3. Se considera la matriz $A = \begin{pmatrix} 1 & 0 \\ 3 & 4 \end{pmatrix}$.

a) Hallar matrices elementales E_1 y E_2 tales que $E_2 E_1 A = I$.
b) Hallar A^{-1}.
c) Expresar A como el producto de matrices elementales.

$$\left[\begin{array}{cc|cc} \overbrace{1 \;\; 0}^{A} & & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right] \underbrace{\xrightarrow{T_1 :\, F_2 \longleftrightarrow F_2 - 3F_1}} \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 4 & -3 & 1 \end{array}\right] \underbrace{\xrightarrow{T_2 :\, F_2 \longleftrightarrow F_2/4}}$$

$Ax = I$ $\qquad E_1 = T_1(Id)$ $\qquad E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1/4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{F_2 \longleftrightarrow F_2 - 3F_1} \boxed{\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} = E_1}$$

$$E_1 A = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \checkmark$$

$$\Rightarrow \underbrace{E_2 \underbrace{E_1 A}_{T_1(A)}}_{T_2(T_1(A))} = I \quad \Rightarrow \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1/4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}}_{A^{-1}} A = I$$

c) Despejo A: $\quad E_2 E_1 A = I$

$$\boxed{A = \underset{\downarrow \\ T_1^{-1}}{E_1^{-1}} \underset{\downarrow \\ T_2^{-1}}{E_2^{-1}}} \; \cancel{\checkmark}$$

E_1^{-1}: Está asociada a la inversa de T_1, osea, T_1^{-1}

E_2^{-1}: " " a T_2^{-1}

$\Rightarrow E_1^{-1}$: $F_2 \longleftrightarrow F_2 + 3F_1$ $\qquad E_1^{-1} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

E_2^{-1}: $F_2 \longleftrightarrow 4F_2$ $\qquad E_2^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$

Si invierto E_1:

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ -3 & 1 & 0 & 1 \end{array}\right] \xrightarrow{F_2 \longleftrightarrow 3F_1 + F_2} \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & \underbrace{3 \;\; 1}_{E_1^{-1}} & \end{array}\right]$$

b) Determine el rango de las siguientes matrices

a) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ $\quad$ b) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$ $\quad$ c) $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \cancel{2} & \cancel{2} & \cancel{2} & \cancel{2} & \cancel{2} & \cancel{2} & \cancel{2} \\ \cancel{\sqrt{5}} & \cancel{\sqrt{5}} & \cancel{\sqrt{5}} & \cancel{\sqrt{5}} & \cancel{\sqrt{5}} & \cancel{\sqrt{5}} & \cancel{\sqrt{5}} \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{4} & \cancel{4} & \cancel{4} & \cancel{4} & \cancel{4} & \cancel{4} & \cancel{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ $\leftarrow F_1$, $F_4 = 4F_1$, F_6

Rango: # de escalones de la matriz escalerizada.

Obs.: Puedo verlo como cantidad de filas linealmente independientes (que no sea C.L. de las otras)

$rg(C) = 2 \longrightarrow 2$ escalones

b) Recomiendo escalerizar

Práctico 4: Determinantes

Def.: Sea $A \in M_{n \times n}$, $A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$

Dado a_{ij}, le llamamos matriz adjunta de la entrada a_{ij}, a la Matriz A^*_{ij} ($\leftarrow$ adjunta) resultante de cancelar la fila i y la columna j de A.

ej.:

$$A = \begin{array}{c} \\ F_1 \\ F_2 \\ F_3 \end{array} \!\!\begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ \cancel{a_{21}} & \cancel{a_{22}} & \cancel{a_{23}} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}_{3\times 3} \end{array} \Rightarrow A^*_{22} = \begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{bmatrix}_{2\times 2}$$

Def.: $n=1 \Rightarrow A_{1\times 1}=(a) \Rightarrow |A|=a \quad (\det(A))$

$n=2 \Rightarrow A=\begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow |A| = ad - cb$

par o impar — adjunta

$$n=3 \Rightarrow A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}_{\substack{n\times n \\ 3 \;\; 3}} \Rightarrow |A| = \sum_{i=1}^{n=3} (-1)^{1+i} a_{i1} |A^*_{i1}|$$

$$= (-1)^{1+1} a_{11}|A^*_{11}| + (-1)^{1+2} a_{21}|A^*_{21}| + (-1)^{1+3} a_{31}|A^*_{31}|$$

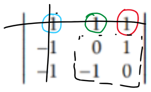
1. Determinantes

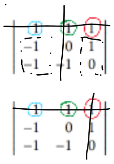
1. Calcular los siguientes determinantes:

$$a)\ \begin{vmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{vmatrix} \qquad b)\ \begin{vmatrix} 0 & 0 & \operatorname{sen}(\theta) \\ \operatorname{sen}(\theta) & \cos(\theta) & \tan(\theta) \\ -\cos(\theta) & \operatorname{sen}(\theta) & -\cos(\theta) \end{vmatrix} \qquad c)\ \begin{vmatrix} e^{2x} & 0 & e^{x} \\ e^{x} & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

