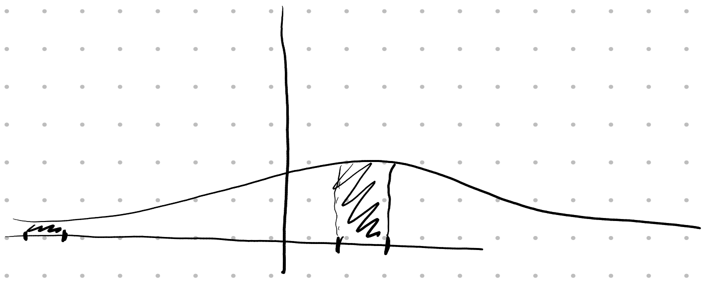
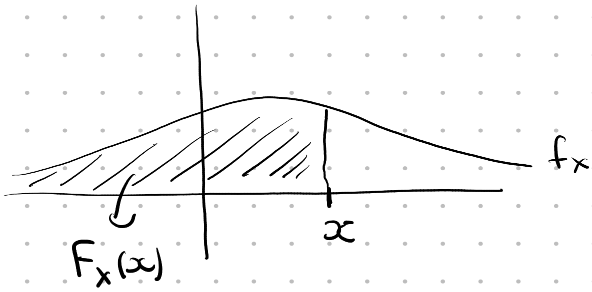
X variable aleatoria absolutamente continua

* f_X función de densidad de X : $f_X : \mathbb{R} \to \mathbb{R}$ positiva, integrable tal que $P(a \leq X \leq b) = \int_a^b f_X(x)\,dx$

* F_X función de distribución de X : $F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_X(x)\,dx$

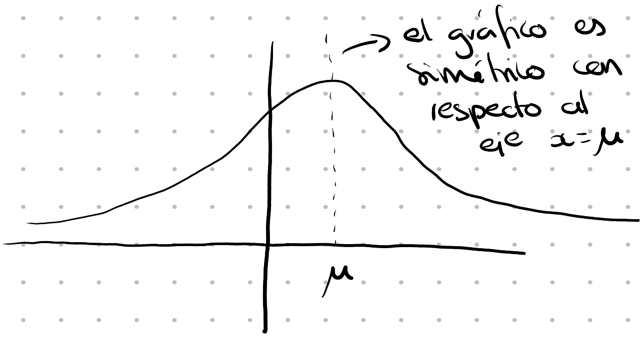


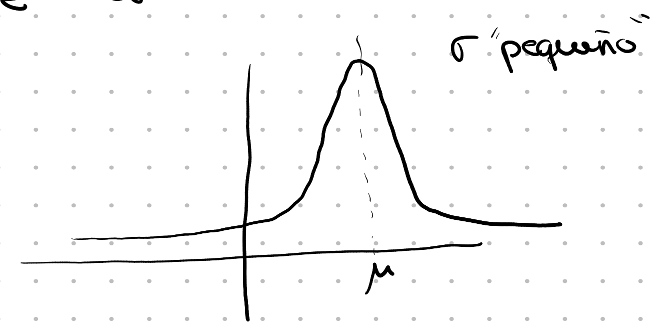


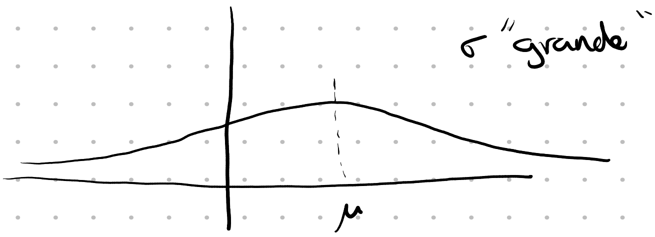
Distribución normal

$X \sim \mathcal{N}(\mu, \sigma^2)$

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



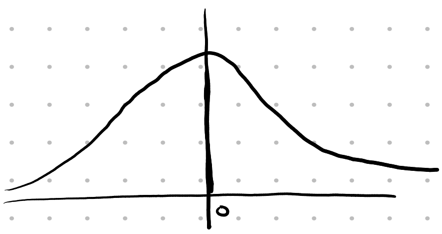




→ F_X no tiene una formula

$Z \sim \mathcal{N}(0,1)$ normal estándar

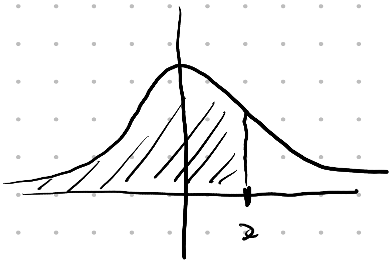
$F_Z(x) = P(Z \leq x) = \phi(x)$



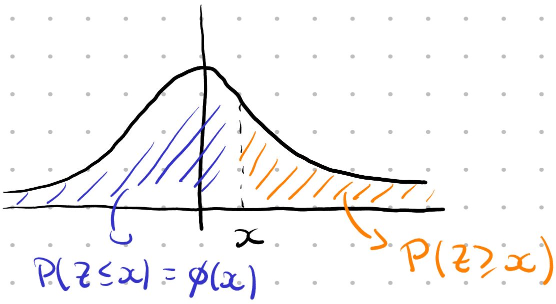
$P(Z \leq 0{,}33) = 0{,}6293$ $\quad P(Z \leq 0) = 0{,}5$

