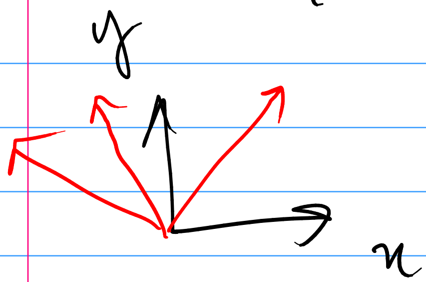
3. Verificar que la función $u(x,t) = e^{-a^2k^2t}\sin(kx)$ satisface la ecuación del calor: $u_t = a^2 u_{xx}$.

$$u(x,t) = e^{-a^2k^2t}\sin(kx)$$

$$\frac{\partial u}{\partial t} = u_t = -a^2k^2 e^{-ak^2t}\sin(kx) = -a^2k^2 u(x,t)$$

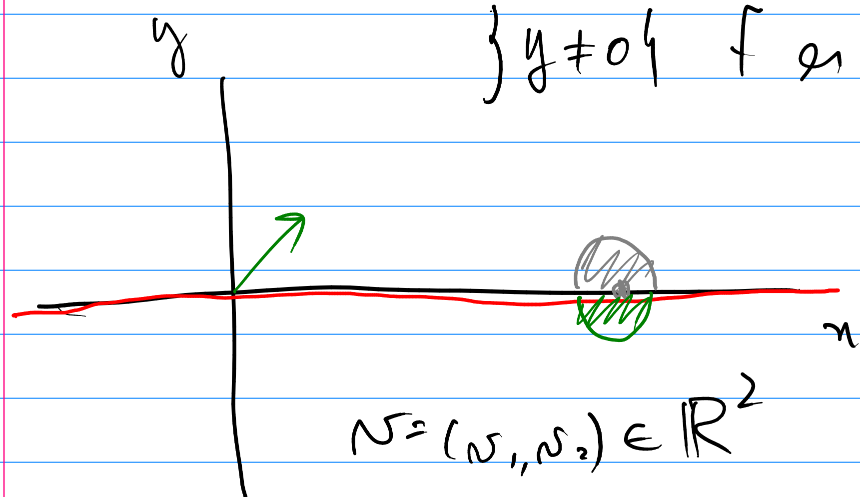
$$\frac{\partial u}{\partial x} = u_x = k e^{-a^2k^2t}\cos(kx)$$

$$u_{xx} = -k^2 e^{-a^2k^2t}\sin(kx) = -k^2 u(x,t)$$



4) c) $f(x,y) = \begin{cases} \dfrac{x^3}{y} & y \neq 0 \\ 0 & y = 0 \end{cases}$

$\{y \neq 0\}$ f es continua : $f = \dfrac{x^3}{y}$



$v = (v_1, v_2) \in \mathbb{R}^2$

$$\frac{\partial f(a)}{\partial v} = \lim_{t \to 0} \frac{f(a+tv) - f(a)}{t}$$

$$\frac{\partial}{\partial v}\frac{x^3}{y} = \lim_{t \to 0} \left(\frac{(x+tv_1)^3}{y - tv_2} - \frac{x^3}{y} \right)\frac{1}{t}$$

$$= \lim_{t \to 0} \frac{y(x+tv_1)^3 - (y+tv_2)x^3}{(y-tv_2)yt}$$

$$= \lim_{t \to 0} \frac{\cancel{y}(\cancel{x^3} + 3x^2\cancel{t}v_1 + 3x(\cancel{t}v_1)^2 + (\cancel{t}v_1)^3) - \cancel{yx^3} - \cancel{t}v_2x^3}{(y-tv_2)yt}$$

