ax = b$

$$Ax = b \iff PAx = Pb \iff L\underbrace{Ux}_{y} = Pb \text{ vector}$$

$$\begin{cases} Ly = Pb \\ Ux = y \end{cases}$$ Son dos sistemas triangulares.

$Ly = Pb$ ← sustitución hacia adelante $O(n^2)$

$Ux = y$ ← sustitución hacia atrás. $O(n^2)$

$$\begin{pmatrix} 1 & 0 & \cdots & 0 \\ & 1 & & \vdots \\ & & \ddots & 0 \\ & & & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} b_1' \\ b_2' \\ \vdots \\ b_n' \end{pmatrix}$$

$$\begin{pmatrix} u_{11} & \cdots & u_{1n} \\ 0 & & \\ \vdots & & \\ 0 & \cdots\ 0 & u_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

Si tenemos que resolver $Ax_1 = b_1$, $Ax_2 = b_2$, ... $Ax_k = b_k$ varios sistemas todos con la misma matriz.

Hallamos la desc LU una vez $O(n^3)$

Después usamos el método de arriba k veces

k veces $O(n^2)$

$\boxed{A \text{ simétrica}}$ $A^T = A$

definida positiva
$$\begin{cases} 1) \; x^t A x = \langle Ax, x\rangle > 0 \quad \forall x \neq 0 \\ 2) \text{ todos los vaps son } > 0 \\ 3) \; \exists R \text{ triang superior} \quad A = \underbrace{R^T R}_{\text{desc Cholesky}} \end{cases}$$

$$R = \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ 0 & r_{22} & \cdots & r_{2n} \\ \vdots & & \ddots & \\ 0 & \cdots & 0 & r_{nn} \end{pmatrix} \qquad A = \begin{pmatrix} r_{11} & 0 & \cdots & 0 \\ r_{12} & r_{22} & & \vdots \\ \vdots & & & 0 \\ r_{1n} & & & r_{nn} \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ 0 & r_{22} & \cdots & r_{2n} \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & & r_{nn} \end{pmatrix}$$

triang inferior   triang superior

Se usa igual que la desc LU, pero se halla más rápido.

$$Ax = b \iff R^T \overbrace{Rx}^{y} = b$$

$$\begin{cases} R^T y = b \\ Rx = y \end{cases} \text{ 2 sist triangulares}$$

- Solo existe para matrices simétricas y definidas positivas.

$$
R^TR = \begin{array}{c} \\ \\ i \\ \\ \end{array}\begin{pmatrix} r_{11} & 0 & \cdots & 0 \\ r_{12} & r_{22} & \cdots & 0 \\ \vdots & & & \\ \cdots & \square & \cdots & \\ r_{1n} & & & r_{nn} \end{pmatrix}
\begin{array}{c} \begin{pmatrix} r_{11} & r_{12} & \cdots j & r_{1n} \\ 0 & r_{22} & \cdots & r_{2n} \\ \vdots & & \square & \\ 0 & \cdots & & r_{nn} \end{pmatrix} k \\ \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{12} & a_{22} & \cdots & a_{2n} \\ & & a_{ij} & \\ & & & a_{nn} \end{pmatrix} \end{array}
$$

$A = A^T$

R triang superior

$\Rightarrow k > i \quad r_{ki} = 0$

$$a_{ij} = \sum_{k=1}^{n} r^T_{ik} r_{kj} = \sum_{k=1}^{n} r_{ki} r_{kj}$$

Vamos a usar solo la parte triangular superior de A

$\Rightarrow i \leq j$

el primero que se anula

$$i \leq j \Rightarrow \left[a_{ij} = \sum_{k=1}^{n} r_{ki} r_{kj} = \sum_{k=1}^{i} r_{ki} r_{kj} \right]$$

Vamos a ir determinando los r_{ij} por filas. $i=1, i=2, \ldots$

Fila 1: $a_{11} = r_{11}^2 \Rightarrow \boxed{r_{11} = \sqrt{a_{11}}}$

$a_{12} = r_{11} r_{12} \Rightarrow \boxed{r_{12} = a_{12}/r_{11}}$

$a_{1j} = r_{11} r_{1j} \Rightarrow \boxed{r_{1j} = a_{1j}/r_{11}}$

✓ Fila 1 de R

Fila 2: $a_{22} = r_{12}^2 + r_{22}^2 \Rightarrow \boxed{r_{22} = \sqrt{a_{22} - r_{12}^2}}$

Fila anterior

$a_{2j} = r_{12} r_{1j} + r_{22} r_{2j} \Rightarrow \boxed{r_{2j} = (a_{2j} - r_{12} r_{1j})/r_{22}}$

✓ Fila 2 de R

Fila i:

$$a_{ii} = \underbrace{r_{1i}^2 + r_{2i}^2 + \dots + r_{(i-1)i}^2}_{\text{Filas anteriores}} + r_{ii}^2 \Rightarrow r_{ii} = \sqrt{a_{ii} - r_{1i}^2 - r_{2i}^2 - \dots}$$

$$a_{ij} = \underbrace{r_{1i}r_{1j} + r_{2i}r_{2j} + \dots}_{\text{Filas anteriores}} + \underbrace{r_{ii}}_{\text{diagonal}} r_{ij} \Rightarrow r_{ij} = \frac{a_{ij} - r_{1i}r_{1j} - \dots - r_{(i-1)i}r_{(i-1)j}}{r_{ii}}$$