$x > 0$

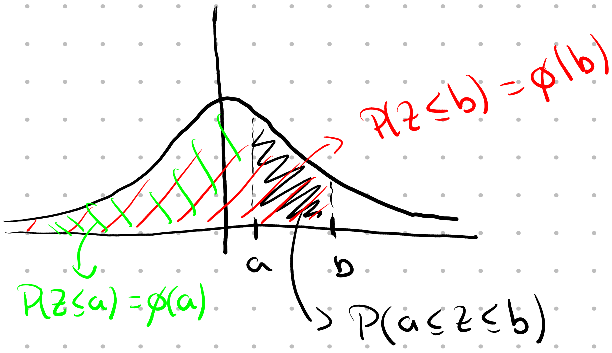
* $P(Z \leq x) = \phi(x)$



* $P(Z \geq x) = 1 - P(Z \leq x) = 1 - \phi(x)$

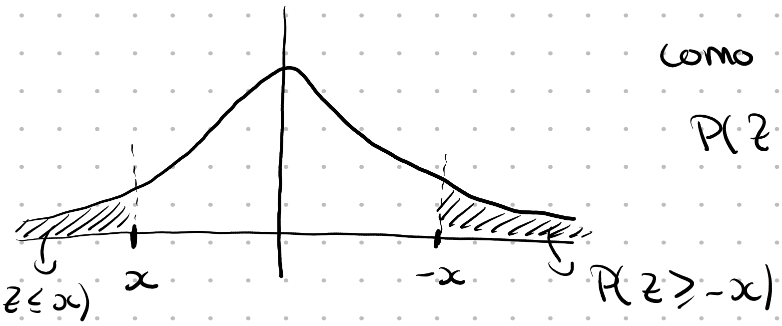


* $P(a \leq Z \leq b) = \phi(b) - \phi(a)$



Si $x < 0$:

$P(Z \leq x) = P(Z \geq -x) = 1 - P(Z \leq -x) = 1 - \phi(-x)$

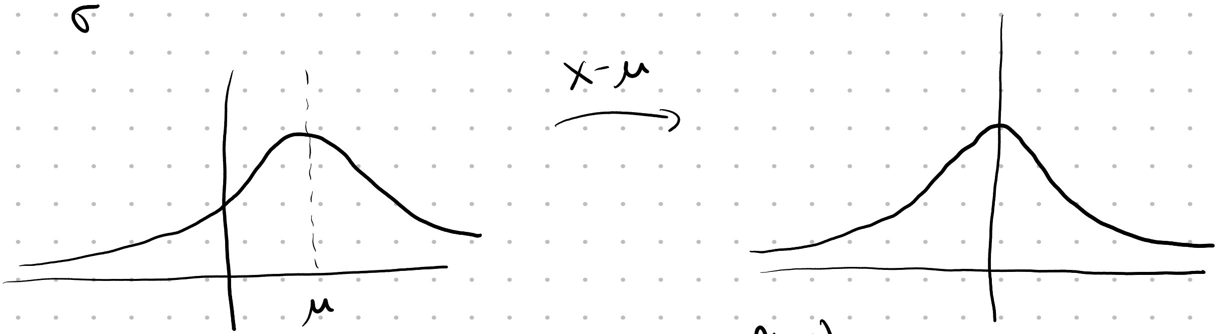


como el gráfico es simétrico

$P(Z \leq x) = P(Z \geq -x)$

si $X \sim \mathcal{N}(\mu, \sigma^2)$

$$\frac{X-\mu}{\sigma} \sim \mathcal{N}(0,1)$$



$$P(a \leq X < b) = P\left(\frac{a-\mu}{\sigma} \leq \underbrace{\frac{X-\mu}{\sigma}}_{\sim \mathcal{N}(0,1)} \leq \frac{b-\mu}{\sigma}\right)$$

