14. Sea $f : (0,+\infty)\times[0,2\pi) \to \mathbb{R}^2 \setminus \{(0,0)\}$ definida por $f(\rho,\theta) = (\rho\cos(\theta), \rho\sin(\theta))$

a) Verificar que es continua y probar que es biyectiva.

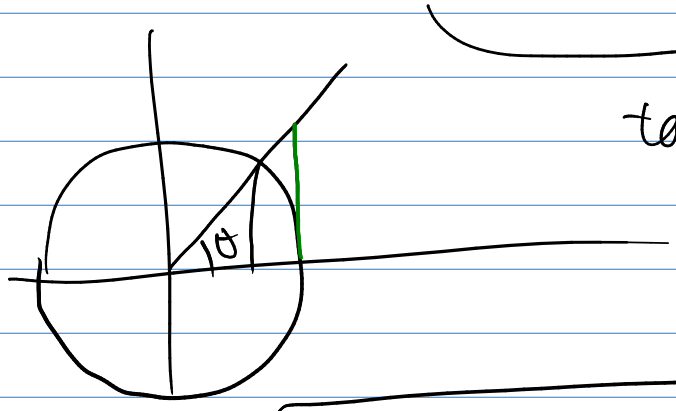
b) Calcular las imágenes de las rectas $\rho = cte$ y $\theta = cte$

c) Calcular la función inversa $f^{-1} : \mathbb{R}^2 \setminus \{(0,0)\} \to (0,+\infty)\times[0,2\pi)$.¿ Es continua f^{-1}?

$(\rho,\theta) \longrightarrow (\rho\cos\theta, \rho\operatorname{sen}\theta)$

Recordar $\underbrace{\operatorname{tang}(\theta) = \dfrac{\operatorname{sen}\theta}{\cos\theta}}$

$\operatorname{tang}^{-1} = \operatorname{arctg}$



$\theta = \operatorname{arctg}\left(\dfrac{\rho\operatorname{sen}\theta}{\rho\cos\theta}\right)$

$\rho = \sqrt{(\rho\cos\theta)^2 + (\rho\operatorname{sen}\theta)^2}$

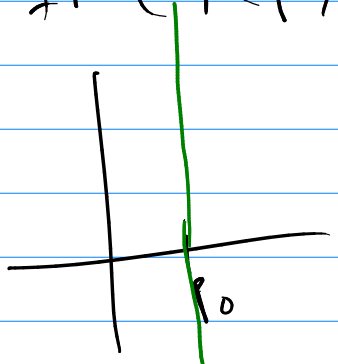
$(x,y) = (\rho\cos\theta, \rho\operatorname{sen}\theta)$

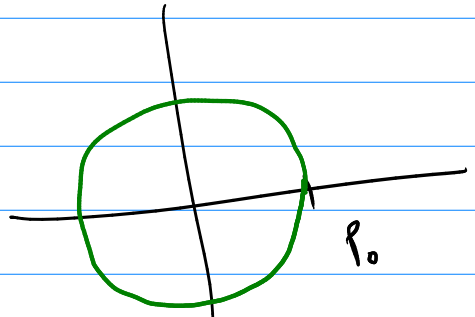
$\underbrace{(\rho\cos\theta, \rho\operatorname{sen}\theta)} \to (\rho,\theta)$

