1   2   4   6   exp(z)

1. Determinar los valores de i^k para todo k entero.

2. Expresar los siguientes números complejos en forma binómica ($a+bi$ con a,b reales) y en notación polar ($re^{i\theta}$ con $r>0$ y θ real).

a) $(1+i)^2$ *b)* $\frac{1}{i}$ *c)* $\frac{1}{1+i}$ *d)* $(2+3i)(3-4i)$ *e)* $(1+i)(1-2i)$ *f)* i^5+i^{16}

g) -1 *h)* $-3i$ *i)* $1+i+i^2+i^3$ *j)* $\frac{1}{2}(1+i)(1-i^{-8})$ *k)* $\frac{1+i}{\sqrt{2}}$ *l)* $\frac{1}{(1+i)^2}$

z es numero complejo $z = a+ib \sim (a,b),\ a,b \in \mathbb{R}$

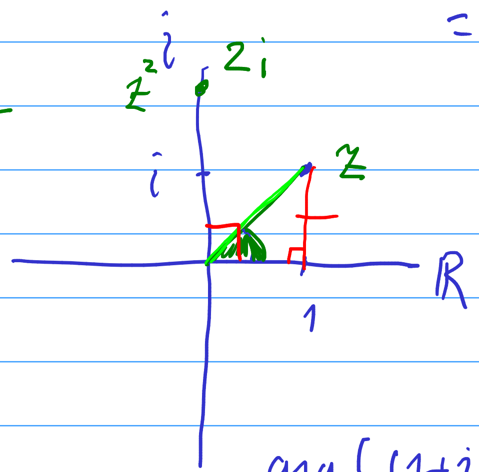
$(i)^2 = -1$

$z = a+ib \quad z' = c+id$

$zz' = (a+ib)(c+id) = ac + a(id) + (ib)c + (ib)(id)$
$= ac + i(ad) + i(bc) + (i)^2(bd)$
$= ac - bd + i(ad+bc)$

a) $(1+i)^2 = (1+i)(1+i) = 1 + 1i + 1i + i^2$
$= 1 + 2i - 1 = 2i$

1+i



$(1,1)$

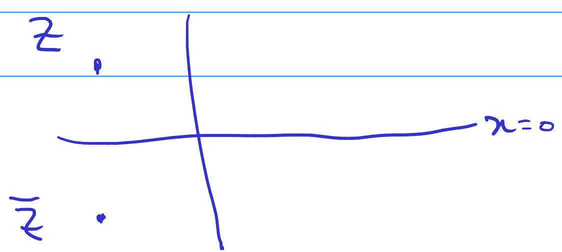
$\arg(1+i) = \pi/4$
$|z| = \sqrt{(1,1)} = \sqrt{2}$

$\arg((1+i)^2) = \arg(2i) = \pi/2$
$|(1+i)^2| = |2i| = 2$

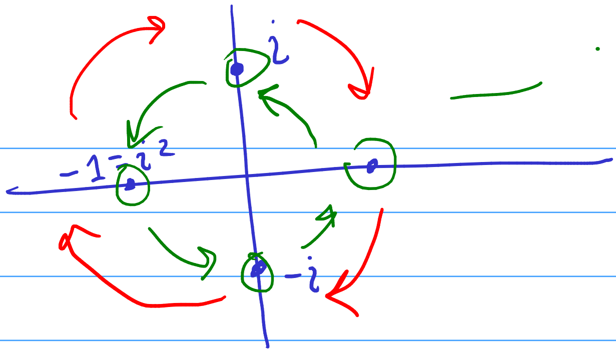
b) $\frac{1}{i} = \frac{1}{i}\cdot 1 = \frac{1}{i}\left(\frac{-i}{-i}\right) = \frac{-i}{-(i)^2} = \frac{-i}{-(-1)} = \frac{-i}{1} = -i$

$z = a+ib \quad \bar{z} = a-ib, \quad z\bar{z} = (a+ib)(a-ib)$
$= a^2+b^2 = |z|^2$

$z\bar{z} \in \mathbb{R}$



Calcular i^k $\forall k \in \mathbb{N}$



$i = 1$

$i^2 = -1$

$i^3 = i^2 \cdot i = (-1)i = -i$

$i^4 = i^3 \cdot i = -i \cdot i = -(i^2) = -(-1) = 1$

$i^5 = i^4 \cdot i = i$

$1 = (i^4)^4$

$i^{17} = i^{16} i = i$

$i^4 = 1$

i^{2k}

$\frac{1}{i} = i^{-1} = -i$

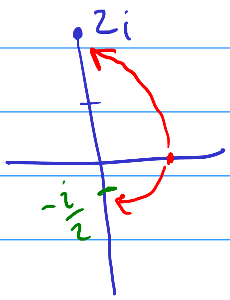
1. Determinar los valores de i^k para todo k entero.

