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>> @(x) sin(5*x)
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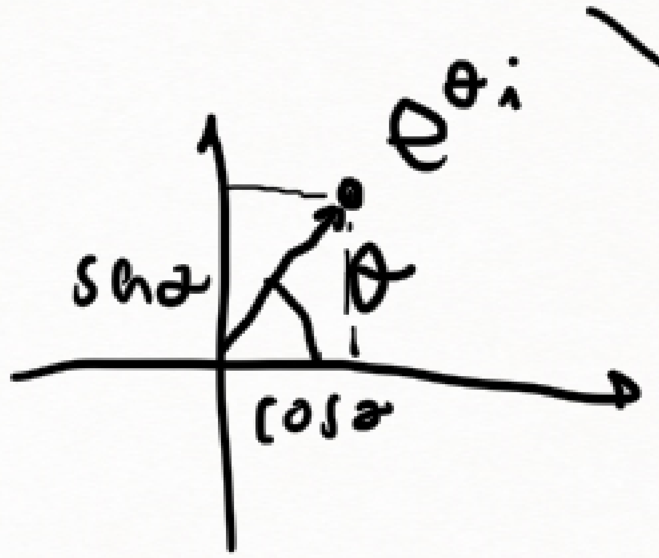
$\sim \quad x \longmapsto \operatorname{sen}(5x)$

$$\frac{\cos(x+h)-\cos(x)}{h}$$



$$e^{(a+b)i} = \cos(a+b) + i\,\text{sen}(a+b)$$

$$\shortparallel$$

$$e^{ai}e^{bi} = (\cos(a)+i\,\text{sen}(a))(\cos(b)+i\,\text{sen}(b))$$

$$= \cos(a)\cos(b) + i\cos(a)\,\text{sen}(b) + i\,\text{sen}(a)\cos(b) + \overbrace{i^2}^{-1}\text{sen}(a)\,\text{sen}(b)$$

$$= \underbrace{\cos(a)\cos(b)-\text{sen}(a)\,\text{sen}(b)}_{\cos(a+b)} + i\,(\underbrace{\cos(a)\,\text{sen}(b)+\text{sen}(a)\cos(b)}_{\text{sen}(a+b)})$$

✓

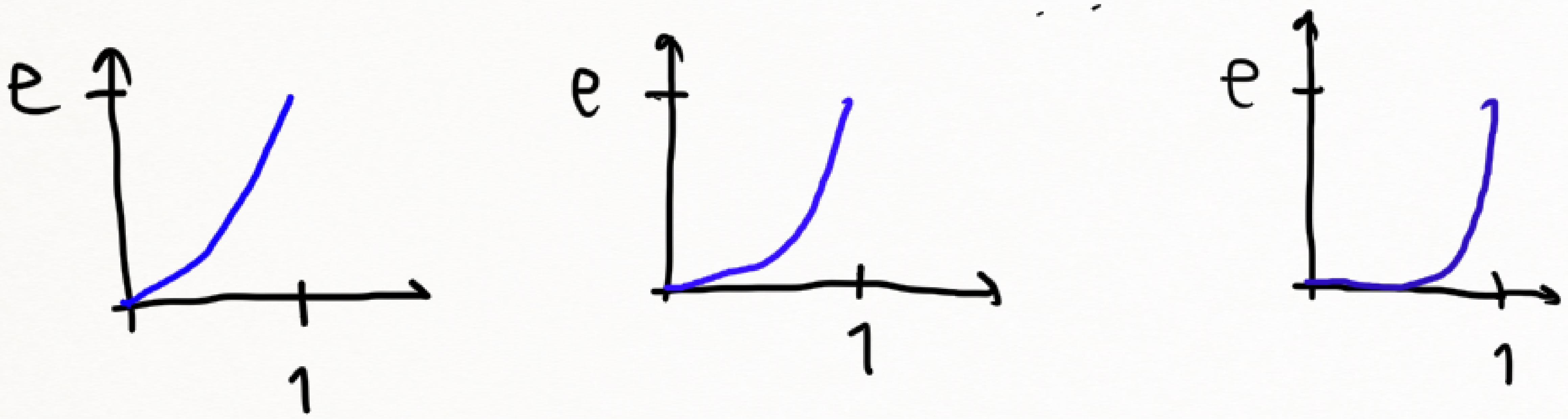
$$\frac{\cos(x+h)-\cos(x)}{h} = \frac{\cos(x)\cos(h)-\text{sen}(x)\,\text{sen}(h)-\cos(x)}{h} = \cos(x)\frac{(\cos(h)-1)}{h} - \text{sen}(x)\frac{\text{sen}(h)}{h}$$

$$\frac{\cos(h)-1}{h} = \frac{\cos^2(h)-1}{h(\cos(h)+1)} = \frac{-\text{sen}^2(h)}{h(\cos(h)+1)}$$ ✓

9) $P_n := \int_0^1 x^n e^x \, dx$ $\qquad P_1 = 1$

xe^x, $n = 1$ $\qquad x^2e^x$, $n = 2$ $\qquad \dots$

$P_1 > P_2 > P_3 > \dots > P_n$



$\lim_{n\to\infty} P_n = 0$

a) $P_n = e - nP_{n-1}$

$$P_n = \int_0^1 \underbrace{x^n}_{f}\underbrace{e^x}_{g'}dx = \underbrace{x^n}_{f}\underbrace{e^x}_{g}\Big|_0^1 - \int_0^1 \underbrace{nx^{n-1}}_{f'}\underbrace{e^x}_{g}dx = e - n\overbrace{\int_0^1 x^{n-1}e^x dx}^{P_{n-1}} = e - nP_{n-1}$$

