Funciones generatrices

Son polinomios con infinitos términos

- $F(x) = 1 + x + x^2 + x^3 + \cdots$

  $a_n = 1 \quad \forall\, n \in \mathbb{N}.$

- $F(x) = 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots$

  $a_n = 1, -1, 1, -1, 1,$

  $a_n = (-1)^n \quad n \in \mathbb{N}.$

$$(1 + x + x^2 + x^3 \qquad)(1 - x) = 1$$

$$\boxed{(1 + y + y^2 + y^3 + \cdots) = \frac{1}{(1-y)}}$$

$f(x) = (1+x)^2 = x^2 + 2x + 1$

$= 1 + 2x + x^2 + 0 \cdot x^3 + 0 \cdot x^4 + 0 \cdot x^5$

$a_n = 1, 2, 1, 0, 0, \ldots$

2 a) es parecido a esto

2 b) $\dfrac{1}{1-x} = \underbrace{1 + x + x^2 + x^3 + \ldots}$

¿ $\dfrac{x^3}{1-x}$?

$\dfrac{x^3}{1-x} = x^3 \cdot (\underbrace{1 + x + x^2 + \ldots})$

$= x^3 + x^4 + x^5 + \ldots$

$a_n = 0, 0, 0, 1, 1, 1$

$$F(x) = \frac{1}{2-x} = \frac{1/2}{\frac{1}{2}(2-x)} = \frac{1/2}{\left(1-\frac{x}{2}\right)} = \frac{1}{1-y}\cdot(1/2)$$

$$y = \frac{x}{2}$$

$$\frac{1}{1-y} = 1 + y + y^2 + y^3 + \cdots$$

$$y = \frac{x}{2} \qquad \frac{x^n}{2^n} = \left(\frac{x}{2}\right)^n$$

$$G(x) = \frac{1}{1-\frac{x}{2}} = 1 + \frac{x}{2} + \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^3 + \cdots$$

$$= 1 + \frac{1}{2}\cdot x + \frac{1}{2^2}x^2 + \frac{1}{2^3}x^3$$

$$+ \cdots + \frac{1}{2^n}\cdot x^n + \cdots$$

$$F(x) = \frac{1}{2}\cdot G(x) \qquad F(x) = \frac{1}{2}G(x)$$

$$F(x) = \frac{1}{2}\left(1 + \frac{1}{2}x + \frac{1}{2^2}x^2 + \frac{1}{2^3}x^3 + \cdots\right)$$

$$F(x) = \frac{1}{2} + \frac{1}{2^2}x + \frac{1}{2^3}\cancel{x^2}\ \frac{1}{2^4}x^2 + \cdots$$

$$a_n = \frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4} \cdots$$

$$a_n = \frac{1}{2^{n+1}}, \quad n \in \mathbb{N}$$

3a) Hallar el coeficiente de x^8 en

$$F(x) = (1 + x + x^2 + x^3 + \cdots)^{10}$$

$$\underbrace{(1 + x + x^2 + \cdots)(1 + x + x^2 + \cdots) \cdots (1 + x + x^2 + \cdots)}_{10}$$

$$= \sum_{n=0}^{\infty} a_n . x^n \quad ¿ a_8 ?$$

$$(1 + x + x^2 + \cdots)(1 + x + x^2 + \cdots) \cdots (1 + x + x^2 \cdots)$$

$$c_i \geq 0 \qquad x^{c_1} \qquad x^{c_2} \quad \cdots\cdots \quad x^{c_{10}}$$

$$= x^{c_1 + c_2 + \cdots + c_{10}} = x^8$$

$$c_1 + c_2 = 1 \qquad 1 + 2x^{1} + x^2$$

1 0

0 1

$$(1+x)(1+x) = 1 + \underbrace{1 . x} + \underbrace{x . 1} + x . x$$

(1) $c_1 + c_2 + \ldots + c_{10} = 8$ $\qquad$ $\underline{a_8}$ es la cantidad de soluciones de (1)

$c_i \in \mathbb{N}$

$(1, 1, 1, \ldots 1, 0, 0)$

sol de (1)

$x^1 \, x^1 \ldots x^1 \cdot 1 \cdot 1$

$\Rightarrow$ a_8 es elegir 8 elementos de 10 $\underline{\text{dores}}$ distintos

$\hookrightarrow$ tipos

$$CR^{10}_{8} = \boxed{C^{17}_{9}}$$

$$\left(CR^{m}_{n} = C^{m+n-1}_{m-1} = C^{m+n-1}_{n} \right)$$

$$a_8 = C^{17}_{9}$$

(b) coeficiente que multiplica a x^n

a_n es la cantidad de soluciones

de $\quad c_1 + c_2 + \ldots + c_{10} = n$

$$a_n = CR_n^{10} = C_9^{10+n-1} = C_9^{9+n}$$

$$F(x) = \sum_{n=0}^{\infty} a_n \cdot x^n = \sum_{n=0}^{\infty} C_9^{9+n} x^n$$

Ejercicio 6

Hallar la fg de [la cantidad de formas que tiene un cajero automático de dar n pesos.]