2. Expresar los siguientes números complejos en forma binómica ($a + bi$ con a, b reales) y en notación polar ($re^{i\theta}$ con $r > 0$ y θ real).

   a) $(1+i)^2$ *b*) $\frac{1}{i}$ *c*) $\frac{1}{1+i}$ *d*) $(2+3i)(3-4i)$ *e*) $(1+i)(1-2i)$ *f*) $i^5 + i^{16}$

   g) -1 *h*) $-3i$ *i*) $1+i+i^2+i^3$ *j*) $\frac{1}{2}(1+i)(1-i^{-8})$ *k*) $\frac{1+i}{\sqrt{2}}$ *l*) $\frac{1}{(1+i)^2}$

$$\frac{1}{(1+i)^2} = \frac{1}{2i} = \frac{1}{2i}\left(\frac{-2i}{-2i}\right) = \frac{-2i}{4} = -\frac{1}{2}i$$



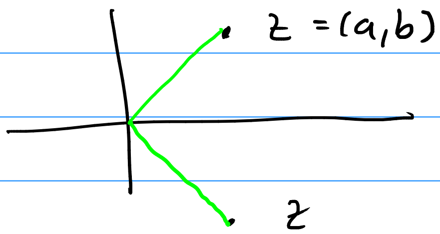
4. Probar que para todo par de números complejos z_1 y z_2

   a) $|z_1| = |\bar{z}_1|$ *b*) $|z_1 z_2| = |z_1||z_2|$ *c*) $|z_1 + z_2| \leq |z_1| + |z_2|$ *d*) si $z_1 \neq 0$ $\left|\frac{1}{z_1}\right| = \frac{1}{|z_1|}$

$z = a + ib$ $\bar{z} = a - ib$

Queremos probar:

$|z| = |\bar{z}|$



$|z| = \sqrt{a^2 + b^2}$

$|\bar{z}| = \sqrt{a^2 + (-b)^2} = \sqrt{a^2 + b^2} = |z|$

$z_1 = a + ib \qquad z_2 = c + id$

$|z_1 z_2| = |z_1||z_2| \leftarrow$ Queremos probarlo

$$|z_1 z_2| = |(a+ib)(c+id)| = |ac - bd + i(ad + bc)|$$

$$= \sqrt{(ac-bd)^2 + (ad+bc)^2}$$

$$= \sqrt{a^2c^2 - \cancel{2acbd} + b^2d^2 + a^2d^2 + \cancel{2acbc} + b^2c^2}$$

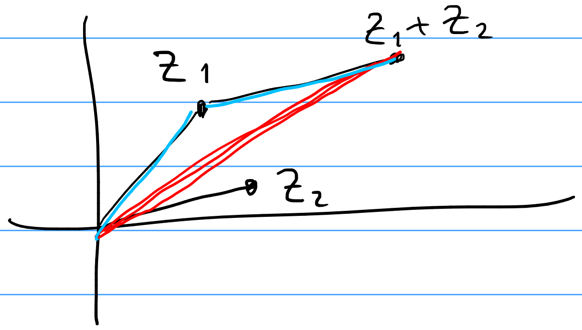
$$= \sqrt{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}$$

$$|z_1||z_2| = \sqrt{a^2+b^2}\sqrt{c^2+d^2} = \sqrt{(a^2+b^2)(c^2+d^2)}$$

$$= \sqrt{a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2}$$

$$|z_1 z_2| = |z_1||z_2|$$

c) $|z_1 + z_2| \leq |z_1| + |z_2|$ desigualdad triangular



d) $\left|\frac{1}{z}\right| = \frac{1}{|z|}$

$$\left|\frac{1}{a+ib}\right| = \left|\frac{1}{a+ib}\left(\frac{a-ib}{a-ib}\right)\right| = \left|\frac{a-ib}{a^2+b^2}\right| = \left|\frac{a}{a^2+b^2} - i\left(\frac{b}{a^2+b^2}\right)\right|$$

$$= \sqrt{\frac{a^2}{(a^2+b^2)^2} + \frac{b^2}{(a^2+b^2)^2}} = \sqrt{\frac{a^2+b^2}{(a^2+b^2)^2}} = \sqrt{\frac{1}{a^2+b^2}}$$

$$= \frac{1}{\sqrt{a^2+b^2}} = \frac{1}{|z|}$$

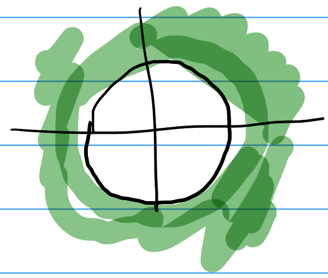
$$z \cdot \frac{1}{z} = 1 \implies |1| = |z|\left|\frac{1}{z}\right|$$

$$\implies \frac{1}{|z|} = \left|\frac{1}{z}\right|$$

6. Encontrar, en cada caso, el conjunto de los $z \in \mathbb{C}$ que satisfacen las siguientes condiciones, y representar geométricamente.