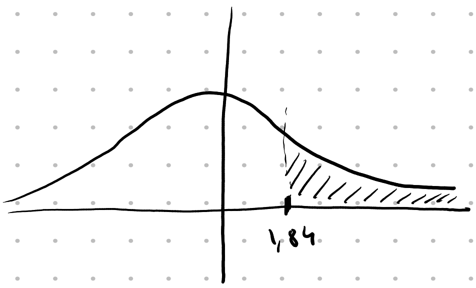
$$= \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)$$

Ejercicio 6

1. En la densidad normal estándar, encuentre el área bajo la curva que está:
   a) a la derecha de $z = 1{,}84$.
   b) entre $z = -1{,}97$ y $z = 0{,}86$.
2. Si $Z \sim N(0,1)$, encuentre los valores de k de tal forma que:
   a) $\mathbf{P}(Z > k) = 0{,}3015$
   b) $\mathbf{P}(k < Z < -0{,}18) = 0{,}4197$
3. En una distribución normal con $\mu = 40$ y $\sigma = 6$, encuentre el valor de x que tiene:
   a) 45 % del área a la izquierda.
   b) 14 % del área a la derecha.

① $Z \sim \mathcal{N}(0,1)$

a)

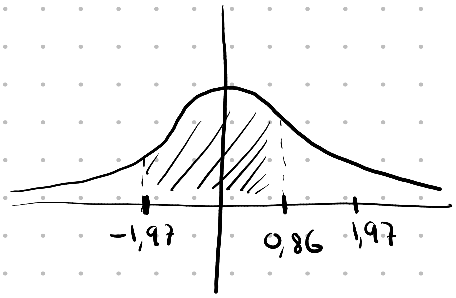


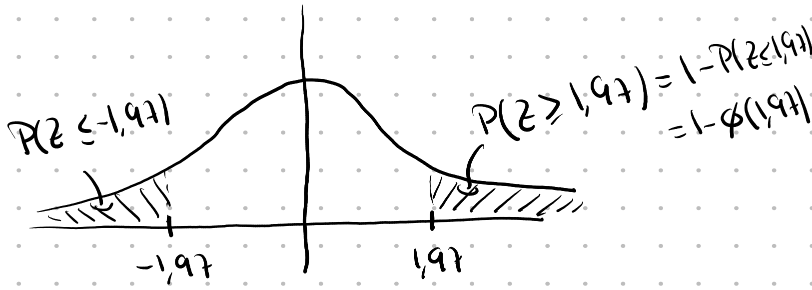
$$P(Z \geq 1{,}84) = 1 - P(Z \leq 1{,}84)$$
$$= 1 - \phi(1{,}84)$$
$$= 1 - 0{,}9671$$

1,84

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

b)

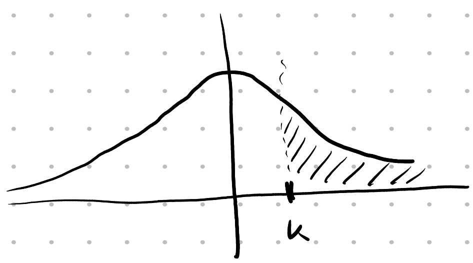


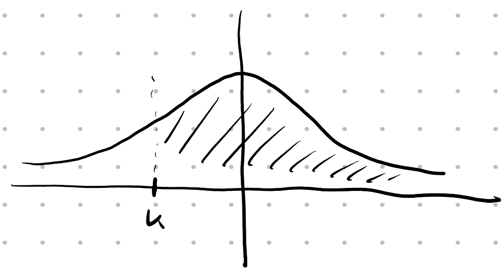


$$P(-1{,}97 \leq Z \leq 0{,}86) = P(Z \leq 0{,}86) - P(Z \leq -1{,}97)$$
$$= \phi(0{,}86) - (1 - \phi(1{,}97))$$
$$= \phi(0{,}86) - 1 + \phi(1{,}97)$$
$$= 0{,}8051 - 1 + 0{,}9756$$

② $Z \sim \mathcal{N}(0,1)$

a) buscamos k tal que $P(Z > k) = 0{,}3015$





$$P(Z > k) > P(Z > 0) = 0{,}5$$

$\Rightarrow$ k es positivo

$P(Z > k) = 0{,}3015 \Rightarrow P(Z \leq k) = 1 - P(Z > k) = 1 - 0{,}3015 = 0{,}6985$

