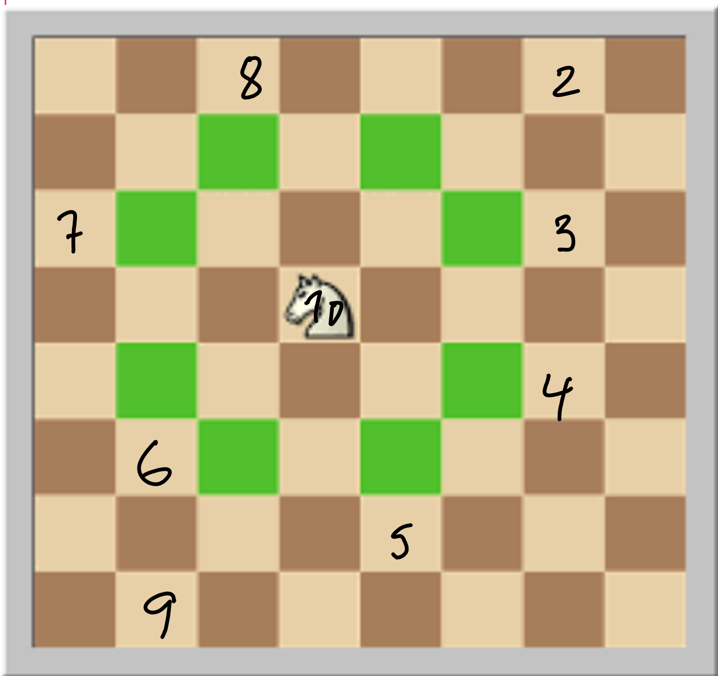
Ejercicio 7
Determinar la mayor cantidad de caballos que se pueden poner en un tablero de ajedrez sin que ninguno pueda saltar hacia la posición de otro.



32 caballos en el mismo color

Tengo a lo sumo 32 caballos que no se 'miran'

Ejercicio 1
Expresar explícitamente en n las sucesiones:

(a) $a_{n+2} = 5a_{n+1} - 6a_n, \quad \forall n \in \mathbb{N}$, con $a_0 = 1, a_1 = 3$.

(b) $b_{n+2} - 6b_{n+1} + 9b_n = 0, \quad \forall n \in \mathbb{N}$, con $b_0 = 5, b_1 = 12$.

(a) $r^2 = 5r - 6$

$r^2 - 5r + 6 = 0$

$(r-2)(r-3)$ raíces son $r=3, r=2$

$a_n = \alpha 2^n + \beta 3^n$

$a_0 = 1 = \alpha + \beta \longrightarrow 1-\beta = \alpha$

$a_1 = 3 = 2\alpha + 3\beta$

$\downarrow$

$3 = 2(1-\beta) + 3\beta$

$3 = 2 - 2\beta + 3\beta$

$1 = \beta$

$1 = \alpha + 1$

$\alpha = 0$

$\underline{a_n = 3^n}$

$a_{n+2} = 5a_{n+1} - 6a_n$

¿ $3^{n+2} = 5 \cdot 3^{n+1} + 6 3^n$

$3^{n+2} - 5 \cdot 3^{n+1} + 2 \cdot 3^{n+1}$

$= 3^{n+1}(3 - 5 + 2) = 0$ ✓

(b) $b_{n+2} - 6b_{n+1} + 9b_n = 0, \quad \forall n \in \mathbb{N}$, con $b_0 = 5, b_1 = 12$.

$r^2 - 6r + 9 = (r-3)^2$

$b_n = 3^n$

$(a + bn)3^n$

$$b_2 - 6\cdot 12 + 9\cdot 5 = 0$$

$$b_2 - 8\cdot 9 + 9\cdot 5 = 0$$

$$b_2 - 9(-8+5) = 0 \rightarrow b_2 = 27$$

$$(a+2b)3^2 - 6(a+b)3 + 9(a+b\cdot 0) = 0$$

$$3^2 a + \cancel{2\cdot 3^2 b} - 18a \cancel{-18b} + 9a = 0$$

$$a(\underline{3^2 - 18 + 9}) = 0 \Rightarrow a = 0$$

$$b_n = n3^n$$

$$b_{n+2} - 6b_{n+1} + 9b_n = 0$$

$$(n+2)3^{n+2} - 6(n+1)3^{n+1} + 9n3^n$$

$$= (n+2)3^{n+2} - 2(n+1)3^{n+2} + n3^{n+2}$$

$$= 3^{n+2}(n+2 - 2(n+1) + n)$$

$$= 3^{n+2}(2(n+1) - 2(n+1)) = 0$$

$$b_n = \alpha 3^n + \beta n 3^n \qquad \alpha = 5$$

$$b_0 = 5 \qquad 12 = 3\alpha + \beta 3$$

$$b_1 = 12 \qquad 12 = 15 + 3\beta$$

$$3(4) = 3(5+\beta)$$

$$4 = 5+\beta \qquad \beta = -1$$

$$b_n = 5\cdot 3^n - n3^n = 3^n(5-n)$$

$$b_0 = 5 \quad , \quad b_1 = 15 - 3 = 12$$

$$b_{n+2} - 6b_{n+1} + 9b_n = 0$$

$$3^{n+2}(5-n-2) - 6\cdot 3^{n+1}(5-n-1) + 9\cdot 3^n(5-n)$$

$$3^{n+2}\left(3-n-2(4-n)+5-n\right)$$

$$3^{n+2}\left(8-n-8+n\right) = 0$$