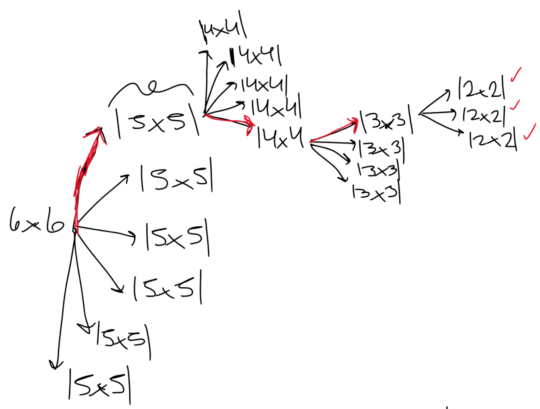
$$d)\ \begin{vmatrix} 1 & 2 & 1 & 2 \\ 3 & 2 & 3 & 2 \\ -1 & -3 & 0 & 4 \\ 0 & 4 & -1 & -3 \end{vmatrix} \qquad e)\ \begin{vmatrix} 2 & 4 & -5 & 2 & 1 & 5 \\ 0 & -1 & 7 & 3 & -3 & 4 \\ 0 & 1 & 1 & 0 & 0 & -1 \\ -3 & 4 & 0 & 1 & 2 & 1 \\ 0 & 5 & 3 & -1 & 4 & 3 \\ 5 & 1 & 3 & 0 & 0 & -1 \end{vmatrix}.$$

$$a)\ \det(M) = 1\cdot \underbrace{\begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix}}_{0\cdot 0-(-1\cdot 1) = 1} - (1)\underbrace{\begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix}}_{-1\cdot 0-(-1\cdot 1) = 1} + (1)\underbrace{\begin{vmatrix} -1 & 0 \\ -1 & -1 \end{vmatrix}}_{-1\cdot(-1)-(-1\cdot 0) = 1} = 1\cdot 1 - \cancel{1\cdot 1} + \cancel{1\cdot 1} = \boxed{1}$$

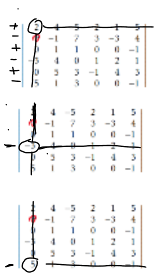




$$e)\ \begin{vmatrix} 2 & 4 & -5 & 2 & 1 & 5 \\ 0 & -1 & 7 & 3 & -3 & 4 \\ 0 & 1 & 1 & 0 & 0 & -1 \\ -3 & 4 & 0 & 1 & 2 & 1 \\ 0 & 5 & 3 & -1 & 4 & 3 \\ 5 & 1 & 3 & 0 & 0 & -1 \end{vmatrix}$$

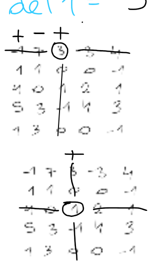


Uso columna 1:



$$2\cdot \underset{\text{det1}}{\begin{vmatrix} -1 & 7 & 3 & -3 & 4 \\ 1 & 1 & 0 & 0 & -1 \\ 4 & 0 & 1 & 2 & 1 \\ 5 & 3 & -1 & 4 & 3 \\ 1 & 3 & 0 & 0 & -1 \end{vmatrix}} - (1)\cdot(-3) \underset{\text{det2}}{\begin{vmatrix} 4 & -5 & 2 & 1 & 5 \\ -1 & 7 & 3 & -3 & 4 \\ 1 & 1 & 0 & 0 & -1 \\ 5 & 3 & -1 & 4 & 3 \\ 1 & 3 & 0 & 0 & -1 \end{vmatrix}}$$

$$(-1)\cdot 5 \underset{\text{det3}}{\begin{vmatrix} 4 & -5 & 2 & 1 & 5 \\ -1 & 7 & 3 & -3 & 4 \\ 1 & 1 & 0 & 0 & -1 \\ 4 & 0 & 1 & 2 & 1 \\ 5 & 3 & -1 & 4 & 3 \end{vmatrix}}$$



$$\text{det1} = \overset{(i+j=1+3)}{3}\cdot \underbrace{\begin{vmatrix} 1 & 1 & 0 & -1 \\ 4 & 0 & 2 & 1 \\ 5 & 3 & 4 & 3 \\ 1 & 3 & 0 & -1 \end{vmatrix}_{4\times 4}}_{\text{det 1.1}} + (1)\,1 \underbrace{\begin{vmatrix} -1 & 7 & -3 & 4 \\ 1 & 1 & 0 & -1 \\ 5 & 3 & 4 & 3 \\ 1 & 3 & 0 & -1 \end{vmatrix}_{4\times 4}}_{\text{det 1.2}} + (-1)(-1) \underbrace{\begin{vmatrix} -1 & 7 & -3 & 4 \\ 1 & 1 & 0 & -1 \\ 4 & 0 & 2 & 1 \\ 1 & 3 & 0 & -1 \end{vmatrix}}_{\text{det 1.3}}$$