$\Rightarrow k = 0{,}52$

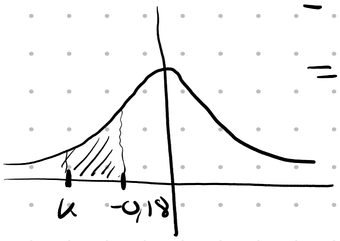
b) $P(k < Z < -0{,}18) = 0{,}4197$

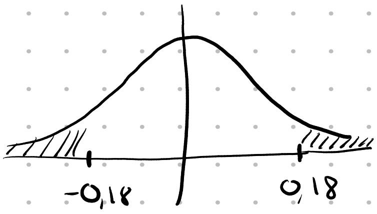
$k < 0$

$P(k < Z < -0{,}18) = P(Z \leq -0{,}18) - P(Z \leq k)$

$= 1 - \phi(0{,}18) - (1 - \phi(-k))$

$= \cancel{1} - \phi(0{,}18) - \cancel{1} + \phi(-k)$





$P(k < Z < -0{,}18) = -\phi(0{,}18) + \phi(-k)$

$\phi(-k) = P(k < Z < 0{,}18) + \phi(0{,}18)$

$= 0{,}4197 + 0{,}5714$

$= 0{,}9911$

$\Rightarrow -k = 2{,}37 \quad \Rightarrow k = -2{,}37$

	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07		
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Ejercicio 7

En un proceso industrial el diámetro de un balero es parte importante de un componente. El comprador establece en sus especificaciones que el diámetro debe ser $3{,}0 \pm 0{,}01$ *cm*. Por lo tanto, no se acepta ningún balero que se salga de esa especificación. Se sabe que en el proceso de producción, el diámetro de un balero tiene una distribución normal con media $\mu = 3{,}0$ *cm* y desviación estándar $\sigma = 0{,}005$ *cm*. En promedio, ¿qué porcentaje de baleros fabricados se descartarán?

X = diametro de un balero

$X \sim \mathcal{N}(3{,}0\ ,\ 0{,}005^2)$

$P(\text{descartar un balero}) = P(\{X > 3{,}01\} \cup \{X < 2{,}99\})$

$$= P(X > 3{,}01) + P(X < 2{,}99)$$

$$= 1 - P(X \leq 3{,}01) + P(X \leq 2{,}99)$$

$X \sim \mathcal{N}(3{,}0,\ 0{,}005^2)$

$\frac{X-3}{0{,}005} \sim \mathcal{N}(0,1)$

$\frac{X-3}{0{,}005} = Z \sim \mathcal{N}(0,1)$

$$= 1 - P\left(\frac{X-3}{0{,}005} \leq \frac{3{,}01-3}{0{,}005}\right) + P\left(\frac{X-3}{0{,}005} \leq \frac{2{,}99-3}{0{,}005}\right)$$

$$= 1 - P\Big(\underbrace{\frac{X-3}{0{,}005}}_{\sim \mathcal{N}(0,1)} \leq 2\Big) + P\Big(\underbrace{\frac{X-3}{0{,}005}}_{\sim \mathcal{N}(0,1)} \leq -2\Big)$$

$$= 1 - P(Z \leq 2) + P(Z \leq -2)$$

$$= 1 - \phi(2) + (1 - \phi(2))$$

$$= 2 - 2\phi(2)$$

$$= 2 - 2 \cdot 0{,}9972$$

$$= 0{,}0056$$

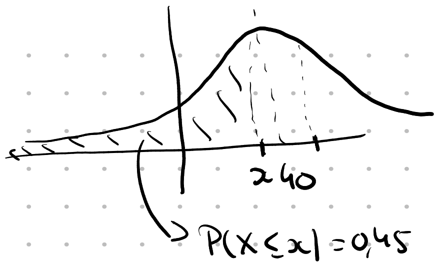
el porcentaje de baleros que se descartan en promedio $0{,}5\%$.

Ejercicio 6

3. En una distribución normal con $\mu = 40$ y $\sigma = 6$, encuentre el valor de x que tiene:

   a) 45 % del área a la izquierda.
   b) 14 % del área a la derecha.

$X \sim \mathcal{N}(40, 6^2)$



$\frac{X-40}{6} = Z \sim \mathcal{N}(0,1)$

a) buscamos x tal que $P(X \leq x) = 0{,}45$

$$P\Big(\underbrace{\frac{X-40}{6}}_{\sim \mathcal{N}(0,1)} \leq \frac{x-40}{6}\Big) = 0{,}45$$

$$P\Big(Z \leq \underbrace{\frac{x-40}{6}}_{\text{negativo}}\Big) = 0{,}45$$

$$\phi\left(\frac{x-40}{6}\right) = 0{,}45$$

$$1-\phi\left(\frac{40-x}{6}\right)=0{,}45$$

$$\phi\left(\frac{40-x}{6}\right)=0{,}55$$

z	0.00	0.01	0.02	0.03	0.04	0.05
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736

$$\left.\begin{array}{l}\phi(0{,}12)=0{,}5478\\ \phi(0{,}13)=0{,}5517\end{array}\right\}\quad 0{,}12<\frac{40-x}{6}<0{,}13$$

$$39{,}22<x<39{,}28$$