a) $|z| > 1$ *b)* $z - \bar{z} = i$ *c)* $|z - i| = |z + i|$ *d)* $Im(z) < 2$ *e)* $|z - \bar{z}| = 2\,Re(z-1)$

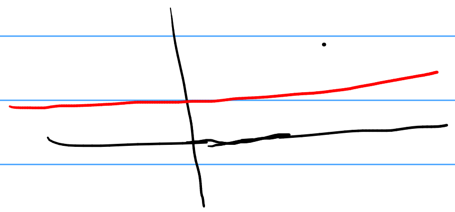
a) $|z| > 1$ $z = a + ib$



$z - \bar{z} = i$

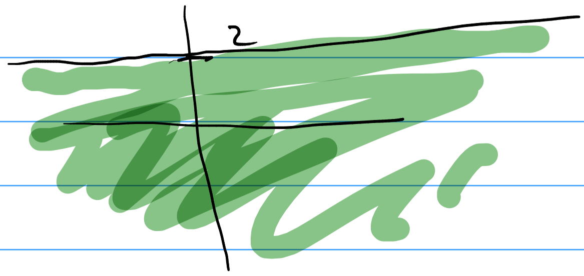
$a + ib - (a - ib) = 2ib = i \rightarrow 2b = 1$

$b = \frac{1}{2}$



$1/2$ $z - \bar{z} = i$

$Im(z) < 2$



$|z - \bar{z}| = 2Re(z-1)$

$|2ib| = 2b = 2Re(z-1)$

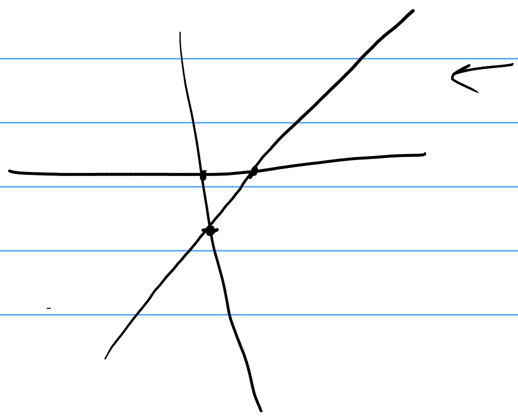
$z - 1 = a + ib - 1$

$= a - 1$

$2b = 2(a-1) \longrightarrow b = a - 1$

$y = x - 1$

$(0, -1)$



$$\exp(z) = 1 + z + \frac{z^2}{2} + \frac{z^3}{3!} + \frac{z^4}{4!} + \frac{z^5}{5!} + \cdots$$

$z \in \mathbb{R}$

$$\exp(iz) = 1 + iz + \frac{i^2 z^2}{2!} + \frac{(iz)^3}{3!} + \frac{(iz)^4}{4!} + \frac{(iz)^5}{5!} + \cdots$$

$$= 1 + iz - \frac{z^2}{2!} - i\frac{z^3}{3!} + \frac{z^4}{4!} + \frac{iz^5}{5!} + \cdots$$

$$\mathrm{Re}(\exp(iz)) = 1 - \frac{z^2}{2} + \frac{z^4}{4!} - \frac{z^6}{6!} \cdots = \cos(z)$$

$$\mathrm{Im}(\exp(iz)) = z - \frac{z^3}{3!} + \frac{z^5}{5!} + \cdots = \mathrm{sen}(z)$$

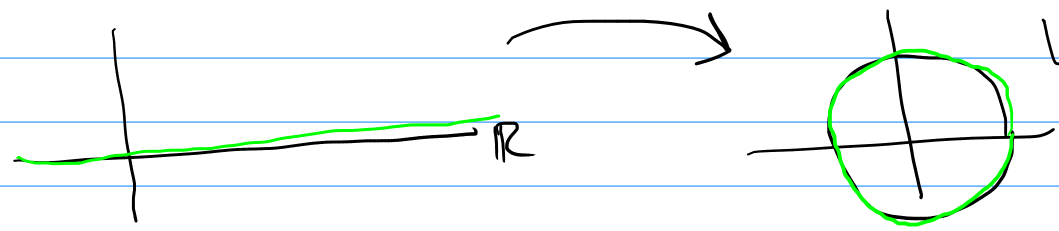
$\theta \in \mathbb{R}$

$$\exp(i\theta) = \cos(\theta) + i\,\mathrm{sen}(\theta)$$

$$|\cos(\theta) + i\,\mathrm{sen}(\theta)| = 1$$

$$\|$$

$$\sqrt{\cos^2\theta + \mathrm{sen}^2\theta} = 1$$



$$e^{a+ib} = e^a(\cos(b) + i\,\mathrm{sen}(b))$$