$$= \lim_{t \to 0} \frac{y(3x^2v_1 + 3xtv_1^2 + t^2v_1^3) - v_2x^3}{(y-tv_2)y}$$

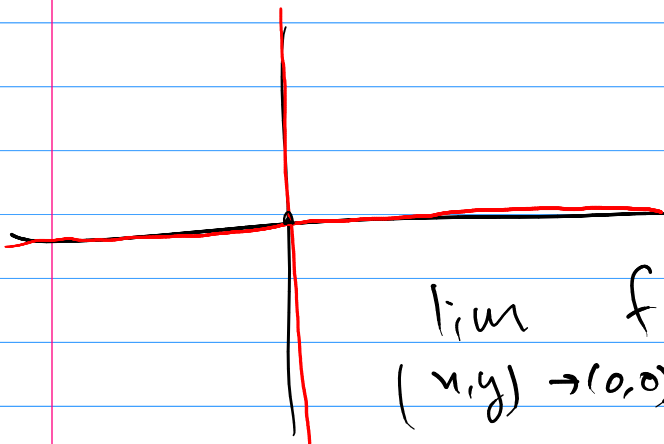
$$= \lim_{t \to 0} \frac{y3x^2v_1 - v_2x^3}{y^2} = \left(\frac{3x^2}{y}\right)v_1 + \left(-\frac{x^3}{y^2}\right)v_2$$

$$\frac{x^3}{y} \qquad f_x = \frac{3x^2}{y}$$

$$f_y = -\frac{x^3}{y^2}$$

$$\boxed{\begin{aligned}\frac{\partial f}{\partial v} &= \langle (f_x, f_y), (v_1, v_2) \rangle \\ &= \left\langle \left(\frac{3x^2}{y}, -\frac{x^3}{y^2}\right), (v_1, v_2) \right\rangle\end{aligned}}$$

4) d) $f(x,y) = \begin{cases} xy\,\text{sen}(1/x)\cos\left(\frac{1}{y}\right) & xy \neq 0 \\ a & xy = 0 \end{cases}$



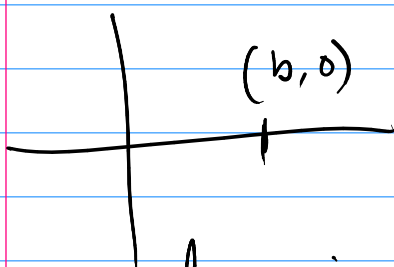
En $\{xy \neq 0\}$ es continua

$\lim_{(x,y)\to(0,0)} f(x,y) = a$ para que f sea continua

$$\lim_{(x,y)\to(0,0)} \underbrace{xy}_{\to 0}\,\underbrace{\text{sen}\left(\frac{1}{x}\right)\cos\left(\frac{1}{y}\right)}_{\in[-1,1]} = 0$$

Para que f sea continua en $(0,0)$ $a=0$

$\{y=0\}$ f es continua?



$$\lim_{(x,y)\to(b,0)} xy\,\text{sen}\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right) = 0$$

lo mismo para $\{x=0\}$

$f(x,y) = \begin{cases} xy\,\text{sen}\left(\frac{1}{x}\right)\cos\left(\frac{1}{y}\right) & xy \neq 0 \\ 0 & xy = 0 \end{cases}$

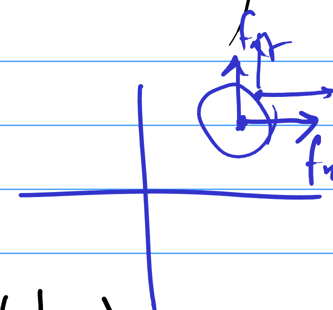
es continua en R^2

(v_1, v_2)

$$\lim_{t\to 0}\frac{f(x+tv_1,\, y+tv_2)-f(x,y)}{t}$$

$$\lim_{t\to 0}\left((x+tv_1)(y+tv_2)\operatorname{sen}\left(\frac{1}{x+tv_1}\right)\cos\left(\frac{1}{y+tv_2}\right) - xy\operatorname{sen}\left(\frac{1}{x}\right)\cos\left(\frac{1}{y}\right)\right)\frac{1}{t}$$

