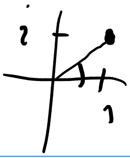
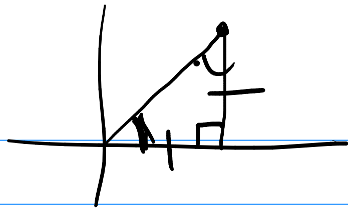
$z = i+1$





$z^{100} = (i+1)^{100} =$

$|z| = \sqrt{2}$

$\arg z = \pi/4$

$i+1 = \sqrt{2}\, e^{i\pi/4}$

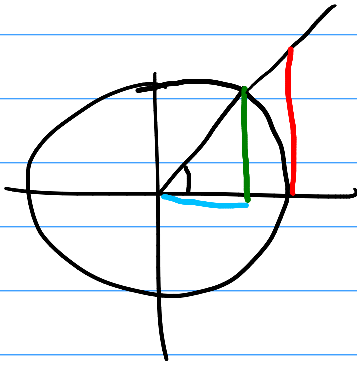
$(i+1)^{100} = \left(\sqrt{2}\, e^{i\pi/4}\right)^{100} = (\sqrt{2})^{100}\, e^{\frac{100}{4} i\pi}$

$= 2^{50} e^{25 i\pi} = 2^{50} e^{i\pi}$

$= -2^{50}$

$e^{i\pi} = -1$

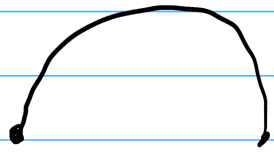
$e^{i\pi} + 1 = 0$



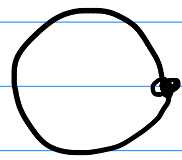
$tg = \dfrac{sen}{cos}$

$\arg(a+ib) = \operatorname{arctg}\left(\dfrac{b}{a}\right)$

$e^{i\pi}$



$e^{2\pi i}$



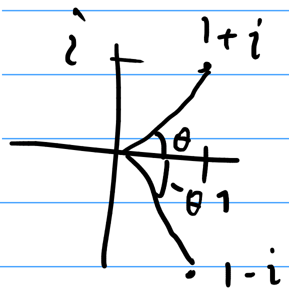
5. Representar geométricamente los complejos:

a) $(1+i)^n - (1-i)^n$ para algunos valores naturales n.

→ b) Las raíces quintas de 1 (es decir, los complejos z tales que $z^5 = 1$).

→ c) Las raíces décimas de 1.

d) Los complejos z tales que $z^6 = 8(\sqrt{3} - i)$.

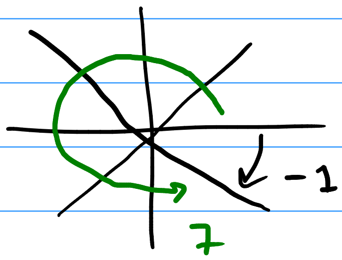


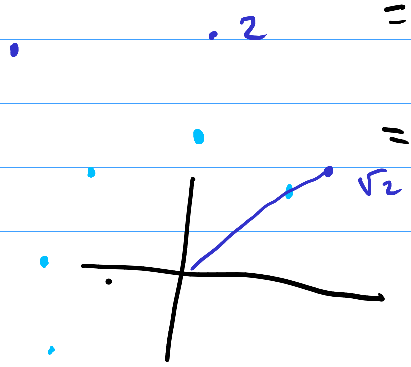
$(1+i)^n - (1-i)^n = \sqrt{2}^n \left(e^{i\frac{\pi}{4}n} - e^{-i\frac{\pi}{4}n}\right)$

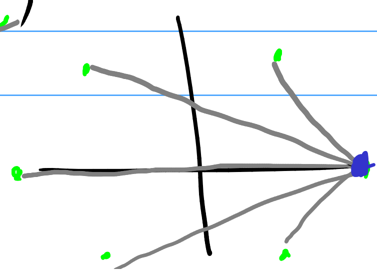
$= \sqrt{2}^n \left(e^{i\frac{\pi}{4}n} - e^{i\frac{7}{4}\pi n}\right)$

$= \sqrt{2}^n \left(e^{i\frac{\pi}{4}n} - e^{i\frac{\pi}{4}n + i\frac{6}{4}\pi n}\right)$