$\rho = \sqrt{(x)^2 + y^2}$

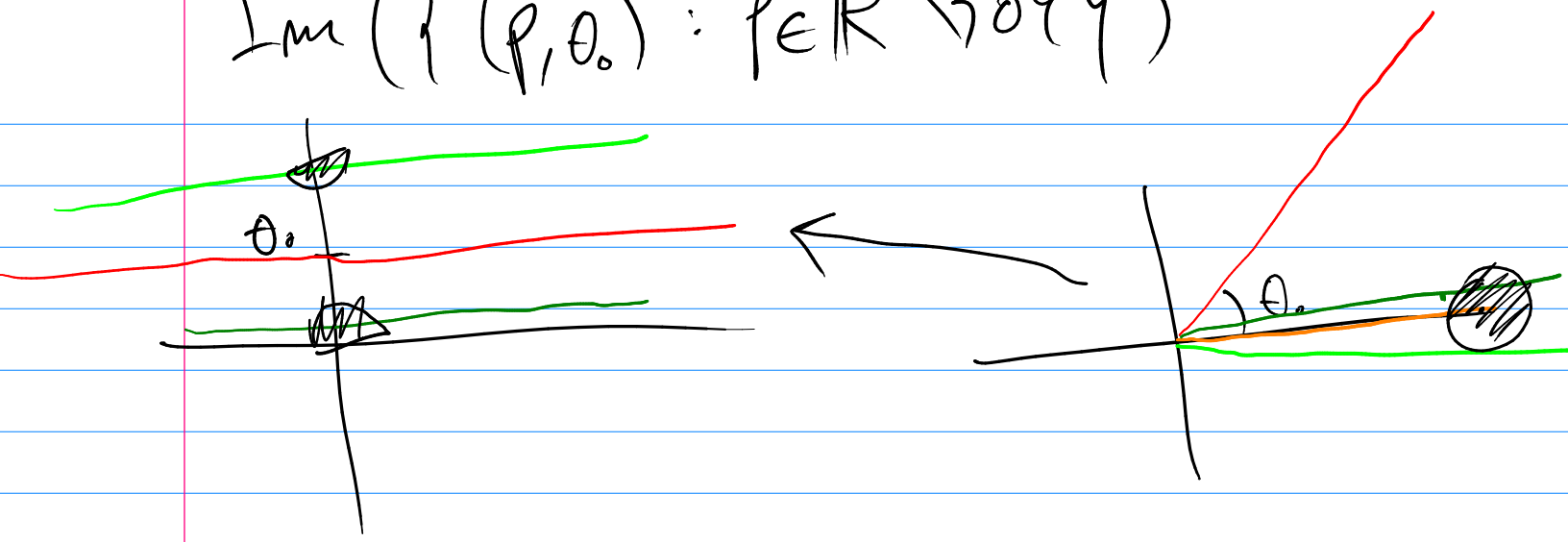
$\theta = \operatorname{arctg}\left(\dfrac{y}{x}\right)$

$\operatorname{Im}(\{(\rho_0,\theta) : \theta \in (0,2\pi]\})$





$Im(f(\rho,\theta_0) : \rho \in \mathbb{R}^+ \setminus \{0\})$

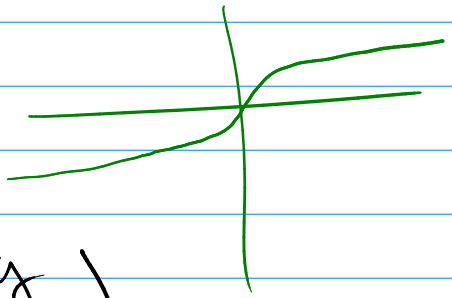


14. Sea $f : (0,+\infty)\times[0,2\pi) \to \mathbb{R}^2 \setminus \{(0,0)\}$ definida por $f(\rho,\theta) = (\rho\cos(\theta), \rho\sin(\theta))$
   a) Verificar que es continua y probar que es biyectiva.
   b) Calcular las imágenes de las rectas $\rho = cte$ y $\theta = cte$
   c) Calcular la función inversa $f^{-1} : \mathbb{R}^2 \setminus \{(0,0)\} \to (0,+\infty)\times[0,2\pi)$.¿ Es continua f^{-1}?

$f^{-1} : \mathbb{R}^2 \setminus \{(0,0)\} \to (0,+\infty)\times[0,2\pi)$

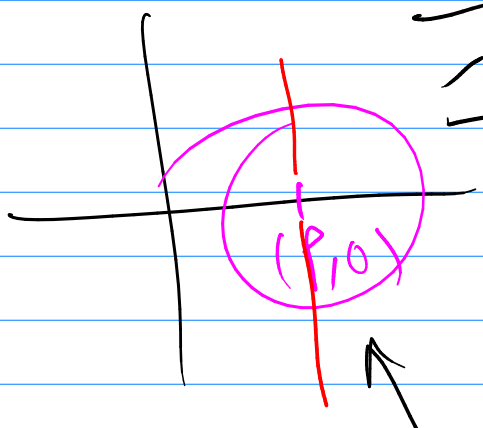
$f^{-1}(x,y) = \left(\sqrt{x^2+y^2}, \operatorname{arctg}\left(\frac{y}{x}\right)\right)$

En $\theta = 0$ da problemas



fijamos $(\rho, 0)$

$\lim_{y\to 0} \operatorname{arctg}\left(\frac{y}{\rho}\right)$



$\lim_{y\to 0} \frac{y}{\rho}$ no existe

No puede ser continua f^{-1}