a_n = la cantidad de formas de dar n pesos

$$¿\, F(x) = \sum_{n=0}^{\infty} a_n x^n \, ?$$

100, 200, 500, 1000

a_n

$$c_1 100 + c_2 200 + c_3 500 + c_4 1000 = n$$

c_1 → cantidad de billetes

- $5.100 = 500$ $\quad c_1 = 5,\ c_2 = 0,\ c_3 = 0,\ c_4 = 0$

- $1.100 + 2.200 = 500$ $\quad c_3 = 0\ \ c_4 = 0,\ c_1 = 1\ \ c_2 = 2$

- $0.100 + 0.200 + 1.500 + 0.1000 = 500$

$c_1 = 0 \quad c_2 = 0 \quad c_3 = 1 \quad c_4 = 0$

- $3.100 + 1.200 + 0.500 + 0.1000 = 500$

$c_1 = 3,\ c_2 = 1,\ c_3 = 0,\ c_4 = 0$

$$a_{500} = 4$$

a_n es la cantidad de soluciones

$$c_1.100 + c_2 200 + c_3 500 + c_4 1000 = n$$

$c_i \in \mathbb{N}$

$$x_1 + x_2 + x_3 + x_4 = n$$

si fuera

$c_1 + c_2 + c_3 + c_4 = n$

a_n de $(1^0 + x^1 + x^2 + \dots)(1^0 + x^1 + x^2 + \dots)(1+x+x^2)(1+x+\dots)$

$$x^{c_1},\ x^{c_2}\ x^{c_3}\ x^{c_4}$$

$$= x^{c_1+c_2+c_3+c_4} = x^n$$

$$F(x) = (\underbrace{x^0 + x^{100} + x^{200} + \dots}_{x^{d_1}}) \cdot (\underbrace{x^0 + x^{200} + x^{400} + \dots}_{x^{d_2}})$$

$$(\underbrace{x^0 + x^{500} + x^{1000} + \dots}_{x^{d_3}}) \cdot (\underbrace{x^0 + x^{1000} + x^{3000} + \dots}_{x^{d_4}})$$

¿Quién es a_n, el coef. que multiplica a x^n, en $F(x)$?

$$x^n = x^{d_1}\ x^{d_2}\ x^{d_3}\ x^{d_4}$$

soluciones

$$\boxed{\begin{array}{l} n = d_1 + d_2 + d_3 + d_4 \\ d_1 = \dot{1}00,\quad d_2 = 2\dot{0}0 \\ d_3 = 5\dot{0}0 \quad d_4 = 1\dot{0}00 \end{array}}$$

$$(*) \quad n = 100 \cdot c_1 + 200\, c_2 + 500\, c_3 + 1000\, c_4$$
$$c_i \in \mathbb{N}$$

a_n de $F(x)$ es la cantidad de soluciones de $(*)$

$d_1 = 100 \cdot c_1 \qquad d_3 = 500\, c_3$

$d_2 = 200 \cdot c_2 \qquad d_4 = 1000\, c_4$

$$F(x) = (x^0 + x^{100} + (x^{100})^2 + (x^{100})^3 + \dots)$$
$$(x^0 + x^{200} + (x^{200})^2 + (x^{200})^3 + \dots)$$
$$(x^0 + x^{500} + (x^{500})^2 + (x^{500})^3 + \dots)$$
$$(x^0 + x^{1000} + (x^{1000})^2 + (x^{1000})^3 + \dots)$$

$a_{100} = 1$

$a_{200} = 2$

$=$

$$\boxed{\frac{1}{1-y} = y^0 + y^1 + y^2 + y^3 + \dots +}$$

$$y = x^{100}$$

$$\frac{1}{1-x^{100}} = (x^{100})^0 + (x^{100})^1 + (x^{100})^2 + \dots$$

$$y = x^{200}$$

$$\frac{1}{1-x^{200}} = (x^{200})^0 + (x^{200})^1 + (x^{200})^2 + \dots$$

$$F(x) = \left(\frac{1}{1-x^{100}}\right) \cdot \left(\frac{1}{1-x^{200}}\right) \cdot \left(\frac{1}{1-x^{500}}\right)\left(\frac{1}{1-x^{1000}}\right. \dots$$

a_n ?

$$\underbrace{F(x)} = \sum_{n=0}^{\infty} a_n x^n$$