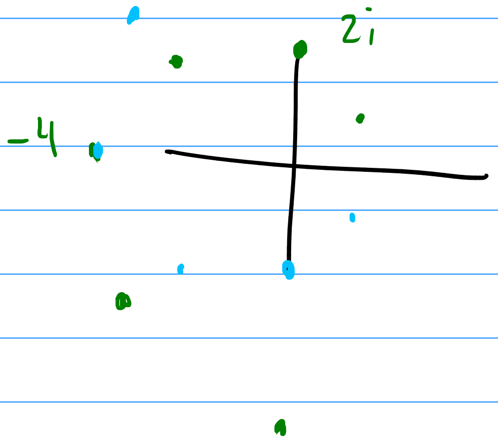
$= \sqrt{2}^n e^{i\frac{\pi}{4}n}\left(1 - e^{i\frac{3}{2}\pi n}\right)$







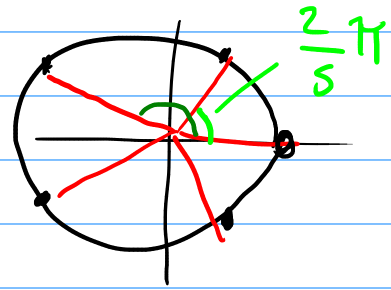
$(1+i)^n - (1-i)^n$



$x^5 = 1$, $x^5 - 1 = 0$

Si $x^5 = 1$ cuanto vale su modulo?
$|x| = 1$
$\arg(x) = \theta$

$5\theta = 2\pi K$
$\theta = \frac{2\pi K}{5}$



$z_1 = e^{i\frac{2}{5}\pi}$ es una raiz de $x^5 - 1$

$K=2$

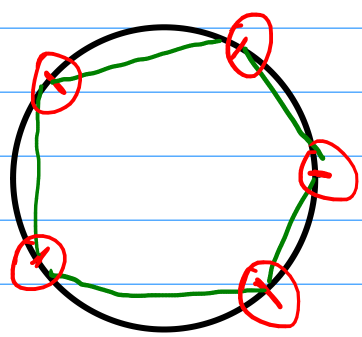
$z_2 = e^{i\frac{4}{5}\pi}$ $\left(e^{i\frac{4}{5}\pi}\right)^5 = e^{i\frac{4}{\cancel{5}}\pi\cancel{5}} = e^{i4\pi} = e^0 = 1$

Queriamos encontrar θ

$z_i = e^{i\theta\pi}$, sabemos $(z_i)^5 = 1$

$(e^{i\theta})^5 = e^{i\theta 5} = e^0 = 1$

$\Rightarrow 5\theta = 0 + 2\pi \cdot K$



$x^5 - 1$

$z_j = e^{i\frac{2}{5}\pi j}$ $j = 0, 1, \ldots 4$

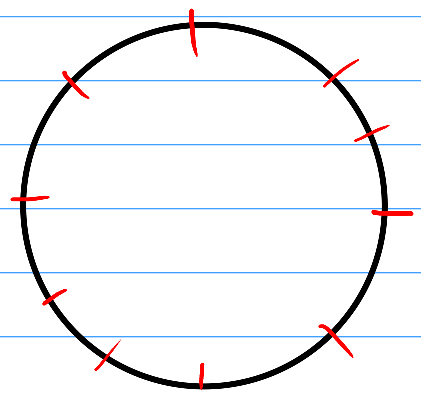
$(z_j)^{10} = ((z_j)^5)^2 = (1)^2 = 1$

Hallar las raices $\nearrow$ $x^{10} - 1$

z_j raiz de $x^{10}-1$, $|z_j^{10}|=1 \Rightarrow |z_j|=1$

$\rightarrow z_j = e^{i\theta\pi}$ y cumple $e^{i\theta\,10} = e^0$

$\rightarrow 10\theta = 2\pi K \Rightarrow \theta = \frac{\pi K}{5}$



5. Representar geométricamente los complejos:

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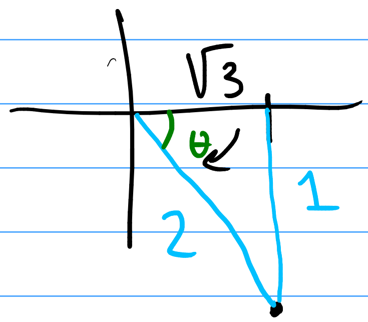
c) Las raíces décimas de 1.

d) Los complejos z tales que $z^6 = 8(\sqrt{3}-i)$.