$P_0 = \int_0^1 e^x dx = e - 1$

$P_1 = e - 1 \cdot P_0 = e - (e-1) = 1 \Rightarrow P_1 = 1$

$\Rightarrow \boxed{P_n = e - nP_{n-1}}$

$$P_n = e - nP_{n-1}$$

$$P_n \xrightarrow{n\to\infty} 0 \Rightarrow e - nP_{n-1} \to 0 \Rightarrow nP_{n-1} \to e \qquad P_{n-1} \sim \frac{e}{n}$$

$$^{*}\ \frac{(-1)^0}{0!} + \frac{(-1)^1}{1!} = 0$$

$$|\varepsilon_{P_n}| \le \frac{nP_{n-1}}{e - nP_{n-1}} |\varepsilon_{P_{n-1}}| = \frac{nP_{n-1}}{P_n}|\varepsilon_{P_{n-1}}|$$

$$\forall n \ge 1 \quad P_n \le 1$$

$$\sum_{i=0}^{\infty} \frac{x^i}{i!} = e^x$$

$n=20 \quad z_{20}$

c) Fijamos P_{20} y resolvemos para atrás. $P_n = e - nP_{n-1} \Rightarrow \boxed{P_{n-1} = \frac{e - P_n}{n}}$

$$P_{n-1} = \frac{e}{n} - \frac{P_n}{n}$$

$$P_{n-2} = \frac{e}{n-1} - \frac{P_{n-1}}{n-1} = \frac{e}{n-1} - \frac{e}{(n-1)n} + \frac{P_n}{(n-1)n}$$

$$P_{n-3} = \frac{e}{n-2} - \frac{e}{(n-2)(n-1)} + \frac{e}{(n-2)(n-1)n} - \frac{P_n}{(n-2)(n-1)n}$$

$\vdots$

$$P_1 = \dots\dots \frac{P_n}{n!}$$

← si $n!$ es mucho más grande que P_n, no afecta al resultado

$$\frac{e}{2} - \frac{e}{3!} + \frac{e}{4!} + \dots = \sum_{i=2}^{\infty} e\frac{(-1)^i}{i!} = e\sum_{i=0^{*}}^{\infty} \frac{(-1)^i}{i!} = e\cdot e^{-1} = 1$$

$$P_1 = \frac{e}{2!} - \frac{P_2}{2!} = \frac{e}{2!} - \frac{1}{2!}\left(\frac{e}{3} - \frac{P_3}{3}\right) = \frac{e}{2!} - \frac{e}{2! \times 3} + \frac{P_3}{2! \times 3} = \frac{e}{2!} - \frac{e}{3!} + \frac{P_3}{3!} = \cdots$$

$p^* \in \mathbb{R}$ número que quermos estimar.

$P_0(h)$ estimador de p^* : $P_0(h) \xrightarrow{h \to 0} p^*$

Ejemplos: $p^* = f'(a)$ $P_0(h) = \Delta_{f(a),h}$ o $P_0(h) = \delta_a(h)$ (Ej 6) son estimadores

$$p^* = P_0(h) + a_0 h^{k_0} + a_1 h^{k_1} + \cdots \qquad 1 \leq k_0 < k_1 \cdots$$

lo conocemos k_0 el ordn del error. término dominante del error

$$\left(P_0(h) - p^* = \underbrace{-a_0 h^{k_0} - a_1 h^{k_1} - \cdots}_{\text{error}} \right)$$

Con la Extrapolación de Richardson construimos un estimador nuevo $(P_1(h))$ que tine error de mayor ordn (k_1)

$$(1)\quad p^* = P_0(h) + \cancel{a_0 h^{k_0}} + a_1 h^{k_1} + \dots$$

Fijamos $t>0$ (x ejemplo $t=2$) y usamos la igualdad con h/t
$t \neq 1$

$$(2)\quad p^* = P_0\left(\tfrac{h}{t}\right) + \frac{a_0}{t^{k_0}} h^{k_0} + \frac{a_1}{t^{k_1}} h^{k_1} + \dots$$

$$(2)\times t^{k_0} : \quad t^{k_0} p^* = t^{k_0} P_0\left(\tfrac{h}{t}\right) + \cancel{a_0 h^{k_0}} + t^{k_0-k_1} a_1 h^{k_1} + \dots$$

$$(2)\times t^{k_0} - (1) : \quad (t^{k_0}-1) p^* = t^{k_0} P_0\left(\tfrac{h}{t}\right) - P_0(h) + \overbrace{(t^{k_0-k_1}-1)}^{\neq 0} a_1 h^{k_1} + \dots$$

$\div\, t^{k_0}-1$:

$$p^* = \underbrace{\frac{t^{k_0} P_0\left(\frac{h}{t}\right) - P_0(h)}{t^{k_0}-1}}_{P_1(h)} + b_1 h^{k_1} + \dots$$

$$P_1(h) := \frac{\overbrace{t^{k_0} P_0\left(\frac{h}{t}\right) - P_0(h)}^{\text{resto de números cercanos}}}{t^{k_0}-1} \quad \text{orden } k_1 > k_0$$