$\{xy\neq 0\}$,



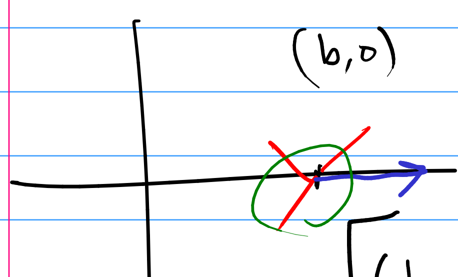
$$f_x = y\operatorname{sen}\left(\tfrac{1}{x}\right)\cos\left(\tfrac{1}{y}\right) - \frac{y}{x}\cos\left(\tfrac{1}{x}\right)\cos\left(\tfrac{1}{y}\right)$$

$$f_y = x\operatorname{sen}\left(\tfrac{1}{x}\right)\cos\left(\tfrac{1}{y}\right) + \frac{x}{y}\operatorname{sen}\left(\tfrac{1}{x}\right)\operatorname{sen}\left(\tfrac{1}{x}\right)$$

f_x, f_y en $\{xy\neq 0\}$ existen y son continuas en un entorno de cualquier entonces f es diferenciable en $\{xy\neq 0\}$ $\Rightarrow$ existen todas las der. direccionales

Estudiar las derivadas direccionales en $\{xy=0\}$

$\{x\neq 0,\, y=0\}$ $\quad \dfrac{\partial f}{\partial v} =$



$$\lim_{t\to 0}\frac{f(b+tv_1,\, tv_2)-f(b,0)}{t}$$

$$\lim_{t\to 0}\left[(b+tv_1)(tv_2)\cos\left(\frac{1}{b+tv_1}\right)\operatorname{sen}\left(\frac{1}{tv_2}\right) - 0\right]\frac{1}{t}$$

$$\lim_{t\to 0} \frac{(b t v_2 + t^2 v_1 v_2)\cos\left(\frac{1}{b+tv_1}\right)\operatorname{sen}\left(\frac{1}{t v_2}\right)}{t}$$

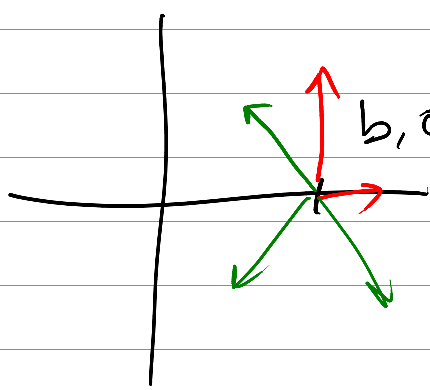
$$\lim_{t\to 0} b v_2 \cos\left(\frac{1}{b+tv_1}\right)\operatorname{sen}\left(\frac{1}{t v_2}\right) + \underbrace{t v_1 v_2 \cos\left(\frac{1}{b+tv_2}\right)\operatorname{sen}\left(\frac{1}{t v_2}\right)}_{\to 0}$$

$v_2 = 0$

Entonces

$$\lim_{t\to 0} b v_2 \cos\left(\frac{1}{b+tv_1}\right)\operatorname{sen}\left(\frac{1}{t v_2}\right) = 0$$

En el conjunto $\{(x,y) : x \neq 0, y = 0\}$ solo existen las direccionales por $(v_1, 0)$

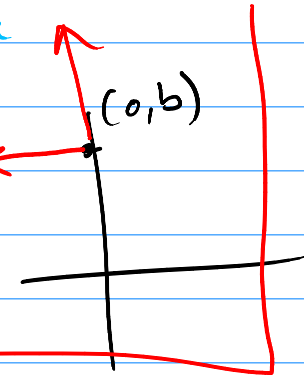


$\exists f_y$

$$\lim_{t\to 0} \frac{f(b+t,0) - f(b,0)}{t} = 0$$

$\exists f_x$

$\{(x,y) \;\; x = 0, y \neq 0\}$



Ejercicio!! calcular $\frac{\partial f}{\partial v}$

En el punto $(0,0)$ tomo $v=(v_1,v_2)$

$$\frac{\partial f}{\partial v}(0,0) =$$

$$\lim_{t\to 0}\left[(tv_1)(tv_2)\cos\left(\frac{1}{tv_1}\right)\operatorname{sen}\left(\frac{1}{tv_2}\right) - 0\right]\frac{1}{t}$$

$$= \lim_{t\to 0} t v_1 v_2 \cos\left(\frac{1}{tv_1}\right)\operatorname{sen}\left(\frac{1}{tv_2}\right) = 0$$

$$\exists \frac{\partial f}{\partial v}(0,0) \quad \forall v \in \mathbb{R}^2 \setminus \{0,0\}$$