$z^6 - 8(\sqrt{3}-i) = 0$

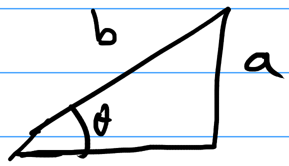
$\arg(\sqrt{3}-i) = -\pi/6$

$|\sqrt{3}-i| = 2$



$\sin\theta = -\frac{1}{2}$

$\theta = -\pi/6$



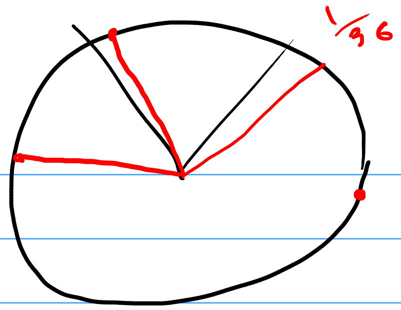
$\sin\theta = \frac{a}{b}$

Paso intermedio

$z^6 - e^{-i\pi/6} = 0$

$z_j = e^{i\theta}$ $\quad e^{i\theta 6} = e^{-i\pi/6} \rightarrow 6\theta = \frac{-\pi}{6} + 2\pi K$

$6\theta = \pi\left(\frac{-1+12K}{6}\right)$



$\theta = \frac{\pi(12k-1)}{36} =$

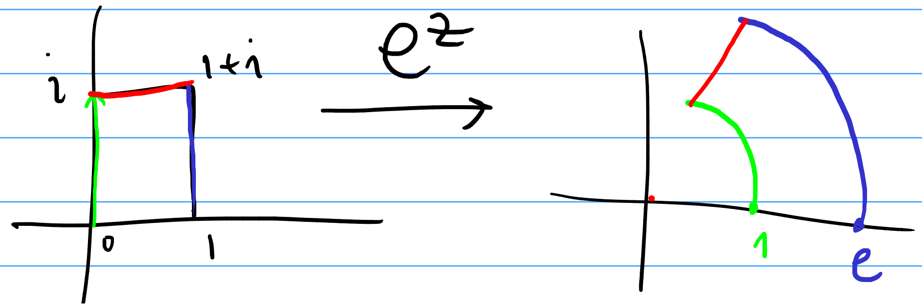
$z_j = e^{\frac{\pi}{36}(12j-1)}$ $\quad j = 0, 1, \dots 5$

son raíces de $z^6 - e^{-i\pi/6}$

$z^6 - 16e^{-i\pi/6}$ $\quad x_i^6 = 16$

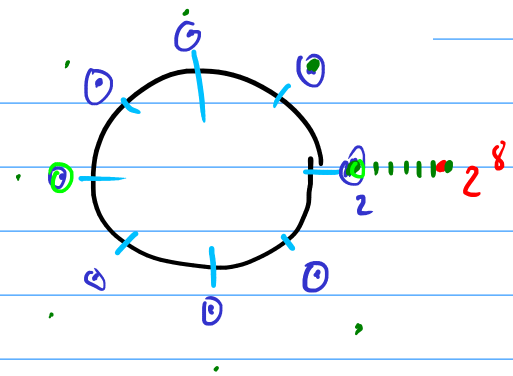
$\sqrt[6]{16} = \sqrt[3]{\sqrt{16}} = \sqrt[3]{4} =$

$\sqrt[3]{4}\, z_j$ son las raíces de $z^6 - 16e^{-i\pi/6}$



8. En $\mathbb{C}$, se consideran $\{z_1, \cdots, z_8\}$ las raíces octavas de 2^8, es decir aquellas que cumplen $z_k^8 = 2^8$ para cada $k = 1, \cdots, 8$. Determinar cuáles de las siguientes afirmaciones son verdaderas y cuáles son falsas:

   a) $z_i = 2$ para todo $i = 1, \cdots, 8$. Falso
   b) Existen al menos dos raíces z_j, z_k tales que $z_j = -z_k$. ✓
   c) Existen al menos dos raíces z_l, z_m tales que $\bar{z}_l = z_m$. ✓
   d) Se cumple $z_1 z_2 z_3 z_4 z_5 z_6 z_7 z_8 = 2^8$.



$x^8 - 1$

podemos hallar raíces

$z_j : (z_j)^8 = 1$

$(2z_j)^8 = 2^8$

$Z_j = 2z_j$

Si sabemos que $z_1, z_2, \dots, z_8$ son raíces

$z^8 - 2^8 = (z-z_1)(z-z_2)\cdots(z-z_8)$

coeficiente constante

$z^8 - 2^8 = \quad z_1 z_2 z_3 \cdots z_8$

11. Considere el polinomio $P(z) = \underbrace{z^4 - 2z^3 + 6z^2 - 8z + 8}$. Sabiendo que $P(z)$ tiene una raíz imaginaria pura halle todas sus raíces.

10, 11, 12 ← Hacer estos ej.

$P(z)$ tiene ix $x \in \mathbb{R}$ $p(ix)=0$

$P \in \mathbb{R}[x]$ tenemos que $p(-ix)=0$

$\underbrace{(z-ix)(z+ix)}$ divide al polinomio p

$$(z^2+x^2)(a+bz+cz^2) = z^4 - 2z^3 + 6z^2 - 8z + 8$$

$c=1$

$b=-2$