$$\begin{array}{ccccc} \cancel{4} & \cancel{0} & 1 & \cancel{2} & \cancel{1} \\ 5 & 3 & -1 & 4 & 3 \\ 1 & 3 & 0 & 0 & -1 \end{array}$$

$$\begin{array}{ccccc} -1 & 7 & 6 & -3 & 4 \\ 1 & 1 & 0 & 0 & -1 \\ 4 & 0 & 1 & 2 & 1 \\ \cancel{5} & \cancel{3} & -1 & \cancel{4} & \cancel{3} \\ 1 & 3 & 0 & 0 & -1 \end{array}$$

$$= -11 - (-1) = -10$$

$$\det 1.1 = (-1)\cdot 2 \underbrace{\begin{vmatrix} 1 & 1 & -1 \\ 5 & 3 & 3 \\ 1 & 3 & -1 \end{vmatrix}}_{\det 1.1.1\ \checkmark} + (1)\cdot 4 \underbrace{\begin{vmatrix} 1 & 1 & -1 \\ 4 & 0 & 1 \\ 1 & 3 & -1 \end{vmatrix}}_{\det 1.1.2}$$

$$\begin{vmatrix} 1 & 1 & 0 & -1 \\ \cancel{4} & \cancel{0} & 2 & \cancel{1} \\ 5 & 3 & 4 & 3 \\ 1 & 3 & 0 & -1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 & 0 & -1 \\ 4 & 0 & 2 & -1 \\ \cancel{5} & \cancel{3} & 4 & \cancel{3} \\ 1 & 3 & 0 & -1 \end{vmatrix}$$

$$\det 1.1.1 = 1\cdot \underbrace{\begin{vmatrix} 3 & 3 \\ 3 & -1 \end{vmatrix}}_{\underbrace{-3-9}_{-12}} - 5\cdot \underbrace{\begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix}}_{\underbrace{1-(-3)}_{4}} + 1\cdot \underbrace{\begin{vmatrix} 1 & -1 \\ 3 & 3 \end{vmatrix}}_{\underbrace{3-(-3)}_{6}} = -12 - 20 + 6 = +26$$

eeeeee

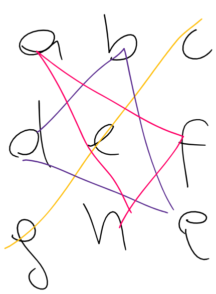
continuar.

$$\begin{array}{ccc} 1 & \cancel{1} & \cancel{-1} \\ 5 & 3 & 3 \\ 1 & 3 & -1 \end{array}$$

$$\begin{array}{ccc} 1 & 1 & -1 \\ 5 & \cancel{3} & \cancel{3} \\ 1 & 3 & -1 \end{array}$$

$$\begin{array}{ccc} 1 & 1 & -1 \\ 5 & 3 & 3 \\ 1 & \cancel{3} & \cancel{-1} \end{array}$$

$$\left|\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right| = a\cdot e\cdot i + bfg + dhc - [gec + dbi + afh]$$



Prop.: $\det(\alpha M) = \alpha^n \det(M)$, $M \in M_{n\times n}$.

Ejemplo: $\det\begin{pmatrix} \alpha a & \alpha b \\ c & d \end{pmatrix} = \alpha \det\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\det\begin{pmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{pmatrix} = \alpha \det\begin{pmatrix} a & b \\ \alpha c & \alpha d \end{pmatrix} = \alpha^2 \det\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Prop.: Intercambio filas o columnas $\longrightarrow$ cambia el signo el determinante

Prop.: $\det(A.B) = \det(A)\cdot\det(B)$

Prop.: si sustituyo una fila /columna por una cole de esta con otras $\Rightarrow$ det n... camb...

2. Sabiendo que

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 5,$$

calcular los siguientes determinantes:

$F_2 - 3F_1$

$$\underbrace{\begin{vmatrix} d & e & f \\ g & h & i \\ a & b & c \end{vmatrix}}_{(-1)(-1)}, \quad \underbrace{\begin{vmatrix} -a & -b & -c \\ 2d & 2e & 2f \\ -g & -h & -i \end{vmatrix}}_{(-1)(-1)(2)}, \quad \underbrace{\begin{vmatrix} a & b & c \\ d-3a & e-3b & f-3c \\ 2g & 2h & 2i \end{vmatrix}}_{2}, \quad \begin{vmatrix} a+5c & 3b & c \\ d+5f & 3e & f \\ 2g+10i & 6h & 2i \end{vmatrix}.